KANTIAN AND NON – KANTIAN LOGICS

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Abstract

In a previous work, certain systems of paraconsistent deontic logic were proposed in order to investigate the problem of inconsistency in the domain of ethics.

In this paper we continue this line of research, studying some paraconsistent systems containing alethic and deontic modalities.

This approach allows us to treat the principles of Kant ($OA \rightarrow \Diamond A$) and Hintikka ($\Box A \rightarrow OA$) from the classical and from the paraconsistent point of view, and to propose systems in which either, both, or neither of these principles is valid.

Introduction

In da Costa and Carnielli 1986, certain systems of paraconsistent deontic logic were studied. The motivation for such a study is twofold: 1) Since the common, classical systems of deontic logic do not allow for the existence of actual moral dilemmas, and as such dilemmas apparently do exist, it seems appropriate to construct deontic logics starting form a paraconsistent basis. In fact, paraconsistent deontic logic does not exclude such dilemmas ab initio. 2) For several reasons, the extant, explicit or implicit, moral codes are normally bound to be inconsistent. One of those reasons is that the concept of moral obligation, as well as other ethical notions, is vague. Therefore, to cope with this situation, while respecting the very nature of such codes, it appears quite natural to employ paraconsistent logic in the domain of ethics, because this kind of logic does not rule out all categories of inconsistency. (These questions are discussed in da Costa and Carnielli 1986; concerning the moral dilemmas in general, see Lemmon 1962, Barcan Marcus 1980, and Routley and Plumwood 1984).

The line of research just sketched leads us naturally to the investigation of paraconsistent calculi involving alethic and deontic operators. Thus, in this note we describe some systems of propositional logic, which contain alethic and deontic modalities. Among the most important interconnections between these two types of modalities, we mention the principles of Kant ($OA \rightarrow \Diamond A$, i.e. what is obligatory is possible) and of Hintikka ($\Box A \rightarrow OA$, what is necessary is obligatory).

In the first section of this paper, we outline some classical systems which contain alethic and deontic notions, describe their semantics, and remark that they are sound and complete relative to the latter. In the second section, we explain how one can extend these results to paraconsistent logic. In the third, we present a tableaux formulation for our paraconsistent calculi, that gives, among other things, an operational meaning to their modal and non modal connectives (a similar procedure is given in Marconi 1980). Finally, in the last section, we make some general observations on these logics.

1. Classical Kantian and Hintikkian logics

To begin with, we introduce a classical system of logic, well-known in the field of deontic logic, let us call it $\mathbb{C}$, obtained by the adjunction of alethic and deontic operators to the classical sentencial logic. Its primitive symbols are those of the latter, plus $\Box$(necessity) and O(obligation). The remaining alethic and deontic operators are defined as usual. The postulates of $\mathbb{C}$ are composed of a complete list of postulates for the common propositional calculus, plus the following:

1) $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$	1′) $O(A \rightarrow B) \rightarrow (OA \rightarrow OB)$
2) $\Box A \rightarrow A$	2′) $OA \rightarrow \sim O \sim A$
3) $A \,/\, \Box A$	3′) $A \,/\, OA$
k) $OA \rightarrow \Diamond A$	h) $\Box A \rightarrow OA$

The notion of syntactic consequence, $\vdash$, is defined as in da Costa and Carnielli 1986.

Theorem 1. In C we have:

$\vdash \ \sim OA \rightarrow \Diamond \sim A$ $\qquad$ $\vdash \ \sim O \sim A \rightarrow \Diamond A$

$\vdash \ \Box A \rightarrow \sim O \sim A$ $\qquad$ $\vdash \ \Box \sim A \rightarrow O \sim A$

Theorem 2. In the preceding axiomatization of $\mathbb{C}$, the axiom scheme $OA \rightarrow \sim O \sim A$ is redundant.

Theorem 3. If $\Gamma \cup \{A\}$ is a set of formulas, we have:

If $\Gamma \vdash A$, then $\Box\Gamma \vdash \Box A$ and $O\Gamma \vdash OA$, where
$\Box\Gamma = \{\Box B : B \in \Gamma\}$ and $O\Gamma = \{OB : B \in \Gamma\}$.

In order to formulate a semantics for $\mathbb{C}$, we need the concept of a $\mathbb{C}$-structure.

Definition. A $\mathbb{C}$-structure is a quadruple structure $\mathfrak{a} = <W, R^{\Box}, R^{O}, \Vdash>$, satisfying the following conditions:

1) W is a non empty set (the set of worlds);
2) $R^{\Box} \subset W \times W$ ($R^{\Box}$ is the $\Box$-accessibility relation);
3) $\phi \neq R^{O} \subset R^{\Box}$ (R^{O} is the O-accessibility relation);
4) $R^{\Box}$ is reflexive;
5) For every w that belongs to the field of R^{O}, there exist $w' \in W$ such that $wR^{O}w'$;
6) $\Vdash$, the forcing relation between worlds and formulas possesses all the usual properties with respect to the connectives ($R^{\Box}$ takes account of $\Box$, and R^{O} of O).

We define in an obvious way the notion of semantic consequence $\models$.

$\mathbb{C}$ is Kantian and Hintikkian, in the sense that the axioms of Kant and Hintikka are valid within it.

Lemma 1. If $\{\Box A_1, ..., \Box A_n, \Diamond D\}$ is a consistent set of formulas, then $\{A_1, ..., A_n, D\}$ is also consistent.

Lemma 2. If $\{\Box A_1, ..., \Box A_n, OB_1, ..., OB_m, \sim O \sim C\}$ is consistent, then so is $\{A_1, ..., A_n, B_1, ..., B_m, C\}$.

Theorem 4. Let $\Gamma \cup \{A\}$ be a set of formulas. Then $\Gamma \vdash A$ if, and only if, $\Gamma \models A$ (soundness and completness).

Proof.- By an obvious adaptation of the standard soundness and completeness proofs, taking advantage, in the case of completeness, of Lemmas 1 and 2.

We can extend the concept of a $\mathbb{C}$-structure through the elimination of the condition that $R^{O} \subset R^{\Box}$. Given such an extended structure, <W,

$R^{\Box}$, R^{O}, $\Vdash$ >, in order that the axioms of Kant and of Hintikka be valid within it we must have, respectively, that:

$$\forall w \in W \ [\forall w' \in W(w \ R^{O} \ w' \rightarrow w' \Vdash A) \rightarrow \exists w' \in W(w \ R^{\Box} \ w' \wedge w' \Vdash A)]$$

and

$$\forall w \in W \ [\forall w' \in W(w \ R^{\Box} \ w' \rightarrow w' \Vdash A) \rightarrow \forall w' \in W(w \ R^{O} \ w' \rightarrow w' \Vdash A)].$$

Thus, we are led to construct sound and complete semantics for logics which are non-Kantian and Hintikkian, Kantian and non-Hintikkian, and non-Kantian and non-Hintikkian.

2. *Paraconsistent Kantian and Hintikkian logics*

To obtain a paraconsistent propositional calculus with alethic and deontic modalities, we replace in $\mathbb{C}$ the classical propositional calculus by C_1 (da Costa 1974, da Costa and Carnielli 1986, and Puga 1985). We shall dub the resulting system $\mathbb{C}_1$. The primitive symbols of $\mathbb{C}_1$ are the following:

1) Connectives: $\rightarrow$(implication), $\wedge$(conjunction), $\vee$(disjunction), $\sim$ (negation), $\Box$ and O; the other connectives are defined as in $\mathbb{C}$; 2) Propositional variables; 3) Parentheses. If A is a formula, then A^{O} is an abbreviation for $\sim(A \wedge \sim A)$. The postulates of $\mathbb{C}_1$ are those of C_1 (cf. da Costa 1974), plus

I) $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$
II) $\Box A \rightarrow A$
III) $A \ / \ \Box A$
IV) $A^{O} \rightarrow (\Box A)^{O} \wedge (OA)^{O}$
V) $O(A \rightarrow B) \rightarrow (OA \rightarrow OB)$
VI) $OA \rightarrow \ \sim O \sim A$
VII) $A \ / \ OA$
VIII) $OA \rightarrow \Diamond A$
IX) $\Box A \rightarrow OA$

Combining the ideas of the preceding section with the semantical considerations of da Costa and Carnielli 1986, we generalize the notion of $\mathbb{C}$-structure, obtaining a semantics for $\mathbb{C}_1$, relative to which it is sound and complete. In $\mathbb{C}_1$ we can handle moral dilemmas and certain kinds of contradictions without the danger of trivialization, simple or deontic (see, for instance, da Costa and Carnielli 1986). In consequence, $\mathbb{C}_1$ is a Kantian and Hintikkian calculus, within the spirit of clauses 1 and 2 of the *Introduction* to this work. On the other hand, modifying the semantics and the axiomatization of $\mathbb{C}_1$, we can build paraconsistent logics that are Kantian and non-Hintikkian, non-Kantian and Hintikkian, and non-Kantian and non-Hintikkian.

Similarly, with the help of da Costa's hierarchy C_n, $1 \leqslant n \leqslant \omega$, we can develop other paraconsistent logics with alethic and deontic modalities (see da Costa 1974).

3. *Tableaux in paraconsistent logic*

Let T and F be two distinct symbols out of the language of C_1. If A is a formula of C_1, TA and FA will be called *signed formulas*. Our new version of C_1, which we will call TC_1, is based on tableau rules that are relations between sets of signed formulas (on the method of tableaux, as developed here, see Fitting 1983, and Carnielli 1986).

In what follows, $\Gamma \cup \{A, B\}$ will denote a set of formulas. Furthermore, Γ; TA, TB, Γ; TA and Γ; FA will denote $\Gamma \cup \{TA, TB\}$, $\Gamma \cup \{TA\}$ and $\Gamma \cup \{FA\}$, respectively. The rules of TC_1 are listed below.

a) Rules of form $\dfrac{\Gamma;\ K}{\Gamma;\ K_1,\ K_2}$:

$$T_\wedge)\ \frac{\Gamma;\ T(A\wedge B)}{\Gamma;\ TA,\ TB} \qquad F_\vee)\ \frac{\Gamma;\ F(A\vee B)}{\Gamma;\ FA,\ FB} \qquad F_\to)\ \frac{\Gamma;\ F(A\to B)}{\Gamma;\ TA,\ FB}$$

b) Rules of form $\dfrac{\Gamma;\ K}{\Gamma;\ K_1 \mid \Gamma;\ K_2}$:

$$F_\wedge)\ \frac{\Gamma;\ F(A\wedge B)}{\Gamma;\ FA \mid \Gamma;\ FB} \qquad T_\vee)\ \frac{\Gamma;\ T(A\vee B)}{\Gamma;\ TA \mid \Gamma;\ TB} \qquad T_\to)\ \frac{\Gamma;\ T(A\to B)}{\Gamma;\ FA \mid \Gamma;\ TB}$$

$$O_1)\ \frac{\Gamma;\ F(A\wedge B)^{\circ}}{\Gamma;\ FA^{\circ} \mid \Gamma;\ FB^{\circ}} \qquad O_2)\ \frac{\Gamma;\ F(A\vee B)^{\circ}}{\Gamma;\ FA^{\circ} \mid \Gamma;\ FB^{\circ}} \qquad O_3)\ \frac{\Gamma;\ F(A\to B)^{\circ}}{\Gamma;\ FA^{\circ} \mid \Gamma;\ FB^{\circ}}$$

c) Rules of form $\frac{\Gamma;\ K}{\Gamma;\ K_1}$:

$$T{\sim}{\sim})\ \frac{\Gamma;\ T({\sim}{\sim}A)}{\Gamma;\ TA} \qquad F_{\sim})\ \frac{\Gamma;\ F({\sim}A)}{\Gamma;\ TA} \qquad F_{O4})\ \frac{\Gamma;\ F({\sim}A)^{O}}{\Gamma;\ FA^{O}}$$

d) Rules of form $\frac{\Gamma;\ K_1,\ K_2}{\Gamma;\ K_3}$:

$$T{\sim})\ \frac{\Gamma;\ T(B^{O}),\ T({\sim}B)}{\Gamma;\ FB}$$

Given a set $\triangle$ of signed formulas, we can easily define what is an application of a rule to $\triangle$. If *configuration* means a finite collection of sets of signed formulas, we can also easily define what we understand by an application of a rule to a configuration. A tableau is a finite sequence of configurations, each one, except the first, obtained from the preceding one by an application of some rule. A set Γ of signed formulas is closed if it contains formulas of the form TA and FA, for some A. A configuration is closed if all its members are closed and a tableau is closed if some of its terms are closed. We say that a formula A of C_1 is a theorem of TC_1 if there exists a closed tableau whose first term is {FA}. This closed tableau constitutes the proof of A in TC_1. We can easily check that a formula of C_1 is a theorem of C_1 if, and only if, it is a theorem of TC_1. This last calculus can be extended by the addition of the operators $\Box$ and O, governed by convenient rules, and the resulting system is equivalent to $\mathbb{C}_1$. It is not difficult to verify that the tableau method may contribute to an elucidation of the meaning of the connectives of C_1, from an operational point of view.

4. *Final remarks*

The propositional calculi with which we have been concerned here are decidable, which may be established by the process of filtrations.

Moreover, we can extend the results of this note to first and higher-order logic. With analogous methods we are able to treat other kinds of logic, such as epistmeic logic, tense logic, and topological logic (in the sense of Rescher 1968). The philosophical counterpart of the present mathematical analysis will appear elsewhere.

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