RESOLUTION IN CONSTRUCTIVISM

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1. Introduction

The aim of this paper is to formalize the *resolution principle* proposed by J.A. Robinson [15, 16] in the framework of constructive logic. As is well-known, this method is based on *indirect proof.* Such a proof, in general, is not permitted in constructive logic because of its lack of the *excluded middle.* Thus, it becomes important to discuss whether the classical interpretation remains valid for resolution from the viewpoint of constructivism. Fortunately, one of the constructive logics, called *strong negation system,* enables us to give a positive answer to the question under some restrictions.

We wish to present an interpretation of the resolution principle on the basis of the strong negation system originating from Nelson [12]. *Strong negation (constructive falsity)* is different from intuitionistic (Heyting) negation. Heyting negation corresponds to the failure of a proof, namely, $\neg$ A implies the derivability of contradiction from A. Strong negation, on the other hand, is a constructive negation. For example, there exist two different interpretations of $\sim \forall x A(x)$. One is the derivability of absurdity from $\forall x A(x)$ on the basis of Heyting negation ($\neg$). The other constitutes the effective method for constructing $\sim A(n)$ for some natural number n on the basis of strong negation ($\sim$).

This paper is organized as follows. In section 2, the strong negation system N is presented. Also, Gentzen-type formulation for N is given in the tableaux method accommodating resolution principle. Secton 3 incluedes our results such as completeness theorem, embedding theorems, accommodation theorems. We also discuss the resolution within intuitionistic logic H. Finally, we shall mention some problems in our formalism in connection with logic programming language such as Prolog.

2. Constructive Falsity and its Proof Theory

Strong negation (constructive falsity) was first investigated by Nelson

[12] in connection with *N-realizability*, being an analogue of Kleene's *recursive realizability* [8]. Around the same time, Markov [10] defined Heyting negation in terms of strong negation and implication. Since then, they have been followed by many works such as Vorob'ev [18], Rasiowa [14], Fitch [5], Thomason [17], Prawitz [13], Ishimoto [6], Almukdad and Nelson [3], Akama [1,2], and others.

The strong negation system (or constructive predicate logic with strong negation) N is the set of well-formed formulas (wffs) defined in terms of six logical symbols, namely, $\&$(conjunction), $\vee$(disjunction), $\supset$ (implication), $\sim$ (strong negation), $\forall$(universal quantifier), and $\exists$(existential quantifier) with a countably infinite list of parameters and variables as well as predicate and function symbols by means of ordinary formation rules.

The Hilbert-version of N is defined as the minimal set of formulas satisfying the following axioms:

(A1) $\vdash \sim A \supset (A \supset B)$,
(A2) $\vdash \sim (A \supset B) \equiv A \& \sim B$,
(A3) $\vdash \sim (A \& B) \supset \sim A \vee \sim B$,
(A4) $\vdash \sim (A \vee B) \equiv \sim A \& \sim B$,
(A5) $\vdash \sim \sim A \equiv A$,
(A6) $\vdash \sim \forall x A(x) \equiv \exists x \sim A(x)$,
(A7) $\vdash \sim \exists x A(x) \equiv \forall x \sim A(x)$,

along with the axioms of *positive logic* being closed under *detachment* and the following quantificational rules:

$$\vdash A \supset B(t) \Rightarrow \vdash A \supset \forall x B(x),$$
$$\vdash A(t) \supset B \Rightarrow \vdash \exists x A(x) \supset B.$$

$A \equiv B$ is an abbreviation for $(A \supset B) \& (B \supset A)$. Here, $\vdash_N$ ($\models_N$) denotes the provability (validity) in N. (the subscript is surpressed whenever no ambiguity arises therefrom.) Vorob'ev [18] proposed a *constructive propositional logic with strong negation* in which intuitionistic and strong negation both occur as logical symbols. The following extra axioms are required for his system in addition to the ones for N;

(A8) $\vdash (A \supset B) \supset ((A \supset \neg B) \supset \neg A)$,
(A9) $\vdash \neg A \supset (A \supset B)$,
(A10) $\vdash \sim \neg A \equiv A$.

As mentioned above, Heyting negation could be defined in N in the following way:

$$\vdash \neg A \equiv A \supset \sim A.$$

This result was also presented in Nelson [12], and Almukdad and Nelson [3] in the following different from:

$$\vdash \neg A \equiv A \supset (B \& \sim B).$$

Here, we shall justify the above relation between intuitionistic and strong negations with the next theorem.

Theorem 2.1

$$\vdash \neg A \equiv A \supset \sim A$$

The proof is as follows: $\neg A \supset (A \supset \sim A)$ is clear from $(A9)$. For proving the converse, we must first prove $\sim A \supset \neg A$, namely

(1) $\vdash \sim A \supset (A \supset A)$	by positive logic
(2) $\vdash \sim A \supset (A \supset \neg A)$	by $(A1)$
(3) $\vdash \sim A \supset ((A \supset A) \& (A \supset \neg A))$	by (1), (2), and positive logic
(4) $\vdash \sim A \supset \neg A$	by $(A8)$, (3), and positive logic

Next, the proof of $(A \supset \sim A) \supset \neg A$ is given as follows:

(5) $\vdash (A \supset \sim A) \supset (A \supset \neg A)$	by (4) and positive logic
(6) $\vdash (A \supset A) \supset ((A \supset \neg A) \supset \neg A)$	by $(A8)$
(7) $\vdash (A \supset \sim A) \supset \neg A$	(5), (6), and positive logic

This completes the proof.

We next present the *tableaux method* for N on the basis of Gentzen's well-known sequent calculus. The proposed tableaux method is defined in terms of a number of *reduction rules*. In the tableaux method, rules are successively applied up side down to a formula to be proved.

We introduced a sequent calculus GN corresponding to N. Our proposed system GN is almost same as the ones developed by Almukdad and Nelson [3] and Vorob'ev [18]; see Akama [2]. Thomason [17] also proposed a similar sequent system. In his system, $\forall x(A(x) \vee B) \supset (\forall x A(x) \vee B)$(x is not free in B) is provable. But, unfortunately, it is not provable in N; see Fitch [5].

A *sequent* is the form $\Gamma \to \Delta$ where Greek letters Γ, Δ etc. denote finite (possibly empty) sequences of formulas. It is shown that $\vdash_N$ A iff $\vdash_{GN} \to$ A, and that if $\vdash_{GN}$ $A_1, \ldots, A_m \to B_1, \ldots, B_n$ then $\vdash_N$ $A_1 \& \ldots \& A_m \supset (B_1 \vee \ldots \vee B_n)$. Henceforth, we may omit $\vdash$ in the case of sequent calculus.

The axioms of GN

(1) $A \to A$, (2) $A, \sim A \to B$

The reduction rules of GN

Thinning $\dfrac{\Gamma \to \Delta}{\Gamma, A \to \Delta}$ $\dfrac{\Gamma \to \Delta}{\Gamma \to \Delta, A}$

Contraction $\dfrac{\Gamma, A, A \to \Delta}{\Gamma, A \to \Delta}$ $\dfrac{\Gamma \to \Delta, A, A}{\Gamma \to \Delta, A}$

Interchange $\dfrac{\Gamma, A, B, \Delta \to \Theta}{\Gamma, B, A, \Delta \to \Theta}$ $\dfrac{\Gamma \to \Delta, A, B, \Theta}{\Gamma \to \Delta, B, A, \Theta}$

$(\vee \to)$ $\dfrac{\Gamma, A \vee B \to \Delta}{\Gamma, A \to \Delta \quad \Gamma, B \to \Delta}$ $(\to \vee)$ $\dfrac{\Gamma \to \Delta, A \vee B}{\Gamma \to \Delta, A, B}$

$(\& \to)$ $\dfrac{\Gamma, A \& B \to \Delta}{\Gamma, A, B \to \Delta}$ $(\to \&)$ $\dfrac{\Gamma \to \Delta, A \& B}{\Gamma \to \Delta, A \quad \Gamma \to \Delta, B}$

$(\forall \to)$ $\dfrac{\Gamma, \forall x A(x) \to \Delta}{\Gamma, A(t) \to \Delta}$ $(\to \forall)$ $\dfrac{\Gamma \to \forall x A(x)}{\Gamma \to A(b)^*}$

$(\exists \to)$ $\dfrac{\Gamma, \exists x A(x) \to \Delta}{\Gamma, A(b)^* \to \Delta}$ $(\to \exists)$ $\dfrac{\Gamma \to \exists x A(x)}{\Gamma \to A(t)}$

$(\supset \to)$ $\dfrac{\Gamma, \Delta, A \supset B \to \Lambda, \Theta}{\Gamma \to \Lambda, A \quad \Delta, B \to \Theta}$ $(\to \supset)$ $\dfrac{\Gamma \to A \supset B}{\Gamma, A \to B}$

$(\sim \sim \to)$ $\dfrac{\Gamma, \sim \sim A \to \Delta}{\Gamma, A \to \Delta}$ $(\to \sim \sim)$ $\dfrac{\Gamma \to \Delta, \sim \sim A}{\Gamma \to \Delta, A}$

$(\sim \supset \to)$ $\dfrac{\Gamma, \sim (A \supset B) \to \Delta}{\Gamma, A, \sim B \to \Delta}$ $(\to \sim \supset)$ $\dfrac{\Gamma \to \Delta, \sim (A \supset B)}{\Gamma \to \Delta, A \quad \Gamma \to \Delta, \sim B}$

$$(\sim\vee\to)\ \frac{\Gamma,\ \sim(A\vee B)\to\Delta}{\Gamma,\ \sim A,\ \sim B\to\Delta} \qquad (\to\sim\vee)\ \frac{\Gamma\to\Delta,\ \sim(A\vee B)}{\Gamma\to\Delta,\ \sim A \quad \Gamma\to\Delta,\ \sim B}$$

$$(\sim\&\to)\ \frac{\Gamma,\ \sim(A\& B)\to\Delta}{\Gamma,\ \sim A\to\Delta \quad \Gamma,\ \sim B\to\Delta} \qquad (\to\sim\&)\ \frac{\Gamma\to\Delta,\ \sim(A\& B)}{\Gamma\to\Delta,\ \sim A,\ \sim B}$$

$$(\sim\forall\to)\ \frac{\Gamma,\ \sim\forall xA(x)\to\Delta}{\Gamma,\ \sim A(b)\to\Delta^*} \qquad (\to\sim\forall)\ \frac{\Gamma\to\Delta,\ \sim\forall xA(x)}{\Gamma\to\Delta,\ \sim A(t)}$$

$$(\sim\exists\to)\ \frac{\Gamma,\ \sim\exists xA(x)\to\Delta}{\Gamma,\ \sim A(t)\to\Delta} \qquad (\to\sim\exists)\ \frac{\Gamma^*\to\ \sim\exists xA(x)}{\Gamma\to\ \sim A(b)}$$

Here * denotes so-called *eigenvariable.* The formula to which the logical symbols are introduced by a reduction rule is called *principal formula.*

Also we can add three axioms for equality to N, namely,

$$\to\alpha=\alpha$$
$$x_1=y_1,\ \ldots,\ x_n=y_n\to f(x_1,\ \ldots,\ x_n)=f(y_1,\ \ldots,\ y_n)$$
$$x_1=y_1,\ \ldots,\ x_n=y_n,\ R(x_1,\ \ldots,\ x_n)\to R(y_1,\ldots,\ y_n)$$

for each n-ary function symbol f and n-ary predicate symbol R of N.

Theorem 2.2 (Cut-elimination theorem)

$$\vdash\Gamma\to\Lambda,\ A \textit{ and } \vdash A,\ \Theta\to\Delta\Rightarrow\ \vdash\Gamma,\ \Theta\to\Lambda,\ \Delta.$$

This is Gentzen's well-known result for classical and intuitionistic logic. We can prove the theorem for GN by transfinite induction on *grade* and *rank* of the *uppermost cut*; see Akama [2].

Here, we do not give semantics for N. The reader is referred to Akama [1] for detailed accounts of Kripke semantics for N. Of course, the above theorem is available by a model-theoretic approach.

3. Resolution as Constructive Proof

At present, resolution principle is considered within a proof system of classical logic. Namely, the deduction is carried out by two inference rules of *factoring* and *unification.* The empty clause is the only axiom. In other words, it is based on *the rule of double negation elimination* in classical logic making use of the excluded middle.

We shall show an outline of resolution method. When we want to prove a formula A in this method, resolution method enables us to derive an empty clause (i.e. contradiction) from $\sim A$ and the set Γ of facts and rules in classical sense, namely

$$\vdash_C \Gamma \cup \{\sim A\} \Rightarrow \text{contradiction} \Leftrightarrow \Gamma \vdash_C \sim\sim A \qquad (1).$$

In the last step, we conclude A by means of the rule of double negation elimination rule: it is on the basis of the excluded middle.

We here show this important point in Hilbert-style proof as follows:

$$A \vdash_C \sim\sim A \supset A \qquad (2)$$
$$\sim A \vdash_C \sim\sim A \supset A \qquad (3)$$

In classical logic, we have:

$$A \vee \sim A \vdash_C \sim\sim A \supset A \qquad (4)$$

from (2) and (3). This implies the resolution method assumes the excluded middle.

However, the intuitionist rejects this procedure, for the excluded middle is not an axiom of intuitionistic logic. Thus, intuitionistic logic is not appropriate as a basis for resolution. The above procedure, on the other hand, is possible in strong negation system. For the reader has only to recall $\sim\sim A \supset A$ is provable in N despote a failure of $A \vee \sim A$.

The inference mechanism of refutation is similar to the behavior of the proposed tableaux method GN. This is why we are going to accomodate resolution to the strong negation system. For so doing, several fundamental theorems are in order.

Theorem 3.1 (Consistency theorem)

N is consistent.

If inconsistent, $\rightarrow A$ and $\rightarrow \sim A$ hold. Suppose the following proof figure.

$$Cut\ \dfrac{\rightarrow \sim A \qquad Cut\ \dfrac{\rightarrow A \qquad A \& \sim A \rightarrow}{\sim A \rightarrow}}{\rightarrow}$$

By elimination theorem, we have $\vdash \rightarrow$. But, $\rightarrow$ is not an axiom of GN. No rule applies to this sequent: a contradiction.

Theorem 3.2 (Completeness theprem)

$\models \Gamma \rightarrow \Delta \Leftrightarrow \vdash \Gamma \rightarrow \Delta$

It is immediate from cut-elimination theorem. Semantical completeness is also proved by Akama [1] on the basis of Henkin construction. Next theorem shows an embedding of classical predicate logic C into N.

Theorem 3.3 (Embedding theorem C)

$\vdash_C A \Rightarrow \vdash_N \sim A \rightarrow$

where A does not contain the implication, and all the logical symbols in $\sim A \rightarrow$ are interpreted constructively.

This theorem is proved by induction on the length of the proof. Also, N can be embedded into intuitionistic predicate logic H.

Theorem 3.4 (Embedding theorem H)

$\vdash_N T\Gamma \rightarrow T\Delta \Leftrightarrow \vdash_H \Gamma \rightarrow \Delta$

where T is a translation analogue to Gödel's defined as below:

$$\begin{array}{ll} \mathrm{TA} = \mathrm{A} \text{ for any atomic A}, & \mathrm{T(A \vee B)} = \mathrm{TA \vee TB}, \\ \mathrm{T(A \supset B)} = \mathrm{TA \supset TB}, & \mathrm{T(\neg A)} = \mathrm{T(A \supset \sim A)}, \\ \mathrm{T(\forall x A(x))} = \mathrm{\forall x T(A(x))}. & \end{array}$$

This theorem is proved by induction on the complexity of translated formulas.

Now, we are in the position to formulate resolution in N. We assume the usual terminology in resolution logic. We here call a *goal clause* the formula to be proved, namely, conjunction of atomic formulas.

Theorem 3.5 (Accommodation theorem 1)

$\vdash_C \Gamma \cup \{\sim G\} \rightarrow$ iff $\vdash_{GN} \Gamma \& \sim G \rightarrow$ *where G is a goal clause properly generalized if it contains free variables, and* Γ *is a set of clauses. All the logical symbols occurring in GN, namely* $\vee$, &, $\sim$, $\forall$ *are interpreted as symbols in N, and the reduction rules to be applied in GN are restricted to* $\vee\rightarrow$ *and* $\forall\rightarrow$.

Sufficiency is trivial. The proof of necessity is as follows. It is clear that $\Gamma \cup \{\sim G\} \rightarrow$ iff $\Gamma \supset G$ or $\sim \Gamma \vee G$ classically. By embedding theorem in C, the following can be proved

$$\sim(\sim \Gamma \vee G) \rightarrow \tag{1}$$

in GN, where all the logical symbols involved are interpreted as the ones in constructive logic.
By reduction rule in GN (de Morgan's law),

$$\Gamma \& \sim G \rightarrow \sim(\sim \Gamma \vee G) \tag{2}$$

which is provable in GN. By cut-elimination theorem, we have:

$$\Gamma \& \sim G \rightarrow \tag{3}$$

Theorem 3.5 describes the deduction in N for resolution but the last one step. Next result is a generalization of resolution within constructive logic.

Theorem 3.6 (Accommodation theorem 2)

$\vdash_C \Gamma \cup \{\sim G\} \rightarrow \Leftrightarrow \vdash_{GN} \Gamma \rightarrow G \vee \sim G$ *and* $\vdash_{GN} \Gamma \& \sim G \rightarrow \ \Rightarrow \ \vdash_{GN} \Gamma \rightarrow G.$

This theorem is immediate from cut-elimination theorem and theorem 3.5. By theprem 3.5, we have:

$$\vdash_C \Gamma \cup \{\sim G\} \text{ iff } \vdash_{GN} \Gamma \& \sim G \rightarrow \tag{1}$$

From the assumption of the excluded middle and $\rightarrow \vee$ in GN:

$$\Gamma \rightarrow G, \ \sim G \tag{2}$$

Apply the rule $\& \rightarrow$ to the right hand side of (1)

$$\Gamma, \ \sim G \rightarrow \tag{3}$$

By cut-elimination theorem, we have:

$$\Gamma, \Gamma \rightarrow G \tag{4}$$

from (2) and (3). From (4), by way of contraction rule, the desired sequent is obtained,

$$\Gamma \rightarrow G.$$

The converse does not present any difficulty. This completes the proof of theorem 3.6.

The right side of this theorem corresponds the so-called *restricted rule of negation introduction* in Fitch [5]. In other words, resolution principle is interpreted within constructive logic assuming the excluded middle. It is of interest from a constructive view.

Some type of refutation procedure (e.g. *SLD-resolution*) can be formalized constructively without the excluded middle. An *SLD-resolution* is an abbreviation for *SL-resolution for Definite clause*. As is well-known, Horn clause is also called a *definite clause*. Here the definition of SLD-resolution is in order.

Let Γ be a set of Horn clauses and G a goal clause where G is of the form $A_1\&, \ldots, \&A_n$ (n > 0) each of A_i ($1 \le i \le n$) is an atom. An *SLD-derivation* of $\Gamma \cup \{\sim G\}$ is a finite (or infinite) sequence $G_0, G_1, \ldots$ ($G_0 = \{\sim G\}$) of *goal clauses*, a sequence $d_1, d_2, \ldots$ of variants of clauses in Γ (the *input clauses* of the derivation), and a sequence $\theta_1, \theta_2, \ldots$ of substitutions called *most general unifier* (mgu). It is assumed that no input clause d_i of a derivation has a variable in common with goal clause G_i. Each non-empty G_i has an atom A_i (the *selected atom*). The goal clause G_{i+1} is derived from G_i and d_i with substitution θ_i. Let

$$G_i \equiv \leftarrow A_1, A_2, \ldots, A_k, \ldots, A_n \ (n \ge 1)$$

with A_k as a selected atom, and

$$d_k \equiv A \leftarrow B_1, B_2, \ldots, B_m \ (m \ge 0)$$

be an input clause in Γ such that A and A_k are unifiable, i.e. $A\theta \equiv A_k\theta$ for some mgu θ. Then, G_{i+1} is

$$\leftarrow (A_1, A_2, \ldots, A_{k-1}, B_1, B_2, \ldots, B_m, A_{k+1}, \ldots, A_n)\theta$$

called the *resolvent of* G_i and d_i. An *SLD-refutation* is a derivation whose last goal is an empty clause $\square$.

For more information, the reader is referred to Lloyd [9],

Theorem 3.7 (Accommodation theorem 3)

$$\vdash_C \Gamma \cup \{\sim G\} \rightarrow \ \Leftrightarrow \ \vdash_{GN} \Gamma \rightarrow G.$$

Sufficiency is trivial. Necessity is proved as follows. By theprem 3.5,

$$\vdash_{\overline{GN}} \Gamma \& \sim G \rightarrow$$

Here we need to prove that $\vdash$ A$\rightarrow$ iff $\vdash \rightarrow \sim$A.

Lemma 3.8

$\vdash A \rightarrow$ iff $\vdash \rightarrow \sim A$

where A is atomic.

We can prove this lemma by induction on the length of the proof. Notice that the proof figure of $\Gamma \& \sim$G$\rightarrow$ has the following properties due to the assumptions of SLD-resolution.

(a) All the logical symbols in the proof are restricted to $\vee$, &, $\sim$, $\forall$.
(b) The proof is carried out only by using the rules $\vee\rightarrow$ and $\forall\rightarrow$.
(c) In the last step of the proof (i.e. $G_k \& \sim G_k \rightarrow$), G_k is one of the sub-formulas in Γ.
(d) Γ is a set of Horn clauses.

The proof is by induction. The inductive steps are carried out without any difficulty by $\vee\rightarrow$ and $\forall\rightarrow$.

In SLD-resolution, there must be a selected literal appearing in the goal clause and input clause. And theorem 3.7 holds only of a goal clause and a set of clauses consist of Horn clauses. Hence, the generalization of lemma 3.8 is not possible. In other words, lemma 3.8 does not hold for any A. To see this, suppose A = B$\vee\sim$B, it is obvious that $\sim$(B$\vee\sim$B)$\rightarrow$ is provable in GN. By lemma 3.8, we have:

$$\rightarrow B \vee \sim B \qquad (1)$$

Nevertheless, (1) is not provable in N since B$\vee\sim$B is not a thesis of N. Lemma 3.9 cannot be extended to the following form, either:

$$\vdash A \rightarrow \Delta \text{ iff } \vdash \rightarrow \sim A, \Delta \qquad (2)$$

for any set of formulas Δ. Now, suppose Δ = A, we have:

$$\rightarrow A \vee \sim A$$

from (2), which is not provable in N. Thus, because of such restrictions, SLD-resolution enables us to guarantee a constructive proof in the sense that it is formalized in GN, without the principle of excluded middle.

Theorem 3.9 (Accommodation theorem 4)

If $G_0 = \sim G, G_1, \ldots, G_n = \square$ *is* an SLD-derivation, then $\vdash_{GN} \Gamma \& G_0 \rightarrow G_n$.

The proof is by induction on the length of the SLD-derivation. Suppose n = 0, namely $G_0 = \sim G$ then $\Gamma \& \sim G \rightarrow \sim G$ holds in N since it is an axiom in N. Induction steps are divided into two parts. The one is an inference, called *factoring*, from

$$G_k: \sim A_1 \vee \sim A_2 \vee \ldots \vee \sim A_i \vee \ldots \vee \sim A_m \ (1 \leq i \leq m) \quad (4)$$
$$d_k: A_i \vee \sim B_1 \vee \sim B_2 \vee \ldots \vee \sim B_n \ (0 \leq n) \quad (5)$$

to *resolvent* G_{k+1} of G_k and d_k. It is proved by cut-elimination theorem for N.

$$G_{k+1}: \sim A_1 \vee \sim A_2 \vee \ldots \vee \sim A_{i-1} \vee \sim B_1 \vee \ldots \vee \sim B_n \vee \sim A_{i+1} \vee \ldots \vee \sim A_m \quad (6)$$

By inductive hypothesis,

$$\Gamma \& G_0 \rightarrow \sim A_1 \vee \ldots \vee \sim A_i \vee \ldots \vee \sim A_m \quad (7)$$
$$\Gamma \& G_0 \rightarrow A_i \vee \sim B_1 \vee \ldots \vee \sim B_n \quad (8)$$

By lemma 3.8 to (8)

$$\Gamma \& G_0 \& \sim A_i \rightarrow \sim B_1 \vee \ldots \vee \sim B_n \quad (9)$$

By cut-elimination theorem to (7) and (9)

$$\Gamma \& G_0 \rightarrow \sim A_1 \vee \sim A_2 \vee \ldots \vee \sim A_{i-1} \vee \sim B_1 \vee \ldots \vee \sim B_n \vee \sim A_{i+1} \vee \ldots \vee \sim A_m \quad (10)$$

The other is the procedure of answer substitution by way of *unification*, namely, from

$$\forall x_1 \ldots x_k A \quad (11)$$

to

$$A\theta \quad (12)$$

where θ denotes a *most general unifer* (mgu) substituting for the free variables in A, namely x_i $(1 \leq i \leq k)$ for suitable terms t_i $(1 \leq i \leq k)$. By induction hypothesis, from

$$\Gamma \& G_0 \rightarrow \forall x_1 \ldots x_k \mathrm{A} \tag{13}$$

and

$$\forall x_1 \ldots x_k \mathrm{A} \rightarrow \mathrm{A}\theta \tag{14}$$

we get

$$\Gamma \& G_0 \rightarrow A\theta \tag{15}$$

The last step in SLD-resolution is carried out as regards a *complementary pair* such that

$$\mathrm{A}\theta \tag{16}$$

and

$$\sim A\theta \tag{17}$$

Of course, A is an atomic formula. By induction hypothesis,

$$\Gamma \& G_0 \rightarrow \mathrm{A} \text{ and } \Gamma \& G_0 \rightarrow \sim \mathrm{A} \tag{18}$$

hold. From (18) and reduction rule $\rightarrow \&$

$$\Gamma \& G_0 \rightarrow \mathrm{A} \& \sim \mathrm{A} \tag{19}$$

By (19) and axiom (20)

$$\mathrm{A} \& \sim \mathrm{A} \rightarrow \tag{20}$$

we have a final goal from cut-elimination theorem, namely

$$\Gamma \& G_0 \rightarrow \tag{21}$$

This completes the proof.

Finally, we formalize resolution in H as a corollary to the above results.

Theorem 3.10 (Accommodation theorem 5)

$\vdash_C \Gamma \cup \{\sim G\} \rightarrow \;\Leftrightarrow\; \vdash_{LJ} \Gamma \& \neg G \rightarrow$.

Theorem 3.11 (Accommodation theorem 6)

$\vdash_C \Gamma \cup \{\sim G\} \rightarrow \Leftrightarrow \vdash_{LJ} \Gamma \rightarrow G \vee \neg G$ and $\vdash_{LJ} \Gamma \& \neg G \rightarrow \Rightarrow \vdash_{LJ} \Gamma \rightarrow G$.

Here, we assume Gentzen's LJ as a corresponding sequent calculus to H. These two theorems are proved by the same manner as that described above.

Theorem 3.12 (Accommodation theorem 7)

$\vdash_C \Gamma \cup \{\sim G\} \rightarrow \Leftrightarrow \vdash_{LJ} \Gamma \rightarrow \neg\neg G$.

In H, the double negation cannot be eliminated for the lack of excluded middle. Thus, theorem 3.9 fails to hold in H.

4. *Discussions*

We have presented a constructive proof system for resolution principle on the basis of strong negation. Our results can be obtained by means of natural deduction style of proof theory. It is also possible that resolution is interpreted in *normal proof procedure* in natural deduction system for N.

Our system is capable of treating the computation of logic programs [7,9]. Compare the *procedural interpretation* of Prolog with the deduction in tableaux method for N. The similarity is clear from their behaviors. One of the most challenging topics in logic programming is the problem of negation. As is well-known, negative information cannot be derived from a set of clauses in resolution. Thus, negation is implemented as a meta-rule in Prolog. For example, Clark's *negation as failure rule* [4] is defined as a meta-rule in procedural semantics instead of declarative semantics for ordinary logic. Interestingly, it has a constructive flavor as in the case of the so-called *finite failure*. Its relation to N is worth investigating. Since such a meta-rule is interpreted only in a procedural way, it is important to formalize a logical meaning of negation as failure from the standpoint of constructive logic.

Our ultimate goal is to develop foundations of *automated theorem-proving* including logic programming within constructive logic. The results in this paper seem to be very instructive since resolution principle plays an important role in the area. If we succeed in the further research in this direction, it would be one of the justifications of the adequacy of constructive logic for computer science. We believe our theory can be a starting point for such an activity comparable to the one proposed by Martin-Löf [11].

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