EQUIPROBABILITY*

David WATT

1. Introduction: Why epistemic probability?

In this paper I shall concentrate on the concept of epistemic probability, i.e. probability relative to knowledge. So except where otherwise indicated, when I use the term "probability" or one of its cognates, I shall always be referring to epistemic probability rather than some other concept, such as Carnap's probability 2. Epistemic probability seems to be identical both with Carnap's probability 1, and also with Cohen's inductive probability. [See p. 23 ff. of Carnap's (1950), and p. 40 ff. of Cohen's (1977).] However, I shall not be presupposing any particular analysis of the concept, such as Carnap's or Cohen's. Neither shall I assume that epistemic probabilities satisfy the axioms of the probability calculus. Indeed, in section 3 I shall offer a proof that such probabilities are not even linearly ordered. In other words, I shall argue for the existence of *incomparable* probabilities; probabilities such that the first is neither greater than, nor equal to, nor less than the second. If this argument is sound, then probabilities certainly do not satisfy the axioms of the calculus.

Despite Cohen's (1977), some may still think that there is no point in discussing a concept of probability which does not satisfy these axioms. However, we can hardly lay it down *a priori* that these axioms must be satisfied by any worthwhile probability concept. For instance, suppose that a meteorologist tells us that rain tomorrow is probable. If we know that he has never learnt any of the mathematics of probability, that does not make us ridicule him for using a "confused", "incoherent" probability concept. On the contrary, we pay close attention to his statement, and the following day, we are prepared against the weather. But in view of the meteorologist's lack of mathematical knowledge, it is an open question whether his concept of probability conforms to the Kolmogorov axioms.

* Special thanks to Graham Priest and Michael Partis, who were exposed to an earlier version of this paper.

The meteorologist's probability concept is widely shared, since interest in weather forecasts is quite common, and once one person has heard the forecast, he passes on the information to his wife, children etc. So the meteorologist's concept is used by those who know a great deal of the philosophy and mathematics of probability, and those who know nothing of either. It is used by doctors, lawyers, physicists, philosophers, mathematicians, and primary school children.

However, it might be claimed that there is no reason for supposing that the probability concept which the meteorologist is using is epistemic probability. Why could he not be using some other probability concept? Perhaps he is saying, not that rain is probable *relative to his knowledge*, but that physical laws, facts about cloud cover and prevailing winds etc. bring it about that there is a strong tendency for it to rain, irrespective of anything anyone may know.

The trouble about this analysis of his statement is that it represents him as asserting what he cannot possibly know. At most he knows that, *ceteris paribus*, what he now *knows* about the weather guarantees a strong tendency for there to be rain. (Here I am not presupposing a general scepticism about knowledge of tendencies. All I need is the much weaker premise that in the current state of meteorological science, we never *know* that, taking *everything* into consideration, there is a strong tendency for there to be rain. For we can never take everything into consideration.) There may be unknown physical laws, facts about the present weather conditions etc. which guarantee that despite appearances, it will not rain. So on the assumption that he is talking directly about the tendency for there to be rain (and not just about our knowledge of this tendency) we cannot *know* that his statement is correct. Thus, even if the above assumption is correct, we still need the concept of epistemic probability. We need it to guide our action in regard to his statement. We need some such principle as this: "Since there is a high epistemic probability of his ascription of tendency being correct, we should act as if we *knew* this ascription to be correct".

So we need the concept of epistemic probability, whatever the correct analysis of the meteorologist's statement. If he is using that concept himself, we need the concept in order to understand what he is saying. If on the other hand he is making an ascription of tendency, then given that we do not *know* that his statement is true, we need the concept of epistemic probability in order to be justified in acting as if we *did* know this. Clearly then, the scope of epistemic probability is very wide indeed. Nor does it seem likely that we will soon outgrow this concept. As long as we are interested in propositions which we neither know to be true, nor know to be false, we will be interested in such questions as the rationality of accepting the proposition. And the rationality of accepting it will depend on its epistemic probability. More generally, it seems that the concept of epistemic probability is necessary for any agent who is not omniscient, for any such agent will need some notion of how to act on the basis of incomplete information.

2. *Epistemic probability: maxima and minima*

Epistemic probability is probability relative to knowledge K. Sometimes I speak of an "item of information", meaning a member m of some K which is clear from the context. Where the reference to K is clear, I sometimes speak elliptically of the "probability" of m [or "P(m)", for short], but when precision is required I speak of the "K-probability of m", or "P(m/K)", for short. I assume that K is always a set of propositions. By a proposition, I simply mean the sort of thing which can be understood, believed, asserted, or denied. I do not presuppose any particular philosophical theory of propositions. However, I do assume (*pace* Stalnaker, 1984, p. 3) that interderivable propositions are not necessarily identical. This assumption seems to be in accordance with our ordinary concept of a proposition. For instance, define:

α: $2 + 2 = 4$,
β: Twice the cube root of 8 equals the fifth root of 1024.

Then although α and β are interderivable, we do not infer that a child who asserts α, thereby asserts β. If we *were* entitled to infer this, the teaching of mathematics would be much easier than it is!

Different sentences can express the same proposition. My discussion will be couched in terms of propositions, rather than sentences, to avoid a familiar kind of difficulty with an approach such as Carnap's. Carnap speaks of sentences rather than propositions [cf. e.g. pp. 283-284 of his (1950)], and hence seems committed to the implausible idea that probability depends not only on what we know, but also on the language we use.

Although I assume that the probability of a proposition depends *only* on what we know, I do not assume that it depends on *everything* we know. For instance, If K is my present knowledge set, and p is the proposition that every human being is less than 3 metres tall, then my knowledge that I have never come across a counterexample to p, clearly bears on the K-probability of p, but it is hard to see how the probability is affected by the fact that my knowledge includes Euclid's proposition that there are infinitely many primes. At any rate it seems unwise to assume in advance that, for any knowledge set K and proposition p, every single element of K plays some part in determining the K-probability of p. Those elements of K which *do* play some such part, I describe as "relevant to the K-probability of p". If K and p are clear from the context, I sometimes speak merely of "relevant information". A subset of K is formed by the items of relevant information, but I make no assumptions as to how large a subset this is.

If K is an arbitrary knowledge set, I do not assume that K contains all its logical consequences – such an assumption is clearly false, if K is the knowledge of any mere human being. I do not even assume that K contains all the consequences of any single element of K. Such an assumption is just as unrealistic as the assumption of generalized closure. Perhaps Goldbach's conjecture that every even number is the sum of two primes, follows from Euclid's proposition; even so, Euclid's proposition is an element of my knowledge set, whereas Goldbach's conjecture is not. So if there are any correct closure principles at all, they must be even weaker than Kyburg's "weak deductive closure" [see p. 78 of his (1970)]. For our purposes there will be no need to determine the exact extent to which a knowledge set must be closed.

What is the maximum epistemic probability? I submit that this maximum is certainty, i.e. knowledge. Thus proposition p has maximum K-probability if and only if K contains p. Similarly p has minimum K-probability if and only if K contains $-$p, i.e. if and only if K contains the proposition that p is false. I shall use the expression "probability 1" as short for "the maximum probability"; similarly "probability 0" is short for "the minimum probability". Since I am not assuming that probability is numerical, the above expressions should not be interpreted as presupposing this assumption. If the maximum probability is knowledge, and a knowledge set K need not contain all its deductive consequences, then Keynes, Jeffreys *et al.* are mistaken in claiming that for a proposition p to have maximum K-probability, it is sufficient that K entails p. Their doctrine has the unfortunate consequence that either Goldbach's conjecture (call it G), or its negation $-G$, is certain (assuming that either G or its negation is entailed by axioms which are items of our knowledge). In fact, of course, Goldbach's conjecture is just that – a conjecture – and neither it nor its negation is certain. We should admit this irrespective of whether we are platonists or intuitionists. Perhaps the conjecture is true now, but it will be certain only when and if it is proved.

Not surprisingly, many of Keynes' assertions are inconsistent with his doctrine that probability 1 is entailment. For example, he says (1921, 2.11):

> What is self-evident to me and what I really know, may be only a probable belief to you, or may form no part of your rational beliefs at all. And this may be true not only of such things as *my* existence, but of some logical axioms also. Some men – indeed it is obviously the case – may have a greater power of logical intuition than others.

Quite so; however, this contradicts the above doctrine, at least assuming that the axioms which are only probable for me nonetheless follow from what I know. For if they do, then given the above doctrine they have probability 1 relative to my knowledge, whatever the feebleness of my logical intuition. Another contradiction occurs in section 2 of chapter 21. Keynes says:

> I am inclined to believe... that if we trust the promptings of common sense, we have the same kind of ground for trusting analogy in mathematics that we have in physics...

Again, very reasonable; if we have checked that Goldbach's conjecture holds for all even numbers up to 10^{100}, we have increased the probability of the conjecture. But no such increase is possible if the doctrine I have been criticizing is correct, for then the probability must remain 1 or 0 whatever we do.

Keynes' mistake has been duplicated by many writers, many of whom are quite unsympathetic towards his theory as a whole. I shall mention only Jeffreys:

> What is the 10,000th figure in the expansion of e? Nobody knows; but that does not say that the probability that it is a 5 is 0.1. By follow-

> ing the rules of pure mathematics we could determine it definitely, and the statement is either entailed by the rules or contradicted; in probability language, on the data of pure mathematics it is either a certainty or an impossibility (1967, p. 38).

If, in accordance with pp. vii and 419, we interpret this passage as using an "ordinary commonsense notion" of probability and certainty, then its falsehood is obvious. For if we consider the following proposition:

p: 5 is the 10,000th figure in the expansion of e,

then plainly neither p nor −p is certain. To say that "by following the rules of pure mathematics we could determine [whether or not p] ... definitely ..." is quite irrelevant. We might as well say that because we can wait to discover what tomorrow's weather will be like, the probability of rain tomorrow is either 1 or 0.

Like Keynes, Jeffreys is not faithful to the doctrine that probability 1 is entailment. On the very same page that he asserts it so clearly, he says (p. 38):

> An expert computer does not trust his arithmetic without applying checks, which would give identities if the work is correct but would be expected to fail if there is a mistake. Thus induction is used to check the correctness of what is meant to be deduction. The possibility that two mistakes have cancelled is treated as so improbable that it can be ignored.

But checks are useless unless they increase the probability of our answer, and given the Keynes-Jeffreys doctrine, our answer will retain its probability – 0 or 1 – whether we apply the checks or not.

Here it might be suggested that the doctrine can be easily corrected: instead of saying that proposition p has K-probability 1 if K entails p, we change this antecedent to the condition that K includes the proposition that K entails p.

In regard to this suggestion, we may ask: Does the new antecedent entail that K contains p? If this entailment holds, then the suggestion is incorporated, very simply, by my proposal to identify the elements of K with those propositions having K-probability 1. If, on the other hand, the entailment does not hold, let us consider a case where it breaks down. By hypothesis, therefore, p is not an element of the knowledge set K. Let q be any element of K, and suppose that this knowledge set belongs to Smith, who argues as follows: "p is not something which I know. q, on the other hand, *is* something which I know. Whereas q is certain, p is *not* certain. So if I were forced to bet, and I had a choice of betting either on p, or on q, rationality would dictate choice of q rather than p". The obvious rationality of Smith's reasoning shows that as regards the matter of whether a proposition has maximal probability relative to a particular knowledge set, the fundamental question is whether that knowledge set contains the proposition. The question of whether the knower *knows* that the proposition is entailed by his knowledge, is relevant only in so far as it bears on the fundamental question.

Given knowledge K and proposition p, is there always such a thing as the K-probability of p? This seems doubtful. For if this probability exists, it must be either the maximum, or the minimum, or something in between. So it must be true either that K contains p, or that K contains −p, or that the members of K give p some intermediate probability. And there seem to be cases where none of these alternatives hold. For instance, let p be Goldbach's conjecture, and let K be the knowledge of a new-born child. Then clearly K does not contain p, nor does K contain −p. But neither does K contain the sort of proposition which would give p some intermediate probability; e.g. "Many good mathematicians suspect that p". This example seems to cast doubt on the assumption that, for any K and any p, there is such a thing as the K-probability of p. Whether or not the assumption has thereby been *refuted*, it is best avoided if possible. So we introduce the concept of a *hypothesis*; defined as follows:

> Given knowledge K and proposition p, "p is a K-hypothesis" means "The K-probability of p exists".

(Where no confusion can arise, I shall drop the reference to K.)

3. *Incomparability*

If knowledge K contains hypotheses h and h′, then h and h′ both have maximum K-probability, hence they have the same K-probability, or, as we shall say, they are *K-equiprobable*. Similarly, if K contains both −h and −h′, then h and h′ both have minimum K-probability, so again they are K-equiprobable. So if h′ has an extreme K-probability − maximum or minimum − then for any K-hypothesis h, the K-probability of h is comparable with the K-probability of h′, i.e., the K-probability of h is either greater than, equal to, or less than the K-probability of h′. But is this result still true if we drop our restriction on the probability of h′? In other words, is it quite generally true that

C: All probabilities are comparable,

i.e., that given a knowledge set K, and given K-hypotheses h and h′, the K-probability of h is always comparable with the K-probability of h′? In the second half of this section (pp. 13-17), I shall offer a proof that the answer is no. I shall do this by adapting an example from chapter 4 of Keynes' (1921), an example which was originally intended for a different purpose. Assuming that the proof is sound, it also refutes the idea that probability is numerical, where "numerical" is understood as entailing comparability − the doctrine that all probabilities are comparable. (I shall always use the word "numerical" in this sense.) Comparability is presupposed also by the assumption that probability satisfies the axioms of the calculus.

Later, I shall use such expressions as "P(H/K) ∗> P(h/K)", "P(H/K) ∗= P(h/K)", "P(H/K) ∗⩾ P(h/K)". These expressions may look as if they refer to numbers, but in my usage they are not intended to carry any such implication. They are merely abbreviations, respectively, for the following expressions: "H has a greater K-probability than h has"; "H and h are K-equiprobable"; "Either H has a greater K-probability than h has, or H and h are K-equiprobable". Thus these expressions should not be read as presupposing the doctrine that probability is numerical. The asterisk is intended to guard against any automatic assumption that this doctrine is correct.

In this paper there is space only to consider sufficient conditions for equiprobability, rather than conditions which are both necessary and sufficient. So given knowledge K and K-hypotheses p and q, we are looking for conditions which guarantee that p and q are K-equiprobable. One answer is the following principle:

POI1: K-hypotheses p and q are K-equiprobable if K contains no reason for believing, of one of them, that it is more K-probable than the other.

POI1 is one possibility as to what might be meant by the famous, or infamous, "Principle of Indifference". Exactly what (if anything) is meant by this term is less clear than is sometimes assumed, but in any case both (a) and (b), below, are widely believed:

(a) the term *does* denote a genuine principle,
(b) this principle is refuted by the examples in chapter 4 of Keynes' (1921).

What is less well-known is that even if (a) is accepted, there are replies to the alleged refutations. [See for example Jeffreys' (1922).] In the present paper, there is no space to discuss whether these replies are acceptable. Instead, I shall offer a new refutation of the Principle (interpreted as meaning POI1). For this purpose, we need to introduce the notion of *involvement*, defined as follows:

Proposition p *involves* proposition q, if and only if, in understanding p, we understand q.

For example, the proposition

J: Jones says that it is raining,

involves the proposition

R: It is raining.

For in understanding J, we understand R. Again, the proposition

B: Jones is a bachelor,

involves the proposition

M: Jones is a man.

For B is the proposition that Jones is an unmarried man, so in understanding B, we understand M. Again, the proposition

D: Jones is either a fool or a knave,

involves both the proposition

F: Jones is a fool,

and the proposition

N: Jones is a knave.

Similarly, F and N are both involved in the proposition that Jones is a fool *and* a knave. Finally, every proposition involves itself. For if p is any proposition, an understanding of p is obviously an understanding of p.

Using the notion of involvement, we can see why POI1 is false. For suppose that, for knowledge K and K-hypotheses p and q, K contains p without containing q. In that case, p has maximal K-probability, whereas q does not, hence p and q are not K-equiprobable. Nonetheless, it may still be that K contains no reason either for the following proposition:

(P): $P(p/K) *> P(q/K)$,

or for the following proposition:

(Q): $P(q/K) *> P(p/K)$.

For if K contains such a reason, whether for (P) or for (Q), then K contains a proposition which involves both of the hypotheses p and q. And it may be that K contains no such proposition. Suppose for example that q is not itself a member of K, but is merely involved in some such proposition as this:

(T) There is moderately reliable testimony that q,

where (T) is a member of K. Since (T) involves q, and K contains (T), there is an element of K which involves q. And since every proposition involves itself, and p is an element of K, there is an element of K which involves p. But there need be no item of K involving both the hypothesis p and the hypothesis q. So it may be that no item of K is either a reason for (P), or a reason for (Q). Thus POI1 is not a correct sufficient condition for equiprobability.

However, POI1 is not the only possible interpretation of the Principle of Indifference. There is also the following interpretation:

POI2: K-hypotheses p and q are K-equiprobable if p is neither more nor less K-probable than q.

POI2 is true if and only if the following proposition C is true:

C: All probabilities are comparable.

Keynes is the best known advocate of the doctrine that there are incomparable probabilities. I shall prove this doctrine (and hence refute both C and POI2) by adapting an example from section 4.6 of his (1921), although it does not seem that this was the use for which he intended this particular example.

Define the *specific volume* of a substance as the density of water, divided by the density of that substance, and define the *specific density* of a substance as the density of that substance, divided by the density of water. (Thus specific volume and specific density are reciprocal quantities: their product is always 1.) Let our knowledge be K, and suppose there is a substance S regarding which the following six propositions are K-hypotheses:

H1: The specific volume of S lies between 1 and 2,
H2: The specific volume of S lies between 2 and 3,
H3: The specific density of S lies between 1/3 and 2/3,
H4: The specific density of S lies between 2/3 and 1,
H1′: The specific density of S lies between 1/2 and 1,
H2′: The specific density of S lies between 1/3 and 1/2.

Since specific volume and specific density are reciprocal quantities, we have:

S1: H1 if and only if H1′.

Since H1′ is logically weaker than H4, we have:

S2: If H4 is true, then H1′ is true.

From S1 and S2, it follows that

S3: If H4 is true, then H1 is true.

However, it does not follow that

S4: If H1 is true, then H4 is true.

Similarly, from the fact that H2 is logically stronger than H3, it follows that

S5: If H2 is true, then H3 is true,

but it does not follow that

S6: If H3 is true, then H2 is true.

Suppose we know that either H1 or H2 is true, and also that either H3 or H4 is true. However, we have no more exact information as to the whereabouts of the specific volume or the specific density. Thus our knowledge does not favour H1 over H2 or vice-versa; similarly, it does not favour H3 over H4 or vice-versa. Thus both (a) and (b), below, are true:

(a) $-[P(H1/K) *> P(H2/K)]$ & $-[P(H2/K) *> P(H1/K)]$,
(b) $-[P(H3/K) *> P(H4/K)]$ & $-[P(H4/K) *> P(H3/K)]$.

In addition, suppose K to be such that both the following propositions are true:

(1) $P(H1/K) *> P(H4/K)$,
(2) $P(H2/K) *< P(H3/K)$.

[For present purposes it does not matter exactly how the structure of K might guarantee the truth of the propositions (1) and (2). It is sufficient that they could both be true while (a) and (b) were true. We might suggest that (1) is true if K contains the proposition that H1 is logically weaker than H4. An alternative requirement would be that K contain the following proposition P:

P: K contains S3, but K does not contain S4.

But we do not need to decide which (if either) of these suggestions is correct. *Some* insight as regards H1 and H4 will guarantee that if K contains that insight, then $P(H1/K) *> P(H4/K)$. And whatever the insight may be, it does not make P(H1/K) either greater or less than P(H2/K). Similarly, it does not make P(H3/K) either greater or less than P(H4/K). So the insight is compatible with the joint truth of (a) and (b). And whatever insight ensures the truth of (1), a similar insight will ensure the truth of (2). For example, if the first insight is seeing that H1 is logically weaker than H4, then the second insight is seeing that H2 is logically stronger than H3. If the first insight is seeing that, of S3 and S4, K contains only S3, the second insight is seeing that, of S5 and S6, K contains only S5. Just as, for present purposes, it does not matter exactly what the first insight is, so too it does not matter exactly what the second insight is. In order to show the existence of incomparable probabilities, it is sufficient that a knowledge set K could include both insights, while (a) and (b) remain true. Thus, we need only to suppose that there is a knowledge set K such that (a), (b), (1) and (2) are all true.]

We are arguing against the following doctrine:

C: All probabilities are comparable,

and the proof proceeds by *reductio*. Assume, therefore, that C is true. Then given the truth of (a), we have:

(3) $P(H1/K) * = P(H2/K)$.

All would agree that probability is transitive in the sense that the following proposition holds:
T: If for knowledge set K and K-hypotheses p, q, and r, we have:

$P(p/K) * \geqslant P(q/K)$,

and

$P(q/K) * > P(r/K)$,

then

$P(p/K) * > P(r/K)$.

(1), T, and (3) jointly entail that

(4): $P(H2/K) * > P(H4/K)$.

(4), T and (2) jointly entail that

(5): $P(H3/K) * > P(H4/K)$,

and (5) contradicts the first conjunct of (b).

Thus the doctrine that all probabilities are comparable has been refuted by describing a knowledge set K, and K-hypotheses H1 and H2, such that although H1 is neither more nor less K-probable than H2, a contradiction arises from the assumption that H1 and H2 are K-equiprobable. So P(H1/K) and P(H2/K) are incomparable. A similar argument shows that P(H3/K) and P(H4/K) too are incomparable.

4. *Equiprobability: sufficient conditions*

Thus the Principle of Indifference is false on its second interpretation, as on its first interpretation. Neither interpretation provides a correct set of conditions for equiprobability. However, I suggest that such a set can be formulated by reconsidering Keynes' example. We supposed initially that our knowledge K included the information that H1 or H2 is true. Call this information (i). Intuitively, there is a clear sense in which (i) is symmetrical between the hypotheses H1 and H2.

[A syntactical criterion for this symmetry is as follows. Define a *sub-sentence* S′ of a sentence S, as any sentence which occurs within S. (We allow that every sentence is a sub-sentence of itself.) Call a sentence S a *basic* expression of a proposition P, if and only if, within S, P is the only proposition expressed. (For example, the proposition P that the sun is shining, is expressed within the sentence

J: Jones says that the sun is shining,

but nonetheless J is not a basic expression of P, unlike J's sub-sentence "the sun is shining".) If S is a basic expression of a proposition P, we may say that P is *P-basically* expressed by S, or that S is *P-basic*. Suppose now that proposition P is P-basically expressed by a sentence S_P, within which proposition H and proposition h are each expressed at least once. Suppose too that every sub-sentence of S_P in which H is expressed has a sub-sentence S^i_H which is H-basic. Suppose that there are exactly n of the S^i_H. Similarly, suppose that every sub-sentence of S_P in which h is expressed has a sub-sentence S^i_h which is h-basic, and that there are exactly n of the S^i_h. Suppose finally that within S_P, if we interchange all the S^i_H with all the S^i_h, then regardless of exactly which sentences are interchanged with which, the interchanges will result in a sentence which still expresses the same proposition P. Then P is symmetrical between H and h.]

So if (i) exhausts that part of our knowledge which is relevant either to P(H1) or to P(H2), these probabilities are in fact equal. Admittedly, given a knowledge set K as described in the above discussion of Keynes' example, P(H1) and P(H2) are incomparable and hence not equal. However, this is because information (i) does not exhaust that part of K which is relevant to one or both of P(H1) and P(H2). For we assumed our K to contain some information – call it (i′) – whose presence in K ensured that P(H1/K) $*>$ P(H4/K). And whatever (i′) may be, a trivial consequence of the way it has been defined is that (i′) is relevant to the K-probability of the hypotheses H1 and H4. Hence (i) is not our only information which is relevant to P(H1). This fact would not be fatal to the equiprobability of H1 and H2, if our knowledge included an item of information symmetrically relevant to P(H2), i.e. relevant to P(H2) in the same way that (i′) is relevant to P(H1). (Below I give necessary and sufficient conditions for applying this intuitive symmetry concept.) For example, assume that (i′) is the following proposition:

W: H1 is logically weaker than H4.

[This assumption is made only for the sake of definiteness, and is not essential to what follows. For example, similar points will apply, *mutatis mutandis*, if (i′) is in fact the following proposition:

P: K contains S3, but K does not contain S4.]

Then if in addition to containing W, our knowledge were to contain a proposition which is symmetrically relevant to P(H2):

W′: H2 is logically weaker than H4,

our relevant knowledge would be exactly symmetrical as regards H1 and H2, so H1 and H2 would be equiprobable.

[I suggest the following conditions for applying these intuitive symmetry concepts: A set S of propositions is symmetrical between two propositions h and h′, if and only if some element of S involves at least one of these propositions, and each element e of S satisfies some one or other of the following conditions:

(a) Neither of the propositions h, h′ is involved in e,
(b) e involves both h and h′, and is symmetrical between them,
(c) e involves just one of h and h′, and there is an element e′ of S involving just the other hypothesis, such that e & e′ is symmetrical between h and h′.]

If e is relevant to P(h), and S contains e′ relevant to P(h′), then e is relevant to h in the same way that e′ is relevant to h′, if and only if e & e′ is symmetrical between h and h′.]

However, our knowledge does not contain W′. Indeed, W′ *could* not be an item of our knowledge, since W′ is false. What *is* true is the following proposition:

S: H2 is logically stronger than H3;

so let us suppose that S is an element of our K. Even so, it does not follow that H1 and H2 are K-equiprobable. For S is not relevant to P(H2) in the same way that W is relevant to P(H1). In the first place, W states that

H1 is logically *weaker* than a particular proposition, whereas S states that H2 is logically *stronger* than a particular proposition. Secondly, the particular propositions involved are different in each case. In the case of W, the proposition is H4, whereas in the case of S, the proposition is H3.

Thus if the probabilities P(H1) and P(H2) are incomparable, this is presumably because we have information relevant to one probability, without having information which is symmetrically relevant to the other. W is relevant to P(H1), and S is relevant to P(H2), but W is not relevant to P(H1) in the same way that S is relevant to P(H2). In this, the propositions W and S are unlike the following propositions:

(I) We do not know that H1 is false,
(II) We do not know that H2 is false.

For the way in which (I) is relevant to P(H1) is the same as the way in which (II) is relevant to P(H2). So if our knowledge contains (I) and (II), the hypotheses H1 and H2 may still be equiprobable. Any lack of equiprobability between the two hypotheses must be caused by the existence of information relevant to the probability of one hypothesis, such that there is no information which is symmetrically relevant to the probability of the other hypothesis. This suggests the following sufficient condition for equiprobability:

E: Given knowledge K and K-hypotheses p and q, P(p/K) * = P(q/K) if those elements of K which are relevant either to P(p) or to P(q), form a set which is symmetrical between p and q.

Note that there are two ways in which condition E might fail to be satisfied. Firstly, K might be asymmetrical as regards p and q in such a way as to make one hypothesis more probable than the other. Alternatively, the asymmetry might be such as to make P(p) and P(q) incomparable.

Let us consider an example of the first type of asymmetry. Suppose that our knowledge includes the following elements:

p1: There is fairly reliable testimony that p,
q1: There is very reliable testimony that q.

Here, p1 is relevant to P(p), and q1 is relevant to P(q). But condition E is not satisfied, because the relationship between p1 and p is not the same as the relationship between q1 and q. For in p1 we know only of *fairly* reliable testimony that p, whereas in q1 we know of *very* reliable testimony that q. Hence P(q) * > P(p).

Now let us consider an example of the second type of asymmetry. Let p1 (above) be an item of our knowledge, but suppose that instead of q1, our knowledge contains the following item:

q2: Ball B is in an urn in which 73 % of the balls are black, and proposition q is that ball B is black.

Suppose also that the word "fairly", in p1, is to be understood in such a way that p1 does not support p either more or less strongly than q2 supports q. Does it follow that p1 supports p to the *same* extent that q2 supports q? No; we might plausibly reply, for the *kind* of support provided is different in each case. So we might suggest that the extent to which p1 supports p is neither greater than, not equal to, nor less than the extent to which q2 supports q. Suppose that this suggestion is correct. Suppose also that p1 contains all our information which is relevant to P(p), and that q2 contains all our information which is relevant to P(q). Then we have:

INC: P(p) and P(q) are incomparable.

The possibility that INC is true does not seem to be excluded by E. For p1 and q2 are not symmetrical, so E does not guarantee that the hypotheses p and q are equally probable. But neither does E guarantee that one of these hypotheses is more probable than the other. Hence E allows the possibility that P(p) and P(q) are incomparable. It seems an advantage of E that it allows for this possibility, since, as we have seen, some probabilities are undoubtedly incomparable.

Campion College
99 Studley Park Rd
Kew 3101
Victoria
AUSTRALIE

Dr. David WATT

REFERENCES

Carnap, R. (1950) *Logical Foundations of Probability*. Routledge and Kegan Paul, London.

Cohen, L.J. (1977) *The Probable and the Provable*. Clarendon Press, Oxford.

Jeffreys, H. (1922) Review of Keynes' *Treatise on Probability*. *Nature*, Feb. 2, 132-133.

Jeffreys, H. (1967) *Theory of Probability*. (Corrected impression of the 3rd edition, Clarendon Press, Oxford 1961.)

Keynes, J. (1921) *A Treatise on Probability*. Macmillan, London.

Kyburg, H. (1970) "Conjunctivitis", in *Induction, Acceptance, and Rational Belief*, ed. Swain, M. Reidel, Dordrecht.

Stalnaker, R. (1984) *Inquiry*. MIT Press, London, and Cambridge Massachusetts.