Extending the Antilogism*

Ru Michael SABRE, Ph.D

Elizabeth Ladd-Franklin proved the following theorem: If S is a valid categorical syllogism and S′ is the set of three categorical sentences comprised of the two premises and the negation of the conclusion of the syllogism, then

Ladd-Franklin Theorem: S is a valid categorical syllogism if and only if S′ is an antilogism.

She defined antilogism as follows:

Antilogism =df A set of three categorical sentences such that if any two are true, then the third is false, or the negation of the third is true. Eaton (1931), pp. 132-140 for discussion.

Now consider the following construction. Let S′ be an antilogism and let S′′ be the following:

S′′ =df Replace the negated conclusion of S in S′ with the premises of a valid syllogism, the conclusion of which is the negated conclusion, and the middle term of which is different from that of S.

The construction is a tetrad of sentences and the claim that is specified and substantiated is that the tetrad has antilogistic properties. An example of an S′′ construction is as follows:

S	S′	S′′		S′′
All B are C.	All B are C.	All B are C.		All B are C.
Some A are B.	Some A are B.	Some A are B.	or	All A are D.
Some A are C.	No A are C.	No D are C.		Some A are B.
	No D are C.	All A are D.		No D are C.
	All A are D.			
	No A are C.			

* Henry W. Johnstone, Jr., Professor Emerituys of Philosophy at the Pennsylvania State University called my attention to the concept of the antilogism, and has made invaluable remarks toward the preparation of this article.

With regard to the antilogistic properties of the tetrad, the definition of antilogism is modified as follows:

Antilogistic Tetrad =df A set of four categorical sentences such that if any three are taken as true, the remaining categorical sentence is false, or its negation is true.

The proof of the following theorem establishes the antilogistic tetrad property of S'':

The Antilogistic Tetrad Theorem: S'' is an antilogistic tetrad.

Proof:
1. S' is such that the negated conclusion of S is inconsistent with the premises of S.
2. S'' replaces the negated conclusion with the premises of a valid argument the conclusion of which is that negated conclusion.
3. The two sets of premises are inconsistent because their conclusions are inconsistent.
4. Any subset of three of the four sentences of S'' includes one set of original premises and these are consistent.
5. The one premise from the other argument includes a term not found in the consistent premises.
6. Thus, no contradiction can arise, or the three sentences are consistent.
7. But, the four sentences are inconsistent.
8. S'' is an antilogistic tetrad, for if any three sentences are taken as true, the remaining one is false, or its negation true.

QED

The following theorem is proven by constructing the number of possible S'' constructions (which will be referred to as Antilogistic Tetrads in this paper).

Theorem: There are exactly five different S'' sentences sets (Antilogistic Tetrads).

Proof:
1. Figure 1 contains the 15 valid syllogisms and their reduction to 8 equivalent forms, organized by A, E, I, and O conclusions, with M and N as middle terms of syllogisms with contradictory conclusions.
2. Figure 2 contains two matrices of arguments with contradictory con-

clusions, the cells of each matrix containing the S′′ sentence sets: the number of which is 7.

3. At the bottom of Figure 2 the S′′ sentence sets are standardized by relabelling the terms of the two affimative four-term sentences.
4. The 7 S′′ are reduced to 5 standardized sentence sets.

QED

A-sentence	O-sentence	E-sentence	I-sentence
Barbara 1. All M are P. All S are M. All S are P	Ferio 2. No N are P. Some S are N. Some S are not P.	Celarent 5. No M are P. All S are M. No S are P.	Darii 7. All N are P. Some S are N. Some S are P.
	Boroco 3. All P are N. Some S are not N. Some S are not P.	Ceasare 5. No P are M. All S are M. No S are P.	Disanis 8. Some N are P. All N are S. Some S are P.
	Bokardo 4. Some N are not P. All N are S. Some S are not P.	Camestres 6. All P are M. No S are M. No S are P.	Datisi 7. All N are P. Some N are S. Some S are P.
	Ferison 2. No N are P. Some N are S. Some S are not P.	Camenes 6. All P are M. No M are S. No S are P.	Disaris 8. Some P are N. All N are P. Some S are P.
	Fresison 2. No P are N. Some N are S. Some S are not P.		
	Festino 2. No P are N. Some S are N. Some S are not P.		

Figure 1: The 15 valid syllogisms and their reduction to 8 equivalent forms, classified by A, B, I and O conclusions, M and N as middle terms. The syllogisms with the same number are equivalent.
Eaton (1931), pp. 95-116.

O-sentence conclusion

A-sentence conclusion

2.	3.	4.
No N are P.	All P are N	Some N are not P.
Some S are N.	Some S are not P.	All N are S.
Some S are not P.	Some S are not P.	Some S are not P.

1.	I.	II.	III.
All M are P.	All M are P.	All S are M.	All M are P.
All S are M.	All S are N.	All P are N.	All N are S.
All S are P.	All S are M.	All M are P.	All S are M.
	No N are P.	Some S are not P.	Some N are not P.

I-sentence conclusion

E-sentence conclusion

7.	8.
All N are P	Some N are P.
Some S are N.	All N are S.
Some S are P.	Some S are P.

5.	IV.	V.
No M are P.	All S are M.	All S are M.
All S are M.	All N are P.	Some N are P.
No S are P.	Some S are N.	All N are S.
	No M are P.	No M are P.

6.	VI.	VII.
All P are M.	All P are M.	All P are M.
No S are M.	Some S are N.	All N are S.
No S are P.	All N are P.	Some N are P.
	No S are M.	No S are M.

Standardized with A, B, C, and D as the four terms:

I.	II.	III.	IV.
All A are B.	All A are B.	All A are B.	All A are B.
Some C are D.	All C are D.	All C are D.	All C are D.
All C are A.	All B are C.	All D are A.	Some A are C.
No D are B.	Some A are not D.	Some C are not B.	No B are D.

V.	VI.	VII.
All A are B.	All A are B.	All A are B.
Some C are D.	Some C are D.	All C are D.
All C are A.	All D are A.	Some C are A.
No B are D.	No C are B.	No B are D.

Note: I. is equivalent to V. and IV. to VII and the five standardized forms are underlined

Figure 2: Construction of the 7 Antilogistic Tetrads I through VII and the standardized 5 Antilogistic Tetrads.

Two or more antilogistic tetrads can be related together by the following construction.

S′′ ′′ =df Given S′′ and its two affirmative sentences that have no terms in common, take one of those two affirmative sentences, s, and another affirmative sentence, t, with no terms in common with the original two sentences; and using one of the 5 antilogistic tetrads produce a second antilogistic tetrad with s and t as its two affirmative sentences with no terms in common.

As an example, consider the following:

S′′		S′′ ′′
All A are B.	All A are B. (s)	All A are B.
All C are D.	All E are F. (t)	All E are F.
Some A are C.		All F are A.
No B are D.		Some E are not B.

Note: S′′ ′′ is formed here using form III from Figure 2.

The definition of S′′ ′′ provides the basis of a recursive definition of what is termed a Plurality Schema:

Plurality Schema (recursively defined)
1. An antilogistic tetrad is a plurality schema.
2. If A is a plurality Schema and B the result of S′′ ′′ performed on A, then A and B are a plurality Schema.
3. Sentences form a plurality schema only if they conform to clauses one and two above.

A useful way from the point of view of computer applications and a perspicuous way of looking at a Plurality Schema which could be composed of a large number of antilogistic tetrads, is in terms of its two affirmative sentences with no terms in common. Any antilogistic tetrad can be seen as three distinct sets of categorical sentences one of which, Set 1, is the pair of affirmative four-different-term affirmative sentences; the second set, Set 2 consists of the affirmative sentence which relates terms of the Set 1 pair; and Set 3 has as its member the negative sentence which denies a relationship between the related Set 1 pair of sentences. The Set 2 and Set 3 sentences would conform to the conditions imposed by the five antilogistic tetrads of Figure 2. The Plurality Schema would be composed of these three sets of sentences; with its antilogistic tetrads' sentences being sorted into those three sets. Using the above example of a Plurality Schema, that schema in terms of the three Sets would be as follows:

Set 1:	All A are B.	All A are B.
	All C are D.	All E are F.
Set 2:	Some A are C.	
	All F are A.	
Set 3:	No B are D.	
	Some E are not B.	

The Plurality Schema defined in terms of the three sets of sentences discussed above is shown to have the property of being a plurality antilogism, plurality antilogism being defined as follows:

Plurality Antilogism =df Three sets of categorical sentences such that if any two of the sets' members are true, then any subset of elements of the remaining set, conjoined with the elements of the other two sets produce as many contradictions as there are elements from the remaining sets's subset; or their negation(s) must be true.

For example, using the above Plurality Schema: if Set 1 and Set 3 are true, and "Some A are C." is taken from Set 2 then; if the Plurality Schema is a Plurality Antilogism, then "Some A are C." contradicts Set 1 and Set 2 sentences.

The Plurality Schema is non-recursively defined below and a theorem proven which shows that this redefinition is a Plurality Antilogism. This more perspicuous definition, in terms of large sets of antilogistic tetrads, uses the notion of Sets 1, 2, and 3 discussed informally above, and insures that the antilogistic tetrads which comprise the Plurality Schema are related as the recursive definition indicates. There are two aspects of the recursive definition to be preserved: (1) *all* of the $n \geqq 2$ affirmative sentences are related together by pairs, and (2) each pair (each element of Set 1) has, in effect, one Set 2 sentence and one Set 3 sentence so that an antilogistic tetrad is formed conforming to one of the 5 antilogistic tetrads in Figure 2.

Plurality Schema =df A set of categorical sentences divided into three sets of categorical sentences called Set 1, Set 2 and Set 3.
These three sets are constructed to describe the relationships between $n \geqq 2$ affirmative categorical sentences which have no terms in common.

Set 1: The set of pairs of $n \geqq 2$ affirmative categorical sentences, the pairs produced by the Set 2 rules.

Set 2: The set of affirmative sentences which relate all of the $n \geqq 2$ together by pairs, any pair sharing only one term each via an affirmative sentence.

The affirmative Set 2 sentences are as follows:

If an I-sentence, then the subject terms of the Set 1 pair share an individual in common.

If an A-sentence, then its predicate term is a subject term from its Set 1 pair, and its subject term is any undistributed term of the other Set 1 pair sentence.

Set 3: The set of negative sentences which one-for-one match the Set 2 affirmative sentences and deny a relationship between the non-Set 2 terms of the Set 1 pairs.
The negative sentences are as follows:
If an O-sentence, then its predicate is a predicate term of a Set 1 sentence pair, and its subject the subject of the Set 1 pairs other sentence.
If an E-sentence, then the non-Set 2 terms are terms for the E-sentence.

All of the above such that for each tetrad produced by a Set 2 affirmative sentence, exactly one sentence is particular.

The following lemma is used in the theorem about the Plurality Schema:

Lemma: P is a one element (n=2) Plurality Schema if and only if P is a S′′.

The proof is by the construction of all the possible Plurality Schemata where Set 1 has one element (n=2) and showing a one to one correspondance with Figure 2 S′′.

Proof:

1. By the definition of a Plurality Schema there is only one particular sentence for any tetrad produced by the definition.
   Thus Table 1 is organized by the Set which contains a particular sentence. The Set criteria are then applied.
2. The standardized S′′ forms of Figure 2 are in a one to one correspondance with the Plurality Schemata as displayed in Table 1. The arabic numbers in the left hand column correspond one to each roman numeraled S′′ from Figure 2. The former are summarized at the bottom of Table 1.

QED

Table 1: The 5 possible four-term Plurality Schemata constructed by following its definition, these compared with the Figure 2 Antilogistic Tetrads.

	Set 1 Affirmative	Set 2 Affirmative	Set 3 Negative
1.	A B universal	D A universal	C B *particular* III.
2.	C D universal	B C universal	A D *particular* II.
3.	A B universal C D universal	A C *particular* IV.	B D universal
4.	A B universal C D *particular* I.	C A universal	B D universal
5.	A B *particular* VI. C D universal	D A universal	C B universal

Note: The roman numerals refer to those in Figure 2, the arabic numerals to the five (n=2) Plurality Schemata which are explicitly shown below.

1. All A are B.
All C are D.
All D are A.
Some C are not B.

2. All A are B.
All C are D.
All B are C.
Some A are not D.

3. All A are B.
All C are D.
Some A are C.
No B are D.

4. All A are B.
Some C are D.
All C are A.
No B are D.

5. All A are B.
Some C are D.
All D are A.
No C are B.

The Antilogistic Plurality Theorem: If P is a Plurality Schema then P is a Plurality Antilogism.

The Proof is by induction on the number of elements in Set 1, which starting at one element means that two affirmative sentences are involved. In effect, the proof is by induction on n $\geqq$ 2 affirmative categorical sentences.*

* Recall, with n=2 affirmative sentences with no terms in commun, Set 1 has one element, a pair of affermative sentences. When n=3, Set 1 via S'' has two elements, i.e. two pairs of categorical sentences sharing one sentence in commun.

Basis: Set 1 has one element (n=2).

If P is a Plurality Schema with one element in Set 1 (n=2), then P is a Plurality Antilogism.

Proof:

1. By the lemma, P is a one element (n=2) Plurality Schema is and only if P is a S′′.
2. Thus, P is inconsistent.
3. Any two subsets of P are consistent:
   a. Set 1 and Set 2 are affirmative;
   b. Set 2 and Set 3 have no term in common; and
   c. Set 1 and Set 3 affirm and deny different terms.
4. Thus, the remaining Set produces the contradiction in each case.
5. The negation of the remaining Set is consistent with the other two Sets.
6. Therefore, P is a Plurality Antilogism.

QED

Induction Case:
The hypothesis of induction is a Plurality Schema with n−1 elements in Set 1 (n affirmative categorical sentences with no terms in common) which is a Plurality Antilogism.
The Induction case is adding one affirmative sentence to the n affirmative sentences, n+1, which makes, in effect, n elements in Set 1; and showing that the Plurality Schema produced is a Plurality Antilogism.

Proof:

1. The sentence added is either an A-sentence of an I-sentence.
2. If an A-sentence, a Set 2 sentence is added according to its rules. (Thus, it is paired off with one other sentence from on of the other Set 1 pairs. That pair, selected by Set 2 criteria also conforms to the Set 3 criteria.) The same process applies for the I-sentence case.
3. Any new tetrad constructed is a basis case tetrad.
4. Therefore, since the hypothesis of induction is a Plurality Antilogism, and the new tetrad of the induction case is an antilogistic tetrad, the induction case is proven.

QED

BIBLIOGRAPHY

EATON, Ralph M. *General Logic*. New York: Charles Scribner's Sons, 1931.