A SIMPLE SOLUTION TO THE "DEEPEST" PARADOX IN DEONTIC LOGIC

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ABSTRACT

James Forrester has pointed out that there is a variant of the Good Samaritan Paradox, involving aspectual actions, which seems to be harder to cope with than the other ones. In this paper we show that in the formal system PD_eL one can adequately deal with this paradox in a straigthforward manner, once the necessary formal tools have been added.

1. *Introduction*

During his visit to Amsterdam in May 1985 Hector-Neri Castañeda gave a talk about what he called "the deepest paradox of deontic logic", which was discovered recently by James Forrester. He argued that while the standard version of the Good Samaritan Paradox can be resolved fairly easily, the Forrester variant of the paradox was much harder to deal with.

The standard version of the Good Samaritan is the following paradoxical argument:

(0) Arthur is forbidden to kill anyone.

(1) Arthur ought to bandage Jack whom he will kill a week hence.

(2) Arthur bandages Jack whom he will kill a week hence implies that Arthur will kill Jack a week hence.

(P) If X's doing A implies X's doing B than it is also the case that "X ought to do A implies that X ought to do B".

From (1),(2),(P) we can infer the absurdity

(3) Arthur ought to kill Jack,

which is contradictory to (0).

In Castañeda's view the solution to this paradox is that we must discriminate between propositions like "Arthur will kill Jack a week hence" and practitions (or actions, as we prefer to call them) such as the act of Arthur killing Jack. The principle (P) is only concerned with actions, so (2) cannot be used as its premiss, thus breaking down the argument. (Cf.[C1].)

However, Forrester's Paradox has the following form:

(4) Arthur is forbidden to murder someone.

(5) Still, if Arthur murders someone, he will have to do it gently.

This is an abbreviation of

(5′) Still, if Arthur murders someone, he will have to murder him gently.

Together with

(6) Arthur murders someone gently implies that Arthur murders someone

and the principle (P), which *does* apply in *this* case, we obtain

(7) Arthur ought to murder someone,

again an absurd duty, conflicting with (4).

Obviously in this case Castañeda's solution of the Good Samaritan by discrimination between actions and propositions does not help, since in both (5′) and (6) we are concerned with actions, and no propositions like in the standard Good Samaritan enter the argument. Castañeda tries to solve this paradox by a refined manner of discrimination involving *aspects* cq. adverbs like "gently" in this particular case. (See [C2]).

However, I do not think this is the issue here and I have discovered that in [SA] this opinion on Castañeda's solution is shared. In [SA] a much more elegant solution is given by means of only quantifiers and scope distinctions. But we shall see in the present paper that even these are not necessary to resolve the paradox, since we can stay within propositional deontic logic when dealing with it.

I agree with Castañeda that the Forrester Paradox is somewhat trickier than other, similar paradoxes. In fact, we shall see that also in the original version of the formal deontic system PD_eL we cannot deal with it adequately. However, with a slight extension of PD_eL the paradox becomes almost trivial to cope with.

But first we shall recapitulate the system PD_eL from [M1,M2], which is a variant of *PDL* (Propositional Dynamic Logic; cf.[H]), rendered suitable to express deontic propositions.

2. *The System PD_eL with Its Semantics*

The language of PD_eL (Propositional Deontic Logic) consists of *actions* and *assertions.* The actions α are either atomic $a_1, a_2, a_3, \ldots$ or a negated action $\bar{\alpha}$, a sequential composition $\alpha_1;\alpha_2$ of actions α_1 and α_2, a choice $\alpha_1 \cup \alpha_2$ between α_1 and α_2, or a joint or simultaneous performance $\alpha_1 \& \alpha_2$ of α_1 and α_2. The assertions ϕ are either the propositional variable V, with liability to sanction as intended meaning, logical compositions $\neg \phi$, $\phi_1 \vee \phi_2$, $\phi_1 \wedge \phi_2$, or expressions of the form $[\alpha]\phi$ with intended meaning that ϕ holds after the performance of α. Furthermore, we use as abbreviations $<\alpha>\phi \equiv \neg [\alpha] \neg \phi$, $F\alpha \equiv [\alpha] V$, $O\alpha \equiv F\bar{\alpha}$ and $P\alpha \equiv \neg F\alpha$.

The semantics of PD_eL expressions is based upon state transformations due to the actions α: given a state σ, $[\![\alpha]\!]$ (σ) yields the collection $W_{\alpha,\sigma}$ of states (worlds) which one gets into after having performed the action α entirely. We recapitulate the main clauses in an informal manner (For a rigorous approach see [M2] and the appendix of [M1]):

- $[\![a]\!]$ (σ) yields the set of states one can get into when performing the atomic action a. This has to be given by some fixed function $a'(\sigma)$.
- $[\![\alpha_1;\alpha_2]\!]$ (σ) = $[\![\alpha_2]\!]$ $([\![\alpha_1]\!]$ $(\sigma))$: in the states that are yielded by the performance of α_1 in state σ, we perform α_2.
- $[\![\alpha_1 \cup \alpha_2]\!]$ (σ) = $[\![\alpha_1]\!]$ (σ) $\cup$ $[\![\alpha_2]\!]$ (σ): the set of states one can reach by performing a choice of actions is (more or less) * the union of the sets of states one can reach by performing those actions.
- $[\![\alpha_1 \& \alpha_2]\!]$ (σ) = $[\![\alpha_1]\!]$ (σ) $\cap$ $[\![\alpha_2]\!]$ (σ), the set of states one gets into by both α_1 and α_2, i.e. (more or less) * the intersection of the sets of states one can reach by performing the actions α_1 and α_2 on themselves.
- $[\![\bar{\alpha}]\!]$ (σ) = $[\![\alpha]\!]^{\sim}$ (σ), where $^{\sim}$ is some complementation operator defined on the domain of denotations (see [M1]).

* Actually, this is a slight oversimplification (cf. [M1,M2]), but it is sufficient for our present purposes.

In this paper we will not need an understanding of the semantics of actions beyond what has been given above. The semantics of assertions is as usual for the logical operators; the meaning of $[\alpha]\phi$ is given by

$$[\alpha]\phi \text{ holds in state } \sigma \text{ iff } \forall\, \sigma' \in [\![\alpha]\!]\,(\sigma): \phi \text{ holds in } \sigma',$$

i.e. ϕ holds in every state that one gets into after the performance of α.

For PD_eL expressions we can give a formal system that is sound with respect to the (informally) given semantics (see [M1]): ($\phi_1 \subset \phi_2$ stands for $\phi_2 \supset \phi_1$.)

Axioms

(T)	All tautologies of propositional calculus
(NP)	$V \supset [\alpha] V$
($\Box \supset$)	$[\alpha]\,(\phi_1 \supset \phi_2) \supset ([\alpha]\phi_1 \supset [\alpha]\phi_2)$
(;)	$[\alpha_1;\alpha_2]\phi \equiv [\alpha_1]\,([\alpha_2]\phi)$
($\cup$)	$[\alpha_1 \cup \alpha_2]\phi \equiv [\alpha_1]\phi \wedge [\alpha_2]\phi$
(&)	$[\alpha_1 \& \alpha_2]\phi \subset [\alpha_1]\phi \vee [\alpha_2]\phi$
($\bar{\bar{\ }}$)	$[\bar{\bar{\alpha}}]\phi \equiv [\alpha]\phi$
($\bar{;}$)	$[\overline{\alpha_1;\alpha_2}]\phi \equiv [\overline{\alpha_1}]\phi \wedge [\alpha_1][\overline{\alpha_2}]\phi$
($\bar{\cup}$)	$[\overline{\alpha_1 \cup \alpha_2}]\phi \subset [\overline{\alpha_1}]\phi \vee [\overline{\alpha_2}]\phi$
($\bar{\&}$)	$[\overline{\alpha_1 \& \alpha_2}]\phi \equiv [\overline{\alpha_1}]\phi \wedge [\overline{\alpha_2}]\phi$

Rules

(MP)

$$\frac{\phi \supset \psi\ ,\ \phi}{\psi}$$

(N)

$$\frac{\phi}{[\alpha]\phi}$$

Remarks

(i) Conditional actions such as in [M1] have been deleted, since they play no role in this paper.

(ii) For convenience, we have included the "no pardon" principle (NP) in order to keep axioms ($\cup$), (&), ($\bar{\cup}$) and ($\bar{\&}$) in this simple unconditional form (cf. [M2]).

(iii) In [M2] it is shown that this system can be rendered complete by adding a special predicate that keeps track of the actual performance of atomic action.

3. *Forrester's Paradox in PD_eL*

How are we to translate Forrester's Paradox (4),(5) in PD_eL? First let us see how we treated a similar contrary-to-duty imperative (cf. [M1]):

(8) You are forbidden to go.

(9) Yet, if you go, you have to close the door afterwards.

This is translated without any trouble into PD_eL by:

(10) Fg

(11) $[g]Oc$

where g stands for the action "to go" and c for the action "to close the door".

However, a similar attempt of translating (4),(5), does not work adequately, taking m for the action "to murder" and g for the action "to murder gently" (Note that $m\&g = g$ in this interpretation.):

(12) Fm

(13) $[m]Og$,

since (13) expresses the statement that *after* you have murdered you ought to be gentle. This is not the intention of (5), where g and m have to occur *simultaneously.*

So we try the following formulation:

(14) Fm

(15) $O(\bar{m} \cup g)$,

where (15) expresses that one ought either not to murder or to be gentle (when murdering). Although this seems to be the right formulation in PD_eL, it is not, because in PD_eL (15) is implied already by (14), which is equivalent to $O\bar{m}$. This is a consequence of the fact that we have in PD_eL the theorem

(R) $O\alpha \supset O(\alpha \cup \beta)$ (see [M1]).

This is the (in)famous Ross' Paradox, but regarding the semantics of PD_eL it is a perfectly sound theorem, meaning that if the doing of not-α leads to punishment, the doing of not-α and not-β leads to punishment as well, since not-α has also been done in this case.

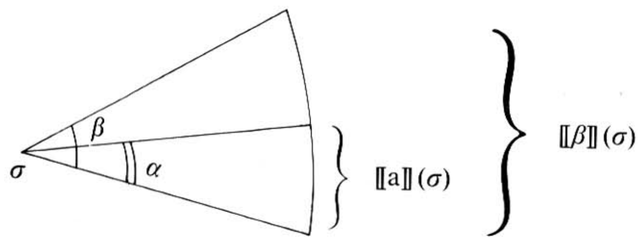
This, by the way, is related to the principle (P), which in PD_eL can be formulated by

$$\text{(P}'\text{)} \qquad \frac{\alpha \blacktriangleright \beta}{O\alpha \supset O\beta}$$

where $\alpha \blacktriangleright \beta$, meaning "the doing of α implies the doing of β" (in an extensional interpretation), has the formal semantics:

$$\forall \sigma : [\![\alpha]\!](\sigma) \subseteqq [\![\beta]\!](\sigma).$$

In a picture:



Now (R) is a special case of the application of (P′), since

$$[\![\alpha]\!](\sigma) \subseteqq [\![\alpha]\!](\sigma) \cup [\![\beta]\!](\sigma) = [\![\alpha \cup \beta]\!](\sigma),$$

so $\alpha \blacktriangleright (\alpha \cup \beta)$ holds.

Furthermore, note the following:
Since $g = g\&m$, we have that

$$[\![g]\!](\sigma) = [\![g\&m]\!](\sigma) = [\![g]\!](\sigma) \cap [\![m]\!](\sigma) \subseteqq [\![m]\!](\sigma) \text{ (for all } \sigma).$$

So $g \blacktriangleright m$. Hence by (P′) we have $Og \supset Om$ as well. But this does not give rise to the same paradoxical argument as in section 1, because here, by (15), we only have $O(\bar{m} \cup g)$ and this does *not* imply Og. Consequently we cannot infer Om, the formal counterpart of the paradoxical conclusion (7) in section 1.

But now back to our original problem. Intuitively we have the feeling that (4),(5) say more than (4) alone, whereas the PD_eL formulas (14),(15) do not express more than (14) on its own. This paradox is caused by the fact that in PD_eL by the definition of $F\alpha$ as $[\alpha]\,V$ only signals the event that doing α leads to some liability to saction. So it only signals violation of some prohibition. It cannot tell whether by doing α perhaps more than one violation of the laws is committed.

So in (15) it signals a violation when $m\&\bar{g}$ has been done. But it already records a violation when m has been done on its own! The additional "crime" of not being gentle when murdering, is not expressed in the PD_eL statement (15).

This gives us the clue of how to do it properly. Apparently it is not sufficient in this case to say that by violation of *some* prohibition a sanction is imposed. We need the expressibility to formulate that when doing $\bar{g}$ – besided doing m – one is liable to an *additional* punishment! We shall see in the next section how easily this is formalised by a slight extension of PD_eL, viz. $PD_eL(n)$.

4. *Forrester's Paradox Resolved: $PD_eL(n)$*

The system $PD_eL(n)$ is the system PD_eL, but instead of *one* propositional variable V, indicating some (state of) liability to sanction, we have n distinct variables $V_1, V_2, \ldots, V_n$, indicating a specific liability to the first to n-th sanction. Furthermore, the abbreviations $F_k\alpha \equiv [\alpha]\, V_k$, $O_k\alpha \equiv F_k\bar{\alpha}$, and $P_k\alpha \equiv \neg F_k\alpha$ are introduced for $k = 1,2,\ldots,n$. Moreover, the axiom (NP) is replaced by (NP_k): $V_k \supset [\alpha]\, V_k$, for $k = 1,2,\ldots,n$. PD_eL is now the special case that $n = 1$, so $PD_eL = PD_eL(1)$.

Now we formalize (4),(5) in $PD_eL(2)$ as follows:

(16) $F_1 m$

(17) $O_2(\bar{m} \cup g)$

Here (16) expresses that murder is forbidden on penalty of sanction No 1, and (17), or equivalently $F_2(\bar{m}\&g)$, states that a not-gently committed murder is forbidden under penalty of sanction No. 2.

An interesting consequence of (16),(17) is the following:
From (16) we obtain $O_1\bar{m}$. By (R) and (MP) we get $O_1(\bar{m} \cup g)$, or equivalently $[m\&\bar{g}]\, V_1$. By (17) we also have $[m\&\bar{g}]\, V_2$. So, since

$$[\alpha]\phi_1 \wedge [\alpha]\phi_2 \equiv [\alpha](\phi_1 \wedge \phi_2)$$

is a theorem of PD_eL (see [M1]) – and so also of $PD_eL(n)$ – we obtain $[m\&\bar{g}](V_1 \wedge V_2)$, meaning that the one who murders in a not-gentle fashion is liable to *both* the sanctions No 1 and 2. Which is a desirable result to have!

5. $PD_eL(n)$ *and the Backward Chisholm Paradox*

In [M1] we have already seen how in PD_eL we could solve the "forward" version of the Chisholm Paradox. The solution of this paradox came down to being able to give a sound treatment of the following two deontic expressions, which we saw already in section 3:

(18) You are forbidden to go.
(19) Still, if you go anyway, you have to close the door afterwards.

In PD_eL this can be expressed in a consistent way like this:

(20) Fg
(21) $[g]Oc$

where g stands for the action of "going" and c for "closing".

From (20),(21) it follows that it holds that $[g](V \wedge Oc)$: after having gone we are liable to punishment, but we are still obliged to do c. This is perfectly sound.

However, in PD_eL we could not yet cope with a "backward" version of (18),(19) like:

(22) You are forbidden to go.
(23) Still, if you go anyway, you will have to open the door first.

By realising that (23) is equivalent to

(24) You are forbidden to not-open-the-door and then go,

we can formulate (22),(23) in PD_eL as:

(25) Fg
(26) $F(\bar{o};g)$.

However, this is not quite good enough, since by PD_eL's rule (N) from (25) we also get

(27) $[\bar{o}]Fg$.

And this expression is equivalent to $[\bar{o}][g]V$, or equivalently $F(\bar{o};g)$, i.e. (26)! So (26) follows from (25), and adds no further information, although it was obviously intended to do so.

In $PD_eL(2)$, however, this problem is resolved by formulating (22),(23) as

(28) $F_1 g$

(29) $F_2(\bar{o};g)$.

Now we can derive from (28),(29) the following:
From (28) we get $[\bar{o}]F_1 g$ (by (N)), i.e. $[\bar{o}][g]V_1$; (29) yields $[\bar{o}][g]V_2$; so by the PD_eL theorem (see [M1])

$$(\Box\wedge)\ [\alpha]\phi_1 \wedge [\alpha]\phi_2 \equiv [\alpha](\phi_1 \wedge \phi_2)$$

we obtain $[\bar{o}][g](V_1 \wedge V_2)$, meaning that if we do not open the door and go, we are guilty of two "crimes": the violation of the prohibition to go and that of not opening the door before going. This is again a desirable statement to have.

6. *Conclusion*

We have seen that by extending the expressiveness of liability to sanctions we can treat Forrester's Paradox in an easy, almost trivial way. The extended system, $PD_eL(2)$, also proved to be sufficiently expressive to solve the "backward" Chisholm Paradox. These paradoxes are often viewed as the most profound paradoxes in deontic logic.

The extension of the number of propositional variables related to sanctions also raises the question whether it is meaningful to put an ordering (e.g. a partial ordering) on them. $V_i \leqq V_j$, for instance, would express that sanction j is more severe than sanction i. Perhaps even $V_j \supset V_i$ can be chosen as ordering. In this case $[\alpha](V_i \wedge V_j) \equiv [\alpha]\ V_j$, since V_j comprises V_i entirely.

The possibility to consider a measure on sanctions induces priorities on prohibitions and obligations, which may be useful to ethicians and jurists who may apply the system.

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