PREDICATE MODIFIERS IN TENSE LOGIC

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1. *Introduction*

Predicate modifiers can be introduced into tense logics in many ways. Our aim is a very modest one: to explain two ways of revising a simple tense logic by adding predicate modifiers to it. Both revisions have an *ad hominem* interest in relation to the founder of tense logic, Arthur Prior. And both can be regarded as exercises in combining a concern to give sentences analyses close to their surface form, with a metaphysical view of time as "tenseless". (1)

The tense logic we start with is like Kripke's (1963) modal logic: it is bivalent, and the quantifier ranges at each time over just the objects that temporally exist at that time)i.e. exist in the sense in which Reagan exists in 1984 and Lincoln does not). In this tense logic, none of a certain set of formulas relating quantifiers and operators – called mixing formulas – is valid. However, Prior believed that some of these mixing formulas are, intuitively, necessary. And this three-valued system Q captured his intuitions: it rendered valid the mixing formulas he found intuitively necessary. We show in Sections 2 and 3 that by adding predicate modifiers to the Kripkean tense logic, we can recover these intuitions within a complete bivalent system. So we provide a *via media* between Kripke and Prior. (As one would expect, this *via media* exists in modal logic as much as in tense logic; but for brevity we shall only discuss the latter version.)

Our second revision (Section 4) aims to accommodate within tense logic sentences in which temporal expressions qualify singular terms like "Toby was fatter in 1980 than William in 1982". Prior considered such sentences (1967: 170). But his view that all temporal expressions be analysed as sentence operators forced him to give these sentences analyses far from their surface form. We suggest instead the use of predicate modifiers. This second revision is more radical than our first. The first amounts to an extension of the Kripkean tense logic; but the second calls for a substan-

(1) This kind of combination is well recognized now, though in the classic Prior-Quine debate of the 1950's and 1960's it was ignored. For discussion see Butterfield (1984).

tial revision of the truth-definition. We do not have a completeness theorem for the second revision and will only present semantic details.

2. *Mixing Formulas*

The main features of Kripke's (1963) modal logic are: (i) bivalence, i.e. every formula is true or false at every world under every valuation; (ii) the quantifier ranges at each world over just the objects in that world; (iii) the extensions at a world of atomic predicates are allowed to include objects that are not in that world; (iv) there is no restriction on the relations (inclusion, disjointness etc.) of domains of quantification (1963: 65-69).

We believe that a tense logic with analogous features, got by substituting times for worlds and temporal existence at a time for being in a world, is a good starting-point for analysing temporal discourse. So we shall use "(i)" etc. for these temporal analogues also.

Our acceptance of (i) and (iii) arises from our "detenser" view of time. We regard the present as an epistemic notion reflecting our limited access to a temporally extended reality. Objects and states of affairs in the past and future are wholly determinate; and they stand in genuine relations to present objects and states of affairs (cf. "Reagan is shorter than Lincoln"). So we see little reason to analyse temporal discourse with a many-valued logic, nor to confine the extensions of atomic predicates at a time to just the objects that temporally exist at that time.

However, our view that past, present and future form a single reality does not prevent our accepting (ii). No doubt the quantifier should not range beyond what one takes to be real; but it can have a *more* limited range. What its range should be is disputed. But in discussing intuitions about mixing formulas, we do best to accept (ii). For we intuitively interpret the quantifier as present-tensed; e.g. we read "$(\exists x)$" as "there temporally exists now an x such that ...". And given (ii), we must accept (iv) since we do not want to be committed to such claims as that all objects are indestructible or uncreatable (cf. inclusion within future or past domains, respectively), or instantaneous (cf. disjointness).

The relation between our detenser view and (iii) is a little delicate. Given bivalence, the denial of (iii), i.e. the proposal that atomic predicates' extensions at a time are confined to that time, amounts to the Falsehood

Principle; that is, it amounts to the principle that atomic predicates are false of non-existents. Such a proposal must of course be restricted to atomic predicates, since complex predicates cannot be so confined. For example, the complex predicate "does not exist" is satisfied at a time by objects that do not then exist; and given bivalence, the Falsehood Principle makes $\neg \emptyset, \emptyset$ a monadic atomic predicate, satisfied by every non-existent. But provided the proposal is thus restricted, it seems plausible when one focusses on monadic atomic predicates, considering examples like "is red" and "is square": for example, the proposal is motivated by the thought that when an object ceases to exist, it ceases to be red. On the other hand, we take the detenser view to require polyadic atomic predicates with unconfined extensions, as in "Reagan is shorter than Lincoln". So a detenser either needs a logic in which monadic and polyadic predicates are treated differently, or needs to sacrifice the Falsehood Principle, even for monadic predicates. We make the second choice, for two reasons. First, we are concerned to give sentences analyses close to their surface form; and this choice allows us to analyse predicates like "is famous" and "is dead" as atomic. Secondly, our predicate modifiers will enable us to capture some of the intuitive appeal of the Falsehood Principle for monadic predicates. For they will give us valid analogues of some formulas, relating quantifiers and operators, that the Falsehood Principle makes valid for atomic predicates.

So let us ask how the quantifiers and operators interact in a Kripkean tense logic with (i) – (iv). To consider this, we focus on the mixing formulas. There are eight such formulas, each a conditional licensing the transposition of a quantifier (α) or $(\exists\alpha)$ and an operator L or M, interpreted as "at all (some) times". (We use Greek letters α,β etc. as metalinguistic variables.) The formulas come in four pairs: the members of each pair are interdeducible given the standard interdefinability of (α) and $(\exists\alpha)$, and of L and M, and the substitution of $\neg \emptyset$ for $\emptyset$. (We could instead focus on analogous formulas with G and F, or H and P, instead of L and M; the discussion would be parallel.)

The pairs are marked off from each other in the diagram. 3. is the Barcan formula, 5. the Converse Barcan formula; 7. might be called the Buridan formula (Prior 1967: 138).

We believe that if one assumes features (ii) and (iv), i.e. the tensed interpretation of the quantifier and no requirement of inclusion, disjointness etc. on domains, then one intuitively judges validity as in the diagram.

(This has been borne out by classroom surveys.) The judgements of invalidity depend on the non-constancy of domains; except that one intuitively judges 1. and 2. (which are analogous to the invalid predicate calculus formula $(x)(\exists y)\emptyset xy \rightarrow (\exists y)(x)\emptyset xy)$ to be invalid even if one assumes that each time has the same domain. According to the diagram, intuition does not respect interdeducibility within pairs. We suspect that one judges 6. and 8. valid because one consciously considers examples like "is red" and "is square", and so restricts $\emptyset$ to an atomic predicate for which the Falsehood Principle holds. Certainly such a restriction does not similarly vindicate 5. and 7..

The Kripkean tense logic does of course respect interdeducibility within pairs, and it rules all eight formulas invalid. The invalidity of 3. to 8. again turns on the non-constancy of domains; but (iii) means that 6. and 8. are invalid even for atomic predicates.

Formula	*Valid according to intuition*
1. $(\alpha)M\emptyset \rightarrow M(\alpha)\emptyset$	No
2. $L(\exists\alpha)\emptyset \rightarrow (\exists\alpha)L\emptyset$	No
3. $(\alpha)L\emptyset \rightarrow L(\alpha)\emptyset$	No
4. $M(\exists\alpha)\emptyset \rightarrow (\exists\alpha)M\emptyset$	No
5. $L(\alpha)\emptyset \rightarrow (\alpha)L\emptyset$	No
6. $(\exists\alpha)M\emptyset \rightarrow M(\exists\alpha)\emptyset$	Yes
7. $M(\alpha)\emptyset \rightarrow (\alpha)M\emptyset$	No
8. $(\exists\alpha)L\emptyset \rightarrow L(\exists\alpha)\emptyset$	Yes

On the other hand, Prior's three-valued system Q (1957: 34-44; 1967: 154-58) agrees with the intuitions: 6. and 8. are valid, the rest not. For our purposes, the main features of Q are: (1) it "confines" predicates' extensions, i.e. violates (iii); (2) the third truth-value, "unstatable", is infectious, i.e. a formula $\emptyset$ is unstatable at time t under valuation V if $\emptyset$ has a subformula ψ (containing no free variables that are not free in $\emptyset$) that is unstatable at t under V; (3) L is interpreted strongly as "true in all worlds" and is therefore stronger than $\neg M \neg$ (i.e. "false in none");

(4) similarly, M is stronger than $\neg L \neg$. Features (3) and (4) prevent interdeducibility within pairs in the diagram.

The validity in Q of 6. and 8. follows from the result that in Q, if α is free in $\emptyset$, and $\emptyset$ is true or false (i.e. not unstatable) at t under V, then V assigns to α some member of t's domain; this result is readily proved by induction using feature (2) above.

In view of the discussion above, we are reluctant to give up the Kripkean tense logic. So we need to recover the intuitions in some different way from Prior's. We do so by extending the Kripkean tense logic with predicate modifiers that give us some new formulas analogous to 6. and 8. which *are* valid in our extended Kripkean logic. Our conception of modifiers is taken from Baldwin's (1979: Sec 2) suggested simplification of Wiggins' (1976) λ-calculus modifiers.

First, we add a modifier for internal negation: if α is a variable and β is a term (in Section 3's logic, which lacks constants and descriptions: a variable), then $(\neg\, \alpha\beta)$ is a modifier which like a quantifier can be prefixed to any formula $\emptyset$ and binds all occurrences of α in $\emptyset$. $(\neg\, \alpha\beta)\emptyset$ says, roughly speaking, that β exists and is not $\emptyset$. That is, $(\neg\, \alpha\beta)\emptyset$ is true at t under valuation V iff $V(\beta)$ is in t's domain and $\neg\, \emptyset$ is true at t under the valuation V^{β}_{α} that differs from V at most in assigning $V(\beta)$ to α, i.e. $V^{\beta}_{\alpha}(\alpha) = V(\beta)$. The use of V^{β}_{α} captures the idea that replacing α by β in $\neg\, \emptyset$ gives a formula that V makes true. (Indeed, with the usual definition of what it is to replace a variable α by a term β in a formula $\emptyset$, to give a formula $\emptyset^{\beta}_{\alpha}$ say, we have the usual result: for all α, β, $\emptyset$, t and V, $\emptyset$ is true at t under V^{β}_{α} iff $\emptyset^{\beta}_{\alpha}$ is true at t under V.)

We also need modifiers which add existence conditions to L and M, just as $(\neg\, \alpha\beta)$ does to negation. So we add modifiers $(L\alpha\beta)$ and $(M\alpha\beta)$: if $\alpha$is a variable and β is a term, then $(L\alpha\beta)$, $(M\alpha\beta)$ are modifiers. Like a quantifier, each of them can be prefixed to any formula $\emptyset$ and binds all occurrences of α in $\emptyset$. $(L\alpha\beta)\emptyset$ says, roughly speaking, that at all times at which β exists, it is $\emptyset$; $(M\alpha\beta)\emptyset$ that there is a time at which β exists at which it is $\emptyset$. That is, $(L\alpha\beta)\emptyset$ is true at t under V iff for all times t′ whose domain contains $V(\beta)$, $\emptyset$ is true at t′ under V^{β}_{α}. And $(M\alpha\beta)\emptyset$ is true at t under V iff there is a time t′ whose domain contains $V(\beta)$ such that $\emptyset$ is true at t′ under V^{β}_{α}.

Example: $(\exists y)(Lxy)(\neg\, zx)Qz$ is true at t under V iff there is some object *o* in t's domain such that at all times whose domain contains *o*, $(\neg\, zx)Qz$ is true of *o*, i.e. *o* is in the domain and is not Q.

Given the modifiers $(\neg\,\alpha\beta)$ and $(\mathrm{M}\alpha\beta)$, we have these valid analogues of 6. and 8.:

$$6.'\quad (\exists\beta)(\mathrm{M}\alpha\beta)\emptyset \rightarrow \mathrm{M}(\exists\alpha)\emptyset$$
$$8.'\quad (\exists\beta)\mathrm{L}(\neg\,\alpha\beta)\neg\,\emptyset \rightarrow \mathrm{L}(\exists\alpha)\emptyset$$

where β is not free in $\emptyset$, so that antecedent and consequent have the same free variables. Our suggestion is, then, that 6.′ and 8.′ are what prompt us to think that 6. and 8. are valid.

Before giving a formal treatment, we should make three supplementary points. First, Q also renders valid some analogues of 5. and 7.; and 5. and 7. are, according to our informal surveys, more acceptable than 3. and 4., and certainly than 1. and 2.! These analogues are:

$$\mathrm{L}(\alpha)\emptyset \rightarrow (\alpha)\neg\,\mathrm{M}\neg\,\emptyset\,,$$
$$\mathrm{M}(\alpha)\emptyset \rightarrow (\alpha)\neg\,\mathrm{L}\neg\,\emptyset\,.$$

These are valid because if α is not free in $\emptyset$, the formulas are equivalent to $\mathrm{L}\emptyset \rightarrow \neg\,\mathrm{M}\neg\,\emptyset$ and $\mathrm{M}\emptyset \rightarrow \neg\,\mathrm{L}\neg\,\emptyset$ respectively, which are valid by (3) and (4) above; and if α is free, the validity follows from p. 34's result about Q. Fortunately, we can use internal negation to offer valid analogues of these, viz.:

$$5.'\quad \mathrm{L}(\alpha)\emptyset \rightarrow (\alpha)\neg\,\mathrm{M}(\neg\,\beta\alpha)(\emptyset^{\beta}_{\alpha})$$
$$7.'\quad \mathrm{M}(\alpha)\emptyset \rightarrow (\alpha)\neg\,\mathrm{L}(\neg\,\beta\alpha)(\emptyset^{\beta}_{\alpha})\,,$$

where β is not free in $\emptyset$, so that antecedent and consequent have the same free variables. (Here $\emptyset^{\beta}_{\alpha}$ is got from $\emptyset$ by replacing all free occurrences of α by β, relettering bound variables as necessary.)

Secondly, our modifier $(\mathrm{L}\alpha\beta)$ gives natural analyses of the temporal analogue of what Kripke (1971: 137) calls "weak necessity". Thus in "Socrates is necessarily human", "necessarily" expresses, not truth at all worlds, but truth at all worlds in which Socrates exists. Similarly, in "Socrates always lived in Athens", "always" means "throughout Socrates' life" not "at all times" nor "throughout Athens' existence". With our modifier $(\mathrm{L}\alpha\beta)$ we can analyse this sentence without having to write existence conditions which are not apparent in the surface form. Instead of:

$$\mathrm{L}((\exists x)(x = \text{Socrates}) \rightarrow \text{Lives(Socrates, Athens)})\,;$$

or, in order to avoid the problem of vacuous truth:

$$P((\exists x)(x = \text{Socrates})) \ \& \ L((\exists x)(x = \text{Socrates}) \rightarrow \text{Lives(Socrates, Athens)});$$

we can write:

$$P((\text{LxSocrates})(\text{Lives(x,Athens)})).$$

(This advantage of predicate modifiers has of course been noticed in modal logic: Wiggins (1976: 310) motivates his λ-calculus modifiers for weak necessity by their avoidance of such existence conditions. For discussion and formal development of Wiggins' proposals cf. Davies (1978) and Stirling 1982: 202f.)

Uses of "always" as a restricted universal quantifier over times are in fact common. But the restriction is not always fixed by the times at which *one* object exists. When we say "Socrates and Plato were always friends", "always" means "at all times when they both existed". (For more examples of "always" as a restricted universal quantifier, in some cases not even over times, cf. Lewis 1975.) This suggests an analysis:

$$(\text{LxSocratesyPlato})(\text{Friend(x,y)}).$$

That is, it suggests modifiers $(L\alpha_1\beta_1...\alpha_n\beta_n)$ with $\alpha_1, ..., \alpha_n$, variables and $\beta_1, ..., \beta_n$, terms: $(L\alpha_1\beta_1...\alpha_n\beta_n)\emptyset$ is true at t under V iff at all t′ whose domain contains $V(\beta_1), ..., V(\beta_n)$, $\emptyset$ is true under $((...(V^{\beta_1}_{\alpha_1})^{\beta_2}_{\alpha_2})...)^{\beta_n}_{\alpha_n}$. Similarly, $(M\alpha_1\beta_1...\alpha_n\beta_n)$ will require that there be some such time t′. And one can add $(\neg\,\alpha_1\beta_1...\alpha_n\beta_n)$ where $(\neg\,\alpha_1\beta_1...\alpha_n\beta_n)\emptyset$ is true at t under V iff $V(\beta_1), ..., V(\beta_n)$ are all in t's domain and $\neg\,\emptyset$ is true under $((...(V^{\beta_1}_{\alpha_1})^{\beta_2}_{\alpha_2})...)^{\beta_n}_{\alpha_n}$.

But we need not add all these modifiers as primitives; that is fortunate, since proofs by induction are simpler the fewer primitives we have. In fact we can define all these modifiers in terms of $(\neg\,\alpha\beta)$ and the sentential operators L and M. So we can take $(\neg\,\alpha\beta)$ as our sole primitive modifier; and in Section 3 we shall do so. We can do this because of the following three facts. (1) By bivalence,

$$(L\alpha\beta)\emptyset \leftrightarrow L\neg((\neg\,\alpha\beta)\emptyset)$$
$$(M\alpha\beta)\emptyset \leftrightarrow M((\neg\,\alpha\beta)\neg\,\emptyset)$$

are valid, so that $(L\alpha\beta)$ and $(M\alpha\beta)$ are definable from $(\neg\,\alpha\beta)$, and L.

(2) More generally, these formulas are valid:

$$(\mathrm{L}\alpha_1\beta_1...\alpha_n\beta_n)\emptyset \leftrightarrow \mathrm{L}\neg((\neg\,\alpha_1\beta_1...\alpha_n\beta_n)\emptyset)$$
$$(\mathrm{M}\alpha_1\beta_1...\alpha_n\beta_n\emptyset \leftrightarrow \mathrm{M}((\neg\,\alpha_1\beta_1...\alpha_n\beta_n)\neg\,\emptyset\ ,$$

so that $(\mathrm{L}\alpha_1\beta_1...\alpha_n\beta_n)$ and $(\mathrm{M}\alpha_1\beta_1...\alpha_n\beta_n)$ are definable from $(\neg\,\alpha_1\beta_1...\alpha_n\beta_n)$ and L. (3) $(\neg\,\alpha_1\beta_1...\alpha_n\beta_n)$ is definable from our original internal negation by using the valid formulas:

$$(\neg\,\alpha_1\beta_1...\alpha_n\beta_n)\emptyset \leftrightarrow (\neg\,\alpha_1\beta_1)\ \neg\ (\neg\,\alpha_2\beta_2)\ \neg\ ...\ \neg\ (\neg\,\alpha_{n-1}\beta_{n-1})\ \neg\ (\neg\,\alpha_n\beta_n)\emptyset\ ;$$

here the "$\neg$" in "$(\neg\,\alpha_1\beta_1...\alpha_n\beta_n)$" corresponds to the "$\neg$" in "$(\neg\,\alpha_n\beta_n)$", and the "$\neg$"s in $(\neg\,\alpha_1\beta_1)$, ..., $(\neg\,\alpha_{n-1}\beta_{n-1})$ are cancelled by the negations to their right. In the same way we can define modifiers $(\mathrm{P}\alpha\beta)$, $(\mathrm{F}\alpha\beta)$, $(\mathrm{H}\alpha\beta)$, $(\mathrm{G}\alpha\beta)$ and $(\mathrm{P}\alpha_1,\beta_1...\alpha_n\beta_n)$ etc. from P, F, G, H and $(\neg\,\alpha\beta)$.

3. *A Completeness Theorem*

We shall implement the ideas of Section 2 in a first-order tense logic with linear non-beginning, non-ending time, and without constants and descriptions. Then we shall sketch how to adapt the techniques of Thomason's (1970) completeness theorem for his modal system Q3, so as to prove our system complete; we shall be brief since we are adapting Thomason's techniques. They could also be adapted to prove completeness for: systems allowing non-linear time; systems with constants and descriptions; and modal rather than tense systems. (As he points out, Stalnaker & Thomason (1968; 1969a) adapt them to prove completeness for a system in which abstraction is used to mark the scope-distinctions that arise when definite descriptions are treated as primitive; rather than being used, as in Wiggins, to distinguish weak and strong necessity.)

We take as vocabulary: variables, predicate letters, brackets, commas, and the symbols: $\exists$, $\rightarrow$, $\neg$, G, H. We add to the usual wff rules the clause that $(\neg\,\alpha\beta)\emptyset$ is a wff whenever $\emptyset$ is, and α and β are variables. We add to the usual definitions of $\wedge$, $\vee$, $(\forall\alpha)$, P, F, the following:

(1) $(\neg\,\alpha_1\beta_1,\ \alpha_2\beta_2,\ ...,\ \alpha_n\beta_n)\emptyset\ =\ (\neg\,\alpha_1\beta_1)\ \neg\ (\neg\,\alpha_2\beta_2)\ \neg\ ...\ \neg\ (\neg\,\alpha_n\beta_n)\emptyset$.

(2) $L\emptyset = \emptyset \wedge G\emptyset \wedge H\emptyset$; $M\emptyset = \neg L \neg \emptyset$.

(The axioms and rules to follow will impose a linear time structure, giving this L the force of an unrestricted quantifier over times, with the S5 property: $M\emptyset \rightarrow LM\emptyset$.)

(3) For $0 \in \{G,H,L\}$: $(0\alpha_1\beta_1, \alpha_2\beta_2, \ldots, \alpha_n\beta_n)\emptyset = 0\neg(\neg\alpha_1\beta_1, \alpha_2\beta_2, \ldots, \alpha_n\beta_n)\emptyset$;

and the dual of $(0\alpha_1\beta_1, \ldots, \alpha_n\beta_n)\emptyset$ is $\neg(0\alpha_1\beta_1, \ldots, \alpha_n\beta_n)\neg\emptyset$. Thus $(M\alpha_1\beta_1, \ldots, \alpha_n\beta_n)\emptyset = M(\neg\alpha_1\beta_1, \ldots, \alpha_n\beta_n)\neg\emptyset$.

(4) $\emptyset < \psi = L(\emptyset \rightarrow \psi)$; $\emptyset <_H \psi = H(\emptyset \rightarrow \psi)$; $\emptyset <_G \psi = G(\emptyset \rightarrow \psi)$.

We take as axioms the following two groups:

A1. Any sentential calculus tautology instance
A2. $(\forall\alpha)(\emptyset \rightarrow \psi) \rightarrow (\emptyset \rightarrow (\forall\alpha)\psi)$ where α is not free in $\emptyset$
A3. $(\forall\alpha)\emptyset \rightarrow \neg(\neg\alpha\beta)\emptyset$
A4. $(\forall\alpha)\emptyset \rightarrow (\exists\alpha)\emptyset$
A5. $(\forall\alpha)\neg(\neg\beta\alpha)\emptyset \rightarrow (\forall\beta)\emptyset$ α not free in $\emptyset$
A6. $(\neg\alpha\beta)\emptyset \rightarrow \neg\emptyset^{\beta}_{\alpha}$
A7. $((\neg\alpha\beta)\emptyset \wedge \neg\psi^{\beta}_{\gamma}) \rightarrow (\neg\gamma\beta)\psi$
A1.1 $G\emptyset \rightarrow F\emptyset$
A2.1 $G(\emptyset \rightarrow \psi) \rightarrow (F\emptyset \rightarrow F\psi)$
A3.1 $G\emptyset \rightarrow GG\emptyset$
A4.1 $PG\emptyset \rightarrow \emptyset$
A5.1 $(F\emptyset \wedge F\psi) \rightarrow F(\emptyset \wedge \psi) \vee F(F\emptyset \wedge \psi) \vee F(\emptyset \wedge F\psi)$

and their mirror-images A1.2, A2.2, ..., A5.2, obtained by replacing G by H, F by P, P by F, H by G.
Here A1 to A5 are in effect shared with Thomason, his formula $(E\alpha \wedge \emptyset^{\alpha}_{\beta})$ being equivalent to our formula $(\neg\beta\alpha)\neg\emptyset$.

We take as rules:

MP $\dfrac{\emptyset \quad \emptyset \rightarrow \psi}{\psi}$ Gen $\dfrac{\emptyset}{(\forall\alpha)\emptyset}$

RG $\dfrac{\emptyset}{G\emptyset}$ RH $\dfrac{\emptyset}{H\emptyset}$

The following rules, which are useful in the completeness proof, are then derivable:

DR.G $\dfrac{\emptyset \to G\psi}{\emptyset \to G(\forall\alpha)\psi}$ α not free in $\emptyset$

DR.$_G$H $\dfrac{\emptyset \to .\emptyset_1 < . \ldots < .\emptyset_n <_G H\psi}{\emptyset \to .\emptyset_1 < . \ldots < .\emptyset_n <_G H(\forall\alpha)\psi}$ α not free in $\emptyset, \emptyset_1, \ldots, \emptyset_n$

DR.$_G$G $\dfrac{\emptyset \to .\emptyset_1 < . \ldots < .\emptyset_n <_G G\psi}{\emptyset \to .\emptyset_1 < . \ldots < .\emptyset_n <_G G(\forall\alpha)\psi}$ α not free in $\emptyset, \emptyset_1, \ldots, \emptyset_n$

and their mirror images DR.H, DR.$_H$G, and DR.$_H$H. Thomason has the modal analogue of these rules with L replacing G and H throughout (so that the four rules DR.$_G$H etc. collapse into his single rule R5).

In this system we can prove analogues of the intuitively valid formulas that are theorems of Q. Thus we can prove the formulas 6.′, 8.′ and 5.′, 7.′ of Section 2; and their analogues with P or F for M and H or G for L. For example, to prove 6.′, we start with axiom A3: $(\forall\alpha)\neg\emptyset \to \neg(\neg\alpha\beta)\neg\emptyset$, where $\emptyset$ has no free β. L obeys: A/LA, and $L(\emptyset \to \psi) \to (L\emptyset \to L\psi)$. So by Gen and A2, $L(\forall\alpha)\neg\emptyset \to (\forall\beta)L\neg(\neg\alpha\beta)\neg\emptyset$. Contraposing and applying the definition of $(M\alpha\beta)$, we have 6.′: $(\exists\beta)(M\alpha\beta)\emptyset \to M(\exists\alpha)\emptyset$.

The following results about derivability are needed for the completeness theorem:

T1. If $\Gamma \cup \{\emptyset^{\alpha}_{\beta}\} \vdash \psi$, and α is an individual variable not in ψ, not in any member of Γ, then $\Gamma \cup \{(\exists\beta)\emptyset\} \vdash \psi$.

T2. $\vdash (\exists\beta)[(\neg\alpha\beta)\neg((\exists\alpha)\emptyset \to \emptyset)]$ β not free in $\emptyset$

T3. If Γ is consistent and $P(\emptyset_1 \wedge \ldots \wedge \emptyset_n) \in \Gamma$, then $\{\emptyset_1 \ldots \emptyset_n\}$ is consistent. Similarly for F.

T4. $(P\emptyset \wedge \psi) \to P(\emptyset \wedge F\psi)$; and its mirror-image.

An interpretation is defined in the usual way: the set T of times has a linear, non-beginning, non-ending order; each time has a non-empty domain to which quantifiers are restricted; variables are rigid; and the extension assigned to a predicate letter relative to a time can include objects not in that time's domain. The only new feature is that valuations have the clause: $V((\neg\alpha\beta)\emptyset)(t) = 1$ iff $V(\beta)$ is in t's domain and $V^{\beta}_{\alpha}(\emptyset)(t) = 0$. We can prove by induction as usual: $V(\emptyset^{\beta}_{\alpha})(t) = 1$ iff $V^{\beta}_{\alpha}(\emptyset)(t) = 1$. And abbreviating $(\neg\alpha\beta)(P^{\circ} \wedge \neg P^{\circ})$, with P° an arbitrary fixed 0-place predicate letter, by $E\beta$, we have: $V((\neg\alpha\beta)\neg\emptyset)(t) = V(E\beta\wedge\emptyset^{\beta}_{\alpha})(t)$. With respect to this semantics, the axioms are valid and the rules respect validity, so that a satisfiable set is consistent.

We can now prove completeness by Thomason's method. To show that a set of formulas is consistent only if it is satisfiable, we show that (a) if Γ is consistent, then for any vocabulary M' extending our given one by adding denumerably many variables, Γ has an extension saturated in M'; and (b) if Γ is an M'-saturated set, then it is satisfiable.

To define saturation, we need a set of functions f^{λ}_{μ} which play an analogous role to Thomason's f_n. We say an ordered pair $\langle\lambda,\mu\rangle$ divides n (n = 0, 1, ...) iff λ and μ are mutually exclusive and jointly exhaustive subsets of $\{0,1,2,...,n\}$, ordered by the less-than relation; each of λ and μ can be empty. Then we define:

$$f_0((\exists\alpha)\emptyset,\beta) = P(\exists\alpha)\emptyset \rightarrow P((\neg\alpha\beta)\neg\emptyset)$$
$$f^0((\exists\alpha)\emptyset,\beta) = F(\exists\alpha)\emptyset \rightarrow F((\neg\alpha\beta)\neg\emptyset)$$
$$f_{01}((\exists\alpha)\emptyset,\beta,\psi) = P\psi \rightarrow P(\psi\wedge f_0((\exists\alpha)\emptyset,\beta))$$
$$f^{01}((\exists\alpha)\emptyset,\beta,\psi) = F\psi \rightarrow F(\psi\wedge f^0((\exists\alpha)\emptyset,\beta))$$
$$f^0_1((\exists\alpha)\emptyset,\beta,\psi) = P\psi \rightarrow P(\psi\wedge f^0((\exists\alpha)\emptyset,\beta))$$
$$f^1_0((\exists\alpha)\emptyset,\beta,\psi) = F\psi \rightarrow F(\psi\wedge f_0((\exists\alpha)\emptyset,\beta))$$

and generally, if $\langle\lambda,\mu\rangle$ divides n, then we define:

$$f^{\lambda(n+1)}_{\mu}((\exists\alpha)\emptyset,\beta,\psi,...,\psi_n,\psi_{n+1}) = F(\psi_{n+1}) \rightarrow F(\psi_{n+1}\wedge f^{\lambda}_{\mu}((\exists\alpha)\emptyset,\beta,\psi_1,...,\psi_n))$$
$$f^{\lambda}_{\mu(n+1)}((\exists\alpha)\emptyset,\beta,\psi,...,\psi_n,\psi_{n+1}) = P(\psi_{n+1}) \rightarrow P(\psi_{n+1}\wedge f^{\lambda}_{\mu}((\exists\alpha)\emptyset,\beta,\psi_1,...,\psi_n))$$

Thus the role of λ and μ is to keep track of which occurrences out of $2(n+1)$ occurrences of P or F are P and which are F; in f^{λ}_{μ}, λ will tell us which are P, and μ will tell us which are P.

We say a subset Γ of wffs in a vocabulary M' is M'-saturated iff it satisfies all the following:

(1) Γ is consistent;
(2) for all wffs $\emptyset$ in M', $\emptyset\in\Gamma$ or $\neg\emptyset\in\Gamma$;
(3) for all wffs $\emptyset$ in M', and variables α of M': if $(\exists\alpha)\emptyset\in\Gamma$, then for some variable β of M', $(\neg\alpha\beta)\neg\emptyset\in\Gamma$;
(4) for all $n \geqslant 0$, for all wffs $\psi_1, ..., \psi_n$, $(\exists\alpha)\emptyset$ in M', for all divisions $\langle\lambda,\mu\rangle$ of n, there is a variable β of M' such that $f^{\lambda}_{\mu}((\exists\alpha)\emptyset,\beta,\psi_1,...,\psi_n)\in\Gamma$.
(Here it is understood that in the case n = 0, the formulas $\psi_1, ..., \psi_n$ are absent.)

We can now prove the following, making straightforward changes in the proofs of Thomason's corresponding results (his L9, L11, L13):

L1. For all n > 0, for all $\langle\lambda,\mu\rangle$ dividing n, if $\Gamma \vdash \neg f^{\lambda}_{\mu}((\exists\alpha)\emptyset, \beta, \psi_1, ..., \psi_n)$, and β does not occur free in $\psi_1, ..., \psi_n, (\exists\alpha)\emptyset$, or any member of Γ, then Γ is inconsistent.

L2. Any consistent set of formulas in our given vocabulary has, for any vocabulary M′ extending it by adding denumerably many new variables, a M′-saturated extension.

L3. Let Γ be any M-saturated set. Let Δ_1, Δ_2 range over M-saturated sets. Let $\Delta_1 [\Delta_2$ iff: $\{\emptyset : G\emptyset \in \Delta_1\} \subseteq \Delta_2$ and $\{\emptyset : H\emptyset \in \Delta_2\} \subseteq \Delta_1$.
Let K be the closure of $\{\Gamma\}$ under [and its converse.
Then K satisfies: for all $\Delta \in K$, and all formulas $\emptyset$ in M, both (a) if $P\emptyset \in \Delta$ then there is a $\Delta' \in K$ with $\emptyset \in \Delta'$, $\Delta' [\Delta$; and (b) if $F\emptyset \in \Delta$, then there is a $\Delta' \in K$ with $\emptyset \in \Delta'$, $\Delta [\Delta'$.

Given L3, it is straightforward to show that [is a linear nonbeginning, nonending order on K. And then we can prove:

L4. If Γ is M′-saturated, it is satisfiable.

by considering an interpretation with K as the set of times, [as the order, the domain of a time t as $\{\alpha : (\neg \beta\alpha)\emptyset \in t$ for some formula $\emptyset\}$; and by showing by induction that for all formulas $\emptyset$, and times t, $V(\emptyset)(t) = 1$ iff $\emptyset \in t$.

4. *Temporal Adjectives*

We turn to the analysis of sentences in which temporal expressions qualify singular terms, like "Toby was fatter in 1980 than William in 1982".

These sentences are readily enough analysed if we accept temporal terms, i.e. terms standing for instants or periods of time. For we can analyse what seem to be n-place predicates as having 2n places, n of which specify the times at which the objects in the other n places satisfy the predicate. Thus we write "Fatter(Toby,1980,William,1982)". And we can use indexical temporal terms and a 2-place later-than predicate to analyse the verbal tense in "was fatter"; so we can analyse the sentence above as: Fatter(Toby,1980,William,1982) & Later(now,1980). This is essentially Quine's approach (1960: Section 36) except that he prefers to analyse seemingly n-place predicates as indeed n-place, and to accept temporal parts, Toby-at-1980, William-at-1982 etc. (For more details, cf. Needham 1975 : 37-41, 59-60.)

But what about the analysis of these sentences within a tense logic of the Kripkean kind we favour? Here there is a problem. To be sure, feature (iii), i.e. letting a predicate's extension at a time include objects that do not then exist, means we can give relations between objects that never exist simultaneously an analysis close to the surface form: "Reagan is shorter than Lincoln" can be analysed as "Shorter(Reagan,Lincoln)". But the fact remains that if all temporal expressions are analysed as operators, only objects *simpliciter*, not objects at various times, can bear a relation (relative of course to a time) to one another. So to analyse "Toby was fatter in 1980 than William in 1982", one must find objects bearing an appropriate relation *simpliciter*. One might suggest degrees of fatness (which will no doubt be taken to exist at all times), bearing the *greater-than* relation, and write:

$$(\exists x)(\exists y)(T_{1980}(\text{Fat}(x,\text{Toby})) \;\&\; T_{1982}(\text{Fat}(y,\text{William})) \;\&\; \text{Greater}\,(x,y))$$

where "T_{1980}" is the operator "It is true in 1980 that ...". Thus temporal adjectives appear as sentential operators governing a relation between their objects (Toby, William) and a degree of fatness: cf. Prior (1967: 170).

But this suggestion suppresses the information, conveyed by "was" that 1980 is in the past. We cannot express this by conjoining "P(Fat(x,Toby))" within the quantifiers' scope. But we can express it with a *later-than* predicate and indexical terms for times and intervals, each time and interval entering the appropriate domains: we conjoin "Later(now,1980)" to the above. Or we can treat times and intervals as propositions and conjoin "P(1980)" to the above.

But whichever tactic we adopt, we end up with an analysis far from the surface form. First, the past tense of "was" ceases to be just an operator; it requires a separate conjunct, as in the approach using temporal terms. Secondly, the analysis invokes odd objects and relations not apparent in the surface form: and even if degrees of fatness pass muster as objects, surely the hairstyles needed to give a similar analysis of "Toby had the same hairstyle in 1980 as William in 1982" do not.

The situation is ironic. Many have found tense logic attractive because they value closeness to the surface form, and analyses in some simpler e.g. extensional language require us to accept as objects items like temporal parts that are not apparent in the surface form (cf. Clark 1970: 316-9; Parsons 1970: 329; Cresswell 1974: 460; Montague 1974: 41, passim). But when it comes to temporal adjectives, the tables are turned: tense logic has to introduce odder objects and arguably depart further from surface form, than an extensional analysis needs to do.

So advocates of tense logic must try to analyse temporal adjectives more closely than the above suggestion: especially if, like us, they are detensers and therefore have no metaphysical objection to relations between objects at different times. We shall describe how to do this, gradually motivating a formal definition for a Kripkean tense logic. ([2])

In a tense logic, a temporal adjective such as "in 1980" can be analysed as a predicate modifier "(1980x...)" which like a quantifier binds the occurrences of x in its scope; but unlike a quantifier does not reduce the polyadicity of the formula it is applied to, since in "(1980x...)" "..." is an argument-place. Indeed an analysis like this is inevitable if we refuse to add to predicates special argument-places for times. For how then could we qualify "Toby" with "1980" and "William" with "1982" in "Fatter(Toby,William)", so as to move from this present-tensed sentence to an analysis of "Toby was fatter in 1980 than William in 1982"? (Ignore the "was" for the moment.) Operators such as "(1980Toby)" and "(1982William)", giving "(1980Toby)(1982William)(Fatter(Toby,William))" will not do in general: for sometimes only one of two occurrences of "Toby" should be thus qualified, as in "Toby was fatter in 1980 than Toby is now". This suggests using variables to mark occurrences of "Toby" to be qualified by "in 1980"; and if predicates are not to have extra argument-places, "Toby" will have to fill an argument-place introduced by the temporal adjective. Thus we write "(1980xToby)(1982yWilliam)(Fatter(x,y))" for "Toby was fatter in 1980 than William in 1982"; and "(1980xToby)(Fatter(x,Toby)" for "Toby was fatter in 1980 than Toby is now". Likewise, indexical temporal adjectives as in "Toby will one day be fatter than

([2]) In modal logic there has been some discussion of expressing crossworld relations by indexing an Actually operator (Humberstone and Davies 1980: Forbes 1985: 92-94; 1983: 296, fn.24). But we do not know of proposals like ours, for modal tense logic. Indeed, traditional grammarians, as well as semantic analysts, give temporal adjectives less treatment than adverbs: in Quirk et al. (1972), Sections 5.71 and 10.11 to 10.14 are about both but 8.56 to 8.75 are only about adverbs. We suspect the reason for temporal adjectives being ignored is that in analysis we tend to choose examples about observable matters, and observable relations usually hold of objects at one time (cf. Butterfield 1984a).

William (is now)" and "Toby was once fatter than William (is now)" can be analysed as modifiers: "(FxToby)(Fatter(x,William))" and "(PxToby)(Fatter(x,William))".

Indexical temporal adjectives raise two points. First, they usually relate to the time of utterance, even when they are in the scope of an expression carrying us to some other time. Thus in "We knew then that Toby will one day be fatter than William is now", "now" refers to the time of utterance and "will one day" to this time's future – despite their being within the scope of the past-referring "we knew then that ...". (Contrast "We knew then that Toby *would* be fatter than William was *then*" (Quirk et al. 1972: 90,92).) This dominance of the time of utterance has long been recognized in temporal adverbs. And tense logics modelling it have been suggested (Kamp 1971; Vlach 1975). The idea is to keep track of the time of utterance when evaluating a subformula at whatever times we are carried to by the tense operators governing the subformula. Thus we relativize truth-values to two times instead of one: we consider "Ø is true at time t′ when part of an utterance made at t". We shall use the same idea to handle indexical temporal adjectives: the "P" and "F" in "(Px...)" and "(Fx...)" will relate to the time of utterance, not to any subsidiary time to which an operator carries us.

Secondly, these indexical adjectives suggest a generalization to modifiers binding more than one variable (and thus having more than one argument-place). "Toby and Harry once weighed as much together as William does now" cannot be analysed as "(PxToby)(PyHarry)(TogetherWeighSame(x,y,William))". For the two modifiers may "carry" Toby and Harry to different past times, so that the formula comes out true if Harry's 1975 weight and Toby's 1980 weight add up to William's present weight. To cope with such cases, we need modifiers ($Px_1...x_2...x_m...$) and ($Fx_1...x_2...x_m...$) binding m variables and introducing m argument-places, all tied to one past (future) time. Similarly, we could use m-place non-indexical modifiers such as ($1980x_1...x_m...$) to analyse (ignoring the tense of "weighed"). "In 1980 Toby and Harry weighed as much together as William does now" as "(1980xTobyyHarry)(TogetherWeighSame(x,y,William))". (But this will be equivalent to "(1980xToby)(1980yHarry)(TogetherWeighSame(x,y,William))" so that m-place non-indexical modifiers, though permissible, are not essential.)

So far we have ignored verbal tenses functioning as temporal adjectives in conjunction with dates, a phenomenon that is in fact as common as any temporal adjective: nouns are rarely qualified only by a date. The copula in particular commonly acts as an indexical temporal adjective, as in "Toby *was* fatter in 1980 than William *was* in 1982". And as with simple indexical adjectives, the tense is usually chosen to fit the time of utterance: "We knew in 1980 that Toby *will be* fatter in 1990 than William *was* in 1982". It turns out that such complex temporal adjectives can be analysed with a pair of our modifiers: we need only link them by making one modifier bind the variable in the other's argument-place. Thus "Toby was fatter in 1980 than William is now" becomes "(PxToby)(1980yx)(Fatter(y,William))", and interpreting this according to our semantics gives it the right truth-conditions. In effect, the conditions imposed on "Toby" by "P" and "1980" just conjoin. (The order of the modifiers doesn't matter: "(1980xToby)(Pyx)(Fatter(y,William))" is an equivalent formula.) Similarly, "Toby and Harry weighed as much together in 1980 as William does now" can be analysed, *without* ignoring the tense of "weighed", as (PxTobyyHarry)(1980uxvy)(TogetherWeighSame(u,v,William)).

How then should we adjust the semantics of an "ordinary" tense logic in order to incorporate these modifiers? For simplicity, we shall omit individual constants and definite descriptions. Ignoring for the moment the use of two times to handle the dominance of the time of utterance, the rough idea is of course that a formula "(t′xy)Ø" is true at time t iff the formula we get from Ø by substituting y, considered at t′, for x is true at t. To make precise sense of this "considered at t′", the first thing we shall do is give an n-place atomic predicate Q^n an extension relative to an n-tuple of times rather than just one. The extensions of Q^n relative to all these n-tuples then represent what objects considered at what times bear Q^n to each other. This kind of relativisation does not prevent atomic formulas being interpreted as present-tensed; we only need to consider n-tuples all of whose members are the same. Thus we shall have the truth-definition make "$Q^n(\beta_1...\beta_n)$", where $\beta_1...\beta_n$ are (not necessarily distinct) variables, true at time t under valuation V if the objects $V(\beta_1)...V(\beta_n)$ assigned to $\beta_1...\beta_n$ by V are in the extension of Q^n for V relative to the n-tuple $\langle t,t,t,...,t\rangle$ (n occurrences). And it is easy to see how to treat a modified atomic formula "$(t'xy)Q^n(\beta_1...\beta_n)$ where some of the terms $\beta_1...\beta_n$ are "x". This is to be true at t under valuation V if the valuation V′ differing from V at most in that V(y) = V′(x) is such that $V'(\beta_1),...,V'(\beta_n)$ are in the extension of Q^n relative to the n-tuple resulting from ⟨t,t,t,...,t⟩ (n occurrences) by substituting t′ for t in those places corresponding to the β_i which are "x".

So far, so good; but what about modified formulas "(t′xa)Ø" where Ø is not atomic? How can we make precise sense in this case of substituting t′ for t in the places "corresponding to x"? After all, a formula as against a predicate is usually interpreted as having a truth-value at each time; there is no n-tuple of times in sight. The idea of the solution we adopt is to relativize the truth-values of any formula with j free variables (j types, not tokens) to j times in addition to the time of utterance. Or rather: in addition to the two times (one of them the time of utterance) used to handle the dominance of the time of utterance. A formula with j free variables thus gets a truth-value relativized to j+2 times. When the formula is in the scope of a modifier which binds one of its j free variables, the modifier affects at what (j+2)-tuple the scope formula must be true for the entire formula to be true (at a (j+2)-tuple since the modifier's argument-place is filled by a variable). Roughly speaking – and ignoring the use of two times to handle the dominance of time of utterance – we say: let Ø have j free variables, of which "x" is the kth ($1 \leqslant k \leqslant j$), and none of which is y; so that "(t′xy)Ø" also has j free variables, since the modifier binds "x". Then "(t′xy)Ø" is true when uttered at t under valuation V relative to $\langle t_1,...,t_j \rangle$ iff Ø is true when uttered at t under the valuation V′ differing from V at most in that V(y) = V′(x), relative to the j-tuple got from $\langle t_1,...,t_j \rangle$ by substituting t′ in the kth place.

However, adopting this relativization as it stands leads to some cumbersome accounting of how the number and order of free variables in a formula depends on the number and order in its subformulas. It is easier to follow the analogy of Tarski's use of denumerable sequences in the account of satisfaction. Thus we assume we have an indexed set of individual variables $\{x_1,..., x_n,...\}$, and we make the indexing correspond to position in denumerable sequences of times. This allows us to have a single semantic relation: allowing for the dominance of time of utterance, it is "Ø is true, when part of a formula uttered at t, relative to t′ and the denumerable sequence of times σ". In accordance with the above discussion, tense operators are interpreted with the "running" time t′, while the modifiers are interpreted with the time of utterance.

Thus suppose we have, together with the variables $\{x_1,...\}$, predicate letters, connectives and quantifiers, time constants t, t′, ... and the operators P, F, M. To the usual wff rules we add the clause that if Ø is a wff and

x_{i_1}, ..., x_{i_m}, x_{j_1}, ..., x_{j_m} are variables, then $(\theta x_{i_1}x_{j_1}...x_{i_m}x_{j_m})\emptyset$ is a wff also, where θ is either P or F or M or one of the time constants. (Here and below, the fact that the semantics exploits variables' indices makes double subscripting more convenient than the metalinguistic variables α_1, β_1, ..., α_n, β_n, used in Sections 2 and 3.)

An interpretation is a quintuple $\langle T,<,D,E,V\rangle$, where E is a set of times, for which we can again use t, t′ etc, without confusion; < is the usual relation on T; D is a set of individuals; for each $t \in T$ E(t) is a non-empty subset of D; V is a map sending each variable to an element of D and each time constant to an element of T; and for each predicate letter Q^n and n-tuple of times $t_1,...,t_n$ $V(Q^n)(t_1,...,t_n)$ is a subset of D^n. V is then extended as follows thus defining the relation "the formula $\emptyset$ is true, when part of a formula uttered at t, relative to t′ and the denumerably infinite sequence of times $\sigma \in T^\infty$", which we write as: $V(\emptyset)(t,t',\sigma) = 1$. We make use of a little notation in the definition of V. (a) If $\sigma \in T^\infty$, then $\sigma(i)$ = the ith member of σ. (b) As usual, $V^{x_j}_{x_i}$ differs from V at most in the assignment to x_i: $V^{x_j}_{x_i}(x_i) = V(x_j)$. (c) σ^t_i is like σ except for having t in the ith place: $\sigma^t_i(j)$ is $\sigma(j)$ if $j \neq i$, and is t if $j = i$. The clauses of V's definition are then:

(1) $V(Q^n x_{i_1}...x_{i_n})(t,t',\sigma) = 1$ iff $\langle V(x_{i_1}),...,V(x_{i_n})\rangle \in V(Q^n)(\sigma(i_1)...\sigma(i_n))$
(2) $V(\neg\emptyset)(t,t',\sigma) = 1$ iff $V(\emptyset)(t,t',\sigma) = 0$.
(3) $V(\emptyset \rightarrow \psi)(t,t',\sigma) = 1$ iff $V(\emptyset)(t,t',\sigma) = 0$ or $V(\psi)(t,t',\sigma) = 1$.
(4) $V(P\emptyset)(t,t',\sigma) = 1$ iff for some $t'' < t'$, $V(\emptyset)(t,t'',\sigma^{t''}_{i_1}...{}^{t''}_{i_m}) = 1$,
where $x_{i_1},...,x_{i_m}$ are the free variables in $\emptyset$.

Thus the operator P sends the process of evaluating a wff, including the free variables, back from the running time, not the time of utterance.

(5) $V((\exists x_i)\emptyset)(t,t',\sigma) = 1$ iff there is a valuation V′ differing from V at most in that $V'(x_i) \in E(t')$ with $V'(\emptyset)(t,t',\sigma^{t'}_i) = 1$.

The use here of the running time, together with clause (4), implies the usual Kripkean interpretation for quantifiers within the scope of a tense operator.

(6) $V((t''x_{i_1}x_{j_1}...x_{i_m}x_{j_m})\emptyset)(t,t',\sigma) = 1$ iff $V^{x_{j1}...x_{jm}}_{x_{i1}...x_{im}}(\emptyset)(t,t',\sigma^{t''}_{i_1}...{}^{t''}_{i_m}) = 1$.
(7) $V((Px_{i_1}x_{j_1}...x_{i_m}x_{j_m})\emptyset)(t,t',\sigma) = 1$ iff for some $t'' < t$,
$V^{x_{j1}...x_{jm}}_{x_{i1}...x_{im}}(\emptyset)(t,t',\sigma^{t''}_{i_1}...{}^{t''}_{i_m}) = 1$.

Thus the modifier sends one back from the time of utterance, modelling the dominance of the time of utterance in the choice of tense for the copulas etc. that function as temporal adjectives.

Clauses for P and for M as operators and in modifiers are similar to (4) and (7).

Owing to the fact that $(\sigma_i^{t})_i^{t'} = \sigma_i^{t'}$, we have, as we would want: if x_i is not free in $\emptyset$, then $V(\emptyset)(t,t',\sigma) = V(\emptyset)(t,t',\sigma_i^{t''})$. We now stipulate: $V(\emptyset)(t,t') = 1$ iff $V(\emptyset)(t,t',\langle t',t',t',\ldots\rangle) = 1$; and $V(\emptyset)(t) = 1$ iff $V(\emptyset)(t,t) = 1$. We now define $\emptyset$ implies ψ as usual: for all interpretations and times, if $V(\emptyset)(t) = 1$ then $V(\psi)(t) = 1$.

Finally, here are two examples of how these definitions match the motivating discussion. (a) $V(PPQ^1x_1)(t) = 1$ iff there is some t'' and some t', with $t'' < t' < t$, and $\langle V(x_1)\rangle \in V(Q^1)(t'')$. (b) $V(P(Px_1x_2)Q^2x_1x_3)(t) = 1$ iff there is some $t' < t$, and there is some $t'' < t$, with $\langle V(x_2),V(x_3)\rangle \in V(Q^2)(t'',t')$. In this example, the operator P puts t' in the second and third position in the sequence of times, and the modifier then puts t'' in the first position; the clause for atomic formulas then ignores the t' in the second position, since x_2 does not occur in the atomic formula $Q^2x_1x_3$ that we evaluate for $V^{x_2}_{x_1}$ and the times $(t,t',\langle t'',t',t',t,t,t,\ldots\rangle)$. ([2])

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