THE EXPLICATIVE SENSE OF FORMAL LOGIC
Some Remarks on the Semantical Basis of Gentzen - Calculi*

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The present article introduces an elementary conception of material and formal truth. This semantics of intuitionistic first order predicate logic results entirely from a philosophical reflexion on the communicative function of elementary assertions. Non-circular definitions of logical operators similar to those given by P. Lorenzen and K. Lorenz are obtained within a normative framework that avoids the conventionalism of gametheoretical conceptions. In particular this approach results in an individual evaluation of the various structural rules of Gentzen - calculi and their different semantical relations with the according logical rules.

I. *The Philosophical Outset*

I.1 *Mode and Content*

It is a common though not generally accepted fact that things go wrong. In case that it is recognized one is forced to distinguish between those things that seem to go wrong and the ways in which we have access to them. In this manner we learn to decide what reason has brought about the failure in question: we either have made things accessible to us inadequately; or although we seem to know how to do things properly – even with words – they develop in their own way. In thought it is much easier to separate these alternatives than it is in life; but, of course, one could not do without thinking.

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In order to establish and maintain our relations with things we have to achieve representations of them, as well for ourselves as for each other. Now, there is no criterion to assume that any immediate access to reality is at our disposal. Otherwise there would have to be an object of knowledge about which we could not be wrong. Therefore we have to conceive our orientation in reality as a successive production of representational units.

But, in order to fulfil their referential task representations have to be determined in at least two interdependent ways:

On the one hand they must refer to something rather than to something else. Otherwise they could not refer to anything at all. In this respect which concerns their content they have *sense.*

On the other hand the order in which they follow one another must not be altogether arbitrary. For if their succession did not matter it would be equally irrelevant what they convey. It would be indifferent to present something rather than something else. Thus there would be no point in doing so at all.

Conversely, it is inconceivable that the order of their production is determined inspite of an indifferent content. Primarily, representations allow to refer. Therefore, initially, their individuation must depend on their content. If it did not matter in what sense they have reference, and thus to what extent they differ, then the order of their production would have to be pointless too.

The role a single representation plays within the successive production of such units cannot get established outside this activity. For if it did this determination would have to be conveyed by yet another unit – and so ad infinitum. That in view of an achieved reference at least not any other may follow has to be in force only because this particular representation has been produced. And insofar as a unit of this kind is a result of human activity the functional determination at issue must count as an *intrinsic* normative aspect of representation. Accordingly each of the units in question will contribute to their successive production in one of the two following ways:

Its realisation either calls for a transition to further representations or it is produced in *accordance* or *contrast* with such an ordered succession.

This intrinsic normative aspect guarantees that a representation makes sense within the context of other, preceding or subsequent productions of this kind. It will be called the *mode* in which a representational unit has sense. In the case where this kind of norm gets installed the unit has a *proper mode*. If its fulfilment is at issue it has an *inverse mode*. (1)

I.2 *Communicative and Phenomenal Representation*

Preliminary considerations have brought about an elementary account of the representational form of human relations with things. In the present context they shall not be developed into a systematic understanding of the cultural form that conditions *all* human orientation in reality. Instead, these basic remarks will help to set forth an account of a particular and most important kind of representation.

What has been said so far concerned forms of reference in general. Whether representation meant, for instance, the form in which experiential orientation is achieved or the medium of communicative interaction was left open. From a methodological point of view, however, we are forced to concentrate first on an understanding of communicative representation. For it is the form of reference that no one can deny to be acquainted with, and any common acquaintance with other forms of representation relies on this one.

Now, an autonomous communicative unit says something about something. The following sentence will serve as an example:

John is coming.

What this unit says does not matter unless it is determined to what end it is said. That, however, has not been mentioned, and in a sense it cannot be said. In one context the sentence might be used in order to *assert* that John is coming. Then the question is whether it is *true*. In other circumstances it could serve as a *description*. Then the point is whether it is an *adequate* rendering of John's situation. Likewise the sentence could count as a mere *assumption* or even as a *promise*.

In the present line of argument the contextual determination at issue has to be accounted for in modal terms. To say something about something does not make sense unless this sentential production contributes to the normative set-up of a progressive representational activity. The manifestation of a sentence either necessitates to pass on to other representations of the same or of a different form. Then a proper mode is in force. Or,

(1) Subsequently the term 'modal' will serve as an adjective derived from 'mode' rather than from 'modality'.
For further information on this conception of modal determination cf. my article 'The Implicit Teleology of Human Experience and Communication'.

conversely, the sentence contributes to or hinders the required transition. Then an inverse mode is given.

If, for instance, the example 'John is coming' is put forward as the premise of a logical implication then it calls for the production of some other unit of this kind like 'John is coming or Mary is leaving.' As a conclusion this second item has an inverse mode. (2)

Normally, a sentence does not say to what end it says something about something. Only in some peculiar, but important cases a sentence will say what its mode is. The following self-referential unit will serve as an instance:

This is an example of an english sentence.

The way in which a communicative unit conveys its modal determination is nevertheless different from saying something about something. For the sentence does not serve as an example simply because it says that it is one. It may well say so for other purposes; for example in order to assert that it is an example of an english sentence. (3)

A distinction between two kinds of representations turns out to be necessary. In the case of communicative units both of them have to be in force. Otherwise it is impossible to answer the question of *how* a sentence conveys the mode in which it has sense: On the one hand it says something about something. On the other hand it *shows* in which mode it is doing so. How do these forms of representation differ from one another? Could one conceive the crucial relationship between them in modal terms?

What a representation shows is accessible to us in its execution. What it says about something is not. A sentence does show *that* it says something – for instance that John is coming. It shows as well that it is doing so in a certain mode – for instance as a description of the situation that

(2) This example points to an essential aspect of the conception of formal logic at issue: Neither is it meant to provide an altogether syntactic account of logical systems in terms of deducibility. Nor is it designed to rely on entirely semantical considerations and, thus, to characterize a formal system in terms of logical consequence. Cf. Ian HACKING 'What is Logic?'; in particular p. 290. The normative unity of mode and transition makes it possible to avoid the schism of these philosophical credi without becoming obliged to accept the adroit agnosticism of a pragmatic approach.

(3) The analyticity of its truth, however, would not be the same as in the following instance:

The one who is coming is coming.

John is coming. But unless it is an analytic and self-referential unit it cannot show *what* it says. ([4])

Now , what a sentence conveys as something it says about something is not known simply because it is said thereof. For in general things are not accessible as they are said to be like just by saying how they are. Otherwise there would be no point in saying something at all.
Conversely, whatever is said, something must have been shown to be that way beforehand. One cannot understand what something is said though not shown, to be like unless something different but of the relevant kind has been accessible previously. Otherwise it is impossible to establish a common repertoire of signs that allow to form and exchange communicative units. ([5])

On the basis of these remarks it is now possible to distinguish two kinds of representational transition and according modes which do not concern a given content as such but solely the *form* of its presentation.

In the primary case something is shown as something in such a way that the achieved representation necessitates to pass on to another unit which says what the given object has been shown to be like. The initial unit has sense in a proper, *exemplary* mode. The subsequent one conveys its content in an inverse, *explicative* mode.

In the second case a representation says something about something in such a way that its production necessitates to pass on to another unit showing the relevant object as it has been said to be like. The initial item has sense in the *assertoric* mode. ([6]) The subsequent unit is meaningful in the *verificational* mode:

If it is asserted that John is coming then saying it requires to show that he does.

In the following this uniform account of explicative and assertoric units shall be used in order to conceive the validity of assertions in terms of

([4]) This distinction between saying and showing does not coincide with Wittgenstein's differentiation in the Tractatus. Cf. nos. 4.022, 4.1212, 4.461, 5.142/3, 6.1264.

([5]) This does not mean that all kinds of concepts are formed in the same way. It is a central task of epistemology to determine the relationship of theoretical notions like 'elementary particle', 'number' or 'revolution' with concepts that in the first instance derive from a phenomenal representation of things.

([6]) It is a most important question in what way a representation shows the mode in which it conveys its content. Unfortunately, the present article cannot deal with this problem. In order to solve it an according theory of appearance is required.

a successive explication of their particular modal order. But before this task is carried out two further remarks seem apropriate:

A communicative unit had to be understood as a kind of representation in which a difference between saying and showing becomes manifest. The modal relationship between these two forms of reference implies that initially representations which say something derive from others that show what these are saying. In both explicative and assertoric contexts a type of representation has to be presupposed that merely shows something as something. Insofar as they do not say anything they cannot be regarded as communicative units. For this reason they will be called *situational representations* or, briefly, *situations*.

The intended explications may not consist in commentaries on the modal order in question. For the phrases employed for this purpose would have to be expressions that had been introduced previously. In the end their definition would have to refer back to the modal order that an assertion enforces and that requires the execution of other, verifying representations. A definite explication of the assertoric mode thus demands to retrace the transitions ordered by its enforcement. For this reason, instead of saying in the explicative mode what kind of transitions they are, it will be shown in abstraction from any particular assertion *how* they are to be realised. In this way a definite *schematic* explication of the assertoric mode will be reached. It would be a mistake though to induce that thereby the explications are replaced by the assertoric transitions themselves. Certainly, the relevant schemata show an essential similarity with them. But they do so in order to say to which modal order they conform. Inspite of its pictorial immediacy a scheme is a communicative unit.

II. *The Modal Account of Elementary Assertion*

II.1 *Affirmation*

With respect to these philosophical considerations the modal order of an elementary assertion a can be presented in the following way:

Sch 0 $\quad a$

$<> \quad \times a$

The pointed brackets indicate the mode in force; ×a stands for the representational end that has to be reached. Whereas the schematic sign for the elementary assertion and the pointed brackets show what *has been* achieved, ×a shows what *has to be* achieved. Subsequently, units of the first kind will be called *actual* signs. Units of the second type will be referred to as *potential* signs.

a and ×a exhibit the transitional aspects of the elementary assertoric order. The pointed brackets, however, present the modal determination as such. Accordingly, the left and the right side of a line in an assertoric scheme will be called its *transitional* and *modal areas*. In order to separate them clearly a stroke will be put between them:

Sch 1_f

$$\begin{array}{rcl} & \mid & a \\ \langle\rangle & \mid & \quad \times a \end{array}$$

Now, there is one important aspect of the elementary assertoric order which the presented schematic explication does not take into account. The required transition cannot be realised unless those for whom a counts as an assertion dispose of the competence to produce the verifying situation. Whether they actually do is relevant here only insofar as it is the content of a which decides what this disposition amounts to. Scheme 1_f exhibits nothing but the modal order of communicative units which merely in view of their form may be regarded as assertoric. (7) In Scheme 1_m an h between the pointed brackets indicates that in view of its content a admits of a material competence to pass adequately from this elementary assertion to its verifying situation:

Sch 1_m

$$\begin{array}{rcl} & \mid & a \\ \langle h\rangle & \mid & \quad \times a \end{array}$$

Only under this condition the attempt to reach the given end will count as a success or as a failure. Each of these two results is explicated if, respectively, one of the two following schematic rules of elementary validation is applied to the last line of Scheme 1_m:

EVS 1

$$\frac{\langle h\rangle \quad \mid \quad \times a}{\odot \quad \mid \qquad\quad}$$

(7) Future contingencies exemplify this kind of sentential units.

EVS 2

$$\begin{array}{c|c} \langle h \rangle & \times a \\ \hline \otimes & \end{array}$$

The separation of the two lines is meant to convey that they are presented as antecedent and conclusion of a rule whose application in an assertoric scheme makes the second line follow the first one. [8]

II.2 *Negation*

The modal order of an elementary negation which denies something of something has to be presented in terms of the mode of the corresponding affirmation. Thus the following scheme is obtained:

Sch 2_m

$$\begin{array}{cc|cc} & & \tilde{a} & \\ a & & & * \\ & \times a & \langle h \rangle & * \end{array}$$

Another pair of elementary validation rules has to be added:

EVS 3

$$\begin{array}{cc|cc} & \times a & \langle h \rangle & * \\ \hline \odot & & & \end{array}$$

EVS 4

$$\begin{array}{cc|cc} & \times a & \langle h \rangle & * \\ \hline \otimes & & & \end{array}$$

As a potenital sign the asterix shows that there is no proper situation to be reached in order to verify the negative assertion. The only indirect way to do so leads via the elementary affirmation a from which the negation is derived. Subsequently, the asterix occurring on its own will be called the *fictitious argument*.

In view of the last two elementary validation rules it is important to notice that a *definite* evaluation pertaining to the modal structure as a

[8] This modal semantics has been developed in preparation of a theory of meaning that offers a rational reconstruction of assertoric modalities like 'necessary' and 'possible'. In this conception a more detailed account of the parameter h and its specification will provide an understanding of assertoric modalities de re and de dicto. Different kinds of orientation in reality and ways of acquiring them will replace 'possible worlds' and 'accessibility relations' respectively. The results of according investigations will be published under the title *Normative Semantik und Logik assertorischer Modalitäten*.

whole has not yet been reached. According rules covering both elementary and complex developments will be presented at a later stage. ([9])

The case of an elementary negation has brought about actual as well as potential signs in the modal area. The general form of a line in an elementary scheme is thus as follows:

Sch 3.1 $\sigma \quad {}^{\circ}\pi \quad | \quad \delta \quad {}^{\circ}\gamma$

σ and δ indicate the places of actual, ${}^{\circ}\pi$ and ${}^{\circ}\gamma$ the places of potential signs.

This division of both the single lines and the elementary schemata they form, into actual and potential parts must serve as a prototype for the kind of complex schemata that will be introduced shortly. For all further specifications of the assertoric mode have to preserve this basic transitional structure. Otherwise they cannot count as differentiations of this one modal set-up.

The kind of assertoric complexity to be presented will only concern the *order* in which parts of an assertion may themselves occur within a modal scheme, and as such have to be explicated with regard to their modal constitution. Units of this kind will be called *arguments*. Likewise, schemata or parts of them in which they are put forward will be referred to as argumentative presentations. The set of schematic rules that determine how arguments have to be put forward and in what way their successive occurence and validation pertains to the value of the initial, most complex unit will be called an *argumentation procedure*. The general form of an according argumentation line is given by the following scheme:

Sch 3.2 $\Sigma \quad {}^{\circ}\Pi \quad | \quad \Delta \quad {}^{\circ}\Gamma$

It differs from its elementary prototype only insofar as the capital greek letters indicate sections in which not just a single but several either actual or potential arguments may be inscribed.

II.3 *The Elementary Conditions of Complex Argumentation*

In all other respects the new argumentation schemata and thus the according rules have to preserve the structural features of elementary asser-

([9]) Cf. p. 20, section III.22.

toric developments. At any stage within a complex scheme it has to be admissible that an elementary argument is about to be explicated. Otherwise the argumentation at issue may not count as an adequate extention of elementary assertoric presentations. Therefore the rules in question will have to meet the following four requirements:

→1 Insofar as lines issue from one another as explications or validations of an argument contained in their immediate predecessors it is required that the relevant operations alternate between the areas of the lines. Each single explication or validation contributes to the presentation of an ordered *global* transition. If it were allowed to perform several of these operations within the same area, and at the same time, this would amount to a schematic rendering of several global transitions instead of a single one.

→2 Accordingly, each argumentation line may arise from merely one explication or validation.

→3 An argumentation scheme may consist of only a *finite* number of lines. For unless the number of intermediate transitions is finite it is impossible to decide whether a global transition has been realised successfully or not.

→4 An assertoric transition has a normative sequential order. After a finite number of steps the development has to reach an end. If the argumentation procedure in question is applied to a complex assertion of the relevant kind then by and by several arguments may occur in the different actual and potential sections of an argumentation line. The sequential order in which they are inserted is thus an integral part of the assertoric order a scheme presents. Therefore an argumentation line does not only consist of a sequence of actual and potential sections. The latter have to be considered as *sequences of arguments* themselves. Their order may not be done away with unless there is sufficient reason to do so.

Obviously enough these requirements are not in particular due to the assertoric kind of an elementary modal set-up. The reasons for their imposition do not relie on the assertoric determination of this order but issue from its normative constitution as such. The task of the present consideration is solely to provide a preliminary understanding of the explicative sense of formal logic. If, additionally, it were to outline the relationship between material and formal orientations in general then the present observation would turn out to be of greatest importance. ([10])

The first and the third demand on a modal explication of complex assertions, i.e. alternation and finiteness, will have to be met by particular *structural* rules of the argumentation procedure. The general presentation of its other rules will comply with the second and the fourth requirement, i.e. uniformity and sequentiality.

III. *The Modal Account of Formally Complex Assertions*

III.1 *Formal Modalities*

It has already been mentioned that the here relevant kind of complexity only concerns the order in which assertoric units are supposed to occur in an argumentation scheme. In this respect the kind of communicative units in question will be called *formally* complex assertions. Subsequently, it will be necessary to introduce specific regulations on the order in which the argumentative components of an assertion have to contribute to the corresponding transition. These specifications of the assertoric mode will be called its *formal modalities*. Expressions indicating these modalities will be referred to as *logical operators*.

Now, assertoric transitions that occur within a global development of this kind can follow one another in only one of three different ways:

In the first case the transition which is initiated first, i.e. by an actual argument in one of the corresponding sections, is succeeded by a development starting in the complementary area. Then a successful execution of the former transition must count as a final coming into force of the order to realise the second development, and thereby the global transition comprising both of them. The according formal modality prescribes a *bilateral succession* of assertoric transitions.

In the two remaining instances the transitions following one another are initiated in the *same* area. The according formal modalities impose a *unilateral succession* of assertoric developments.

([10]) A genetic theory of the relationship between phenomenal and communicative representation as well as material and formal orientation will be ready for publication in due course. It will be presented in a book with the title *Die Bewegung der Referenz*.

On the one hand the successful realisation of the preceding one may count as a successful execution of the development consisting of both transitions. Then, however, it must be required to carry out the second one if the realisation of the first one fails. Otherwise the global transition coincides with its initial part. This kind of formal modality will be called *alternative succession.*

On the other hand the successful initial development may count as an instigation to realise the succeeding transition, and thereby the one they form together. Then, however, the unsuccessful execution of the first transition must amount to a failure of the whole development. Otherwise an alternative succession would be prescribed. This kind of formal modality will be called *connective succession.*

Now, it has to be guaranteed that in a comprehensive scheme the successive realisation of transitions helps to explicate the modal order of a formally complex assertoric unit. For this reason each argumentation line explicating the assertoric constitution of an actual argument will contain an actual unit in the area opposite to the one in which the preceding line presents the explicated argument. Furthermore, it will include a potential argument in the same area. In this way the form of an elementary explication of the assertoric mode will be preserved in the case of these formal differentiations. Accordingly, *modal argumentation rules* will be presented as follows:

SCH 4

$$\frac{\Sigma \qquad {}^{\circ}\Pi \mid \Delta\ A\ {}^{\circ}\Gamma \qquad \mid}{\Sigma\ \bullet A\ {}^{\circ}\Pi \mid \Delta \qquad {}^{\circ}\Gamma\ {}^{\circ}A \mid}$$

Again, the separation of the argumentation lines indicates that a rule, and not a scheme formed by its application is presented. The stroke at the end of the lines says that the rule is valid for both areas. In order to maintain the general sequential order, each of the relevant arguments must occur at the very end of its section. Furthermore, an argument under consideration must not reappear in the succeeding line. For each actual unit may be explicated only once.

Unless a potential unit is either fictitious or indicates a situational end its actualisation will admit of its explication, and thus allows for the execution of an assertoric transition within a global development. The introduced kinds of succession require different types of argumentative actualisation. They will be presented by *transitive argumentation rules* of the following form:

SCH 5

$$\frac{\Sigma \quad {}^{\circ}\Pi \mid \Delta \qquad {}^{\circ}\Gamma \ {}^{\circ}A \quad \mid}{\Sigma \quad {}^{\circ}\Pi \mid \Delta \ A' \ {}^{\circ}\Gamma \qquad \mid}$$

The three types of succession at issue and the complementary forms of validation will be accounted for in terms of the way in which arguments are inserted into the different sections of a line.

If the order is bilateral then one explicable argument will occur in the opposite actual section whereas the other unit is introduced by a potential item in the same area.

If the succession is unilateral then in both instances there will be an indication of the relevant formal modality in the opposite actual area. This *purely modal* unit cannot be subjected to an explication. In the alternative case the succession will be initiated in the potential section of the area in which the explicated argument was presented. In the connective case the succession will start in the actual section of that area.

In order to present these unilateral modalities adequately it is necessary to introduce a particular kind of *structural argumentation rules.* They will make it possible to split the explication of a modality into a unilateral succession of partial explications separated from one another in a potential or actual section.

These *reduplication rules* will have the following form:

SCH 6.1

$$\frac{\Sigma \quad {}^{\circ}\Pi \mid \Delta \ A^{\sigma} \qquad\qquad {}^{\circ}\Gamma \quad \mid}{\Sigma \quad {}^{\circ}\Pi \mid \Delta \ A^{\sigma-1} \ A^{\sigma-1} \quad {}^{\circ}\Gamma \quad \mid}$$

$\sigma \leqslant n$, and n is an ordinal number.

– in the case of a connective reduplication

SCH 6.2

$$\frac{\Sigma \quad {}^{\circ}\Pi \mid \Delta \quad {}^{\circ}\Gamma \ {*A^{\tau}} \qquad\qquad \mid}{\Sigma \quad {}^{\circ}\Pi \mid \Delta \quad {}^{\circ}\Gamma \ {*A^{\tau-1}} \ {*A^{\tau-1}} \quad \mid}$$

$\tau \leqslant n$, and n is an ordinal number.

– in the case of an alternative reduplication. ([11])

([11]) Semantical rules of this kind justify the contraction rules of a Gentzen - calculus. It turns out that their introduction is not independent of any presentation of its logical rules, but helps to explicate the formal modalities of unilateral seccession.

A transformation that is due to one of these rules does not explicate the assertoric order an argument enforces, but helps to realise a schematic presentation of this kind. Therefore, a reduplication will not count in view of the requirement to modify the areas of argumentation lines alternatively.

The introduced kind of structural transformation increases the number of arguments in a line without lowering their formal complexity. For this reason the according rules must comply explicitely with the requirement that all assertoric transitions consist in a finite number of lines. This is guaranteed by the decreasing reduplication rank σ or τ which is assigned to each candidate of this kind of modification. A supplementary structural rule will specify how these assignments are brought into accord with the sequential order in which the sections of argumentation lines become relevant.

Before the argumentation procedure AR is presented in detail one final remark will concern the requirement that in general arguments have to be arranged sequentially: Actual units will occur in sequences only insofar as they are inserted into their sections in the order in which their potential precursors were lined up. As arguments explicable in their own modal constitution, they are on a par with one another. For this reason it will be admitted to change the sequential order of an actual section. Moreover, as in the present context merely the modal constitution of assertoric units is at issue the argumentation rules will not impose any particular order on a choice between several candidates that are available for the start of a partial transition. As assertoric units they are equal. For the same reason no order is imposed if in the development of a scheme a choice between the application of modal and transitional argumentation rules opens up. (12)

III.2 *The Argumentation Procedure AR*

The preceding considerations now justify to present an argumentation procedure AR consisting of the following set of structural, modal and transitive argumentation rules and according schematic validations.

In their subsequent presentation superscripts 'ρ', 'σ' and 'τ' will mention any reduplication rank. 'r', 's' and 't' will be used where it equals the reduplication measurement of a section. If no superscript is used the rank is 1.

(12) These considerations justify the structural rules of a Gentzen - calculus accounting for the interchangeability of formulae.

III.21 *Argumentation Rules*

$AR_{ST}1$ In a proper argumentative move either a modal or a transitive argumentation rule or a situational validation is applied. A passage of this kind requires that by turns the areas of the lines are either extended by an actual sign or shortened by a potential one.
The first intermediate line is a modal one.

$AR_{ST}2$

$$\frac{\Sigma \quad {}^{\circ}\Pi \quad | \; \Delta \; \{A^{\rho}\; B^{\sigma}\} \quad {}^{\circ}\Gamma \quad |}{\Sigma \quad {}^{\circ}\Pi \quad | \; \Delta \; \{B^{\sigma}\; A^{\rho}\} \quad {}^{\circ}\Gamma \quad |} \qquad \text{(f)}$$

$AR_{ST}3.0$ If an argumentation requires reduplications then at the outset a constructive, if necessary transfinite ordinal number has to be chosen for each of the four sections. The order in which the choice has to be made is fixed by $AR_{ST}1$: first the number is given for the actual section of the transitive area, then for the potential section of the modal area, and subsequently for the remaining sections.
These *reduplication measurements* are to be listed above the first line:

$$\begin{array}{ll|ll} n & k & l & m \\ & & A & \end{array}$$

To each argument a number, its reduplication rank is added. It indicates to what extent the unit in question may be reduplicated:
1. If an argument is listed without being a duplicate of an other unit its number is the reduplication measurement of its section.
2. If an argument results from a reduplication and if the rank of the unit to which the rule was applied is the successor β' of an ordinal number β then the latter is the rank of the new argument.
3. If the rank of an argument which is to be reduplicated is a limit number λ then an ordinal β is assigned to the resulting unit such that $\beta < \lambda$.
4. An argument which has the rank 1 may not be reduplicated.

$\mathrm{AR}_{\mathrm{ST}}3.1$

$$\frac{\Sigma \quad {}^{\circ}\Pi \quad \mid \Delta \; A^{\sigma} \qquad {}^{\circ}\Gamma \quad \mid}{\Sigma \quad {}^{\circ}\Pi \quad \mid \Delta \; A^{\sigma-1} \; A^{\sigma-1} \; {}^{\circ}\Gamma \quad \mid}$$

$\sigma \leqslant s$, and s is the reduplication measurement of the section.

$\mathrm{AR}_{\mathrm{ST}}3.2$

$$\frac{\Sigma \quad {}^{\circ}\Pi \quad \mid \Delta \quad {}^{\circ}\Gamma \; *A^{\tau} \qquad \mid}{\Sigma \quad {}^{\circ}\Pi \quad \mid \Delta \quad {}^{\circ}\Gamma \; *A^{\tau-1} \; {}^{\circ}A^{\tau-1} \quad \mid}$$

$\tau \leqslant t$, and t is the reduplication measurement of the section.

$\mathrm{AR}_{\mathrm{T}}1$

$$\frac{}{\mid A^{s}}$$

$\mathrm{AR}_{\mathrm{T}}2$

$$\frac{\Sigma \quad {}^{\circ}\Pi \quad \mid \Delta \qquad {}^{\circ}\Gamma \; *A^{\tau} \quad \mid}{\Sigma \quad {}^{\circ}\Pi \quad \mid \Delta \; A^{s} \quad {}^{\circ}\Gamma \qquad \mid}$$

$\mathrm{AR}_{\mathrm{T}}3.1$

$$\frac{\Sigma \quad {}^{\circ}\Pi \quad \mid \Delta \qquad {}^{\circ}\Gamma \; *AB^{\tau} \quad \mid}{\Sigma \quad {}^{\circ}\Pi \quad \mid \Delta \; A^{s} \quad {}^{\circ}\Gamma \qquad \mid}$$

$\mathrm{AR}_{\mathrm{T}}3.2$

$$\frac{\Sigma \quad {}^{\circ}\Pi \quad \mid \Delta \qquad {}^{\circ}\Gamma \; *AB^{\tau} \quad \mid}{\Sigma \quad {}^{\circ}\Pi \quad \mid \Delta \; B^{s} \quad {}^{\circ}\Gamma \qquad \mid}$$

$\mathrm{AR}_{\mathrm{T}}4$

$$\frac{\Sigma \quad {}^{\circ}\Pi \quad \mid \Delta \qquad {}^{\circ}\Gamma \; *A(x)^{\tau} \quad \mid}{\Sigma \quad {}^{\circ}\Pi \quad \mid \Delta \; A(n)^{s} \quad {}^{\circ}\Gamma \qquad \mid}$$

$\mathrm{AR}_{\mathrm{M}}1_{m}$

$$\frac{\Sigma \qquad {}^{\circ}\Pi \quad \mid \Delta \; a^{\sigma} \quad {}^{\circ}\Gamma \qquad \mid}{\Sigma \; \langle h \rangle \; {}^{\circ}\Pi \quad \mid \Delta \qquad {}^{\circ}\Gamma \; \times a \quad \mid}$$

$\mathrm{AR}_{\mathrm{M}}2$

$$\frac{\Sigma \qquad {}^{\circ}\Pi \quad \mid \Delta \; \neg A^{\sigma} \quad {}^{\circ}\Gamma \qquad \mid}{\Sigma \; A^{r} \; {}^{\circ}\Pi \quad \mid \Delta \qquad {}^{\circ}\Gamma \; * \quad \mid}$$

$\mathrm{AR}_{\mathrm{M}}3$

$$\frac{\Sigma \qquad {}^{\circ}\Pi \quad \mid \Delta \; A \rightarrow B^{\sigma} \quad {}^{\circ}\Gamma \qquad \mid}{\Sigma \; A^{r} \; {}^{\circ}\Pi \quad \mid \Delta \qquad {}^{\circ}\Gamma \; *B^{t} \quad \mid}$$

$\mathrm{AR}_{\mathrm{M}}4.1$

$$\frac{\Sigma \qquad {}^{\circ}\Pi \quad \mid \Delta \; A\&B^{\sigma} \quad {}^{\circ}\Gamma \qquad \mid}{\Sigma \; 1? \; {}^{\circ}\Pi \quad \mid \Delta \qquad {}^{\circ}\Gamma \; *A^{t} \quad \mid}$$

$\mathrm{AR}_{\mathrm{M}}4.2$

$$\frac{\Sigma \qquad {}^{\circ}\Pi \quad \mid \Delta \; A\&B^{\sigma} \quad {}^{\circ}\Gamma \qquad \mid}{\Sigma \; 2? \; {}^{\circ}\Pi \quad \mid \Delta \qquad {}^{\circ}\Gamma \; *B^{t} \quad \mid}$$

$$\mathrm{AR_M5}\qquad \frac{\Sigma \quad\ \ {}^{\circ}\Pi \mid \Delta\ A\vee B^{\sigma}\quad {}^{\circ}\Gamma \qquad \mid}{\Sigma\ \ ?\ {}^{\circ}\Pi \mid \Delta \qquad\quad {}^{\circ}\Gamma\ {*}AB^{\iota} \mid}$$

$$\mathrm{AR_M6}\qquad \frac{\Sigma \quad\ \ {}^{\circ}\Pi \mid \Delta\ \wedge\times A(x)^{\sigma}\quad {}^{\circ}\Gamma \qquad \mid}{\Sigma\ ?n\ {}^{\circ}\Pi \mid \Delta \qquad\quad {}^{\circ}\Gamma\ {*}A(n)^{\iota} \mid}$$

$$\mathrm{AR_M7}\qquad \frac{\Sigma \quad\ \ {}^{\circ}\Pi \mid \Delta\ \vee\times A(x)^{\sigma}\quad {}^{\circ}\Gamma \qquad \mid}{\Sigma\ \ ?\ {}^{\circ}\Pi \mid \Delta \qquad\quad {}^{\circ}\Gamma\ {*}A(x)^{\iota} \mid}$$

The first structural rule conveys the first structural demand on a schematic explication of formally complex assertions.

The second structural rule expresses the interchangeability of actual arguments. '(f)' indicates that the rule may be applied only finite many times.

The third structural rule presents the reduplication rules that allow for the explication of unilateral formal modalities.

A preliminary instruction secures the finite application of these rules in accordance with the sequential structure of an assertoric scheme.

The first reduplication rule relates to connective successions. The second one concerns alternative successions.

Their general formulation takes into account that bilateral successions and indirect transitions enforced by negative assertions may occur in the context of argumentation schemata presenting unilateral successions as well. Otherwise rules of this kind would have to be introduced for A&B, $\wedge\times A(x)$, $*AB$, and $*A(x)$ individually.

The first transitive argumentation rule brings forward the initial line of an argumentation scheme.

By the second rule of this kind a bilateral or connective succession is established once it has been initiated by the corresponding modal and structural rules.

In the case of a bilateral development the third modal rule must have been applied previously.

In the case of a connective succession applications of the first reduplication rule and the fourth modal rule must go before. If the quantificational version of a connective succession is at issue applications of the sixth instead of the fourth modal rule have to precede.

By the third transitive rule an alterative succession is established if it has been set up by applications of the fifth modal and the second reduplication rule.

The fourth transitive rule helps to realise the quantificational version of an alternative succession. For this purpose the seventh modal rule and the second reduplication rule must have been applied beforehand.

The first modal argumentation rule presents the explication of an elementary affirmation in the context of an argumentative scheme.

The second rule of this kind establishes the indirect transition a negative assertion makes necessary. The case that the assertion at issue may be formally complex is taken into account. From a formal point of view a special rule for the case of an elementary negation is redundant.

As it has already been mentioned the remaining modal rules introduce bilateral and unilateral succession together with their quantificational versions.

III.22 *Evaluations*

Complementary to these explicative rules, and with respect to the indirect, bilateral, alternative or connective form of a global transition the following schematic validation rules have to be added:

STVS 1
$$\frac{\Sigma \; <h> \; {}^\circ\Pi \mid \Delta \; A^\sigma \; {}^\circ\Gamma \; \times\alpha \mid}{\Sigma \qquad {}^\circ\Pi \mid \Delta \; A^\sigma \; {}^\circ\Gamma \qquad \mid} \qquad \odot$$

STVS 2
$$\frac{\Sigma \; <h> \; {}^\circ\Pi \mid \Delta \qquad {}^\circ\Gamma \; {*}A^\tau \; \times\alpha \mid}{\Sigma \qquad {}^\circ\Pi \mid \Delta \; A^s \; {}^\circ\Gamma \qquad \mid} \qquad \otimes$$

These two *strategic validation schemata* account for an intermediate evaluation in the context of a unilateral succession: If an elementary transition or an uninterrupted series of these passages is successful in a connective or unsuccessful in an alternative context it is requried to continue the assertoric development. '$\times\alpha$' instead of '$\times$a' stands for an uninterrupted series of situational ends.

DVS 1
$$\frac{\Sigma \qquad {}^\circ\Pi \mid \qquad {}^\circ\Gamma \; \times\alpha \mid}{\Sigma \qquad {}^\circ\Pi \mid \odot \quad {}^\circ\Gamma \qquad \mid}$$

DVS 2
$$\frac{{}^\circ\Pi \mid \Delta \qquad {}^\circ X \; \times\alpha \mid}{{}^\circ\Pi \mid \Delta \; \otimes \; {}^\circ X \qquad \mid}$$

These two *definite validation schemata* account for a final evaluation in a context of unilateral succession:

If an uninterrupted series of elementary transitions is successful in a context which may be alternative though not connective, then, it is a definitely successful development.

If an uninterrupted series of elementary transitions fails in a context which may be connective though not alternative, then it is definitely a failure. For this purpose '°X' indicates that a sequence of potential arguments either is empty or ends with the fictitious argument.

The remaining four *argumentative validation schemata* embody the evaluation of an assertoric development as a whole with respect to the contextual conditions under which a final argumentation line has been reached.

In the first case a definitely positive validation has been achieved in the transitive area. Then, the argumentative transition as a whole must count as a successful one. For a definitely positive development in the modal area merely would have confirmed the requirement to end a complementary transition with the same result. Accordingly, there are no particular demands on the arrangement of the modal area. (13)

AVS 1

$$\frac{\Sigma \quad {}^{\circ}\Pi \mid \odot \quad {}^{\circ}\Gamma}{\odot}$$

In the second case an unsuccessful development ending in the transitive area counts as a global failure. It is a necessary condition of this result that no further actual arguments are inscribed in the modal area. For otherwise a development ending there without success could have abolished the order to achieve a definitely positive transition on the other side. Accordingly, the modal area of a final argumentation line of this kind may not contain actual arguments.

AVS 2

$$\frac{{}^{\circ}\Pi \mid \Delta \otimes \quad {}^{\circ}X}{\otimes}$$

(13) For this reason the structural rule of dilution or weakening in the antecedent is justified.

In the third case a definitely positive development ending in the modal area counts as an unsuccessful global transition. It is a necessary condition of this result that the transitive area does not offer an opportunity to let follow a development which ends successfully in the transitive area. Therefore this side of the final argumentation line either does not contain any potential arguments at all, or lists the ficitious one last.

AVS 3 $\dfrac{\odot \quad {}^{\circ}\Pi \quad | \quad \Delta \quad {}^{\circ}X}{\otimes}$

In the fourth case an unsuccessful end in the modal area accounts for a positive global development. Under the given circumstances the requirement to realise a transition that ends successfully on the opposite side finally has not been established. Therefore it cannot matter what arguments the transitive area contains, and accordingly there are no demands on its arrangement. ([14])

AVS 4 $\dfrac{\Sigma \quad \otimes \quad {}^{\circ}X \quad | \quad \Delta \quad {}^{\circ}\Gamma}{\odot}$

III.3 *Material Truth*

A formally complex argumentation scheme may develop into a presentation of several global transitions starting from the same initial unit. They can end with different results although, of course, with respect to the same competence h an elementary transition occuring at different places must always have the *same* outcome. The modal order an argumentation scheme exhibits is not itself a material one. That it is presented in space and time does not affect the formal assertoric relations between the presented arguments.

As a formally complex argumentation scheme thus may consist of a finite number of argumentation threads of different value the truth or falsity of an assertion has to be defined with respect to the various argumentative options it opens up. For this purpose *material argumenta-*

([14]) For this reason the structural rule of dilution or weakening in the succedent is justified.

tion strategies for and against formally complex assertions are introduced as follows:

$D_{MA}f$ A material argumentation strategy *for* an assertion A is given if, and only if, the materially successful end of an argumentation thread is reached irrespective of the choice of explicative and evaluative passages realised in the modal area.

$D_{MA}ag$ A material argumentation *against* an assertion A is given if, and only if, the materially unsuccessful end of an argumentation thread is reached irrespective of the choice of explicative and evaluative passages realised in the transitional area.

D_{MT} An assertion A is *materially true* if, and only if, there is a material argumentation strategy for A.

D_{MF} An assertion A is *materially false* if, and only if, there is a material argumentation strategy against A.

IV. *Formal Argumentation*

IV.1 *Formal Validity*

A particular kind of argumentation line is determined by the following constellation:

In the case of a bilateral succession the definite establishment of the assertoric order in the modal area calls for a conclusive execution of a transition that ends on the opposite side and, possibly, consists in a formally complex development. Now, it can happen that all elementary actual arguments which for this purpose are inserted into the transitive area have already been listed in the modal one. Then, the argumentation thread under consideration must be a valid path no matter what the outcome of the relevant elementary transitions would be. Under the described circumstances the confirmation of the bilateral assertoric order embodies the development that in view of its transitive role decides upon the success of the overall transition.

This constellation does not have to be explicated in terms of elementary material transitions; though it must be compatible with an account of this kind. Primarily, however, the necessarily successive presentation of modal and transitive aspects of the assertoric order is responsible for this kind of evaluation. It relies on the *form* of this explication, and therefore neither depends on the content of elementary assertions nor on the existence of an according material competence. For these reasons the introduced kind of validity will be referred to as the *formal validity* of formally complex assertions.

Parallel to the presented material argumentation procedure AR a formal argumentation procedure ARF is now at hand. It contains the same argumentation rules as its material complement except for the elementary modal rule . At present the question whether the content of an elementary unit admits of an adequate material competence is irrelevant. Therefore the explication of an elementary assertoric order is replaced by a schematic presentation merely of its form:

$$AR_M 1_f \qquad \begin{array}{llllll} \Sigma & & {}^\circ\Pi & \mid \Delta\ a^\sigma & {}^\circ\Gamma & \mid \\ \hline \Sigma & <> & {}^\circ\Pi & \mid \Delta & {}^\circ\Gamma \times a & \mid \end{array}$$

Instead of material validation schemata two *formal validation rules* have to be added:

$$STFVS \qquad \begin{array}{llllll} \Sigma\ a & {}^\circ\Pi & \mid \Delta\ a & {}^\circ\Gamma & [T] \\ \hdashline \Sigma & {}^\circ\Pi & \mid \Delta & {}^\circ\Gamma & [T] \end{array}$$

where °Π is not °X or Δ is not empty.

The *strategic formal validation scheme* accounts for a formal constellation comprising a connective context in the transitive area or an alternative one in the modal area. '[T]' indicates that the relevant argumentation line is due to the application of a modal or transitive rule in the transitive area.

$$ARFVS \qquad \begin{array}{lllll} \Sigma\ a & {}^\circ X & \mid a & {}^\circ\Gamma & [T] \\ \hdashline & & + & & \end{array}$$

The *argumentative formal validation scheme* accounts for a formal constellation in which all unilateral contexts are exhausted. Under this condition the potential section of the modal area is empty or ends with the fictitious argument whereas the actual section of the transitive area may only contain the relevant elementary unit.

IV.2 *Formal Truth*

Like in the case of AR argumentation schemata formed on the basis of ARF may consist of a finite number of argumentation threads. Not all of these paths must turn out to be formally valid. Therefore the formal truth or falsity of a unit of assertoric form has to be defined with respect to the various argumentative options it opens up. Accordingly, a formal argumentation strategy for a unit A of assertoric form is determined as follows:

$D_{FA}f$ A formal argumentation strategy *for* a unit A is given if, and only if, the formally successful end of an argumentation thread is reached irrespective of the choice of explicative passages realised in the modal area.

If with respect to elementary actual arguments occuring in both areas the successive presentation of modal and transitive aspects of the assertoric order is reversed a *formally invalid* constellation is reached. It is not necessary, though, to introduce according validation schemata in order to determine the notion of a formal argumentation strategy against a unit A. If the latter is defined in terms of a formal argumentation strategy for $\neg$ A, then, the inverted sequence is accounted for via the original order of succession. For this reason further validation schemata are redundant.

$D_{FA}ag$ A formal argumentation strategy *against* a unit A is given if, and only if, there is a formal argumentation strategy for $\neg$ A.

D_{FT} An assertion A is *formally true* if, and only if, here is a formal argumentation strategy for A.

D_{FF} An assertion A is *formally false* if, and only if, there is a formal argumentation strategy against A.

Some examples will help to make out the kind of formulae that with reference to the introduced formal argumentation procedure ARF represent formally true assertoric units. As the subsequent schemata will show, there is an according argumentation strategy for $\neg(a \& \neg a)$ as well as $a \rightarrow ((a \rightarrow (a \rightarrow b)) \rightarrow b)$. No such strategy is available for $a \vee \neg a$ or $((a \rightarrow b) \rightarrow a) \rightarrow a$.

Sch 7

	2	1	|	1	1
			|	¬(a&¬a)	
	$a\&\neg a^2$		|		*
a&¬a	a&¬a		|		*
	a&¬a	*¬a	|	2?	*
¬a	a&¬a		|		*
	¬a	*a	|	1?	*
¬a	a		|		*
	a	*	|	a	*
			+		

Sch 8

		2	1	|	1	1	
				|	a→((a→(a→b))→b)		
		a^2		|		*(a→(a→b))→b	
		a^2		|	(a→(a→b))→b		
	a^2	$a\to(a\to b)^2$		|		*b	
a	a	$a\to(a\to b)^2$		|		*b	
a	a		*a→b	|	a	*b	[T]
	a		*a→b	|		*b	[T]
	a	$a\to b^2$		|		*b	
	a		*b	|	a	*b	[T]
			*b	|		*b	[T]
		b^2		|		*b	
		b b		|		*b	
		b b		|	b		[T]
				+			

Sch 9

1	1	|	1	2
		|	av¬a	
?		|		$*a\neg a^2$
		|		*a¬a *a¬a
		|	¬a	*a¬a
a		|		*a¬a *
	×a	|	< >	*a¬a *

Sch 10

1	1	|	1	2
		|	((a→b)→a)→a	
(a→b)→a		|		$*a^2$
	*a	|	a→b	$*a^2$
a	*a	|		$*a^2$ $*b^2$

These instances suggest that in the given argumentative perspective just those units can be shown to be formally true that in terms of the game-theoretical semantics of P. LORENZEN and K. LORENZ ([15]) are known to be *effectively true*. Indeed the introduced formal argumentation procedure makes it possible to characterise GENTZEN's calculus IL ([16]) as a set of rules that enumerates all those formally valid argumentation lines with empty potential sections that may contain only one argument in their transitive area.

A proof of this thesis sets out from the introduction of a reduced formal argumentation procedure ARFR that is admissible with respect to the units A it proves to be formally true. Furthermore an inductive definition of formally valid argumentation lines is given. Then, with respect to ARFR a calculus $\mathcal{A}$ is shown to be a consistent and complete enumeration of the formally valid argumentation lines it allows to derive. In both directions the result is obtain by induction on the formal complexity of the arguments. Finally, all potential units are replaced by those actual ones from which they derive by application of an according rule of $\mathcal{A}$. In this way IL is shown to list a subset of the argumentation lines enumerated by $\mathcal{A}$. They are precisely those whose potential sections are empty. ([17])

The outlined normative semantics of formal logic is greatly indebted to Dialogic Logic. From a technical point of view the concept of an argumentation line is closely related to the notion of a *reduced dialogue position* in the formalisation of Dialogic Logic by K. LORENZ. ([18]) But, certainly, there are further reaching similarities. The present conception keeps the non-circular introduction of logical operators for which game-theoretical semantics has become well known. The finite applicability of the argumentation procedure was accounted for in almost the same way as in the case of dialogue rules. Basically, this modal account of assertion adheres to the same criterion of constructivity as Dialogic Logic.

On the other hand there should be differences justifying the effort to deviate from a given model.

([15]) Cf. the various articles contained in: P. LORENZEN and K. LORENZ, *Dialogische Logik*.

([16]) Cf. for instance G. GENTZEN, 'Untersuchungen über das logische Schließen'.

([17]) A detailed presentation of this proof will be contained in a publication on *Der Modus materialer und formaler Wahrheit*. This work is about to be finished.

([18]) Cf. K. LORENZ, 'Dialogspiele als semantische Grundlage von Logikkalkülen' in *Dialogische Logik*, p. 131 - 134.

A pragmatic conception of assertoric validity runs the risk of conventionalism. ([19]) Dialogue games consist of two kinds of rules. The general outline of dialogues is established by set of *structural rules*. The particular kinds of attacks and defenses admitted in a dialogue are given by a set of *argument rules*. A justification of these normative determinations may not depend on the semantical purpose they serve. For the argument rules of Dialogic Logic K. LORENZ has shown that their introduction exhausts the possibilities to present such rules within the given dialogue structure. ([20]) This general context, however, is put forward as a natural order that should be acceptable as long as no weighty reservations come to one's mind.

For the outlined semantics the notion of a mode in which an assertion has sense justifies the admission of all structural rules corresponding to those needed for a dialogic account of both material and formal truth. Right from the start the point was not to design a set of rules for the exchange of arguments and to argue for their being adequate to the end for which sentences are asserted. On the contrary the very fact that representations essentially have an end was made the starting point for a reflexion as well on particular kinds of communicative purposes as on adequate ways of presenting them. The modal unity of these two concerns determined the complementary relationship between explication and assertion on the one hand, mode and transition on the other hand. Formal validity was presented as the point of intersection of both these sequential orders.

In the third and fourth of the above examples the formulae at issue could not be proved to be formally true because of the sequential order in the potential sections. Structural rules admitting to interchange potential arguments would have opened up further strategic possibilities.

For instance the following rules might have been applied:

$$AR_{ST}cl \qquad \frac{\Sigma \ \ {}^\circ\Pi \ \mid \Delta \ \ {}^\circ\Gamma \ \{{}^\circ A \ \ {}^\circ B\} \ \mid}{\Sigma \ \ {}^\circ\Pi \ \mid \Delta \ \ {}^\circ\Gamma \ \{{}^\circ B \ \ {}^\circ A\} \ \mid}$$

– in the case of $a \vee \neg a$.

([19]) Cf. W. STEGMÜLLER, 'Remarks on the Completeness of Logical Systems relative to the Validity-Concepts of P. Lorenzen and K. Lorenz'.

([20]) Cf. K. LORENZ, 'Dialogspiele als semantische Grundlage von Logikkalkülen' in *Dialogische Logik*, p. 109 - 120.

$$AR_{ST}clp \qquad \frac{\Sigma \ {}^{\circ}\Pi \quad | \ \Delta \quad {}^{\circ}\Gamma \ \{*a \ *b\} \quad |}{\Sigma \ {}^{\circ}\Pi \quad | \ \Delta \quad {}^{\circ}\Gamma \ \{*b \ *a\} \quad |}$$

– in the case of $((a \rightarrow b) \rightarrow a) \rightarrow a$.

Their admission amounts to an entire or partial neutralisation of the sequential relationship between actual and potential arguments. Rules of this kind cannot abolish the successive explication of mode and transition in principle. But the neutralisation they embody implies the assumption that in one way or another the order in which a formally complex assertion is to be verified does not matter.

It is an important question whether the outlined semantics allows to prove that every concept of formal validity corresponding to a proper extension of IL results from a weaker version of the basic sequential order. The answer will pertain to the philosophical substance of this modal semantics.

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