SYLLOGISTICS AND SOME OF ITS EXTENSIONS IN THE CONTEXT OF RELATIONAL LOGIC

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1. *Contractions of a Relation Schema*

Let R^2_n be a binary relation schema of dimension n, with $n > 2$; and let $R_1, R_2, \ldots, R_{n-1}, R_n$ be its basic relations, and X the associated domain of elements x on which the basic relations are defined. (1) If we introduce R'_{n-1} as the disjunction $R_{n-1} \vee R_n$ and put $R'_i = R_i$ for $i < n-1$, then the relations $R'_1, R'_2, \ldots, R'_{n-1}$ form the basis of a new relation schema of dimension $n-1$. For, to each of these $n-1$ basic relations there is at least one ordered couple (x, x') of elements of X to which it applies; conversely, to every ordered couple of elements of X one, and only one, of the relations $R'_1, R'_2, \ldots, R'_{n-1}$ applies. Instead of R_{n-1} and R_n we may take any other pair R_j, R_m from the original basis and replace them by their disjunction $R'_j = R_j \vee R_m$, with $j < m$. In each case we obtain a new basis consisting of $n-1$ basic relations, and hence a new relation schema R'^2_{n-1} (of dimension $n-1$). The new basic relations are written and enumerated as follows: $R'_k = R_k$ for $1 \leqslant k \leqslant m-1$, $k \neq j$, $R'_j = R_j \vee R_m$, and $R'_{k-1} = R_k$ for $m \leqslant k \leqslant n$. I shall call such a transition from R^2_n to R'^2_{n-1} a *simple contraction* of R^2_n. If R'^2_{n-1} is thus contracted again, to R''^2_{n-1}, then the latter will be said to result from R^2_n by a twofold contraction. In a similar way threefold, fourfold etc contractions may be defined for relation schemata of appropriate dimensions. A relation schema R^2_n $(n > 2)$ gives rise to $n(n-1)/2$ simple contractions.

(1) The reader is referred to "On the Logic of Relations", *Dialectica*, 34 (1980), 167-82; "On an Application of Truth-functions to the Logic of Predicates", *Logique et Analyse*, 101 (1983), 3-18, and "On the Generalised Converse in Relational Logic", *Logique et Analyse*, 105 (1984), 63-67. In these publications I have explained some of the crucial terms (e.g. 'relation schema', 'generalised converse', 'regular', 'counterposition', 'syllogistic function') in more detail.

Let C_n be the generalised converse (or g-converse, for short) of a relation schema R_n^2 ($n > 2$) and $F_n(2,1)$, $F_n(1,1)$, $F_n(2,2)$ and $F_n(1,2)$ its syllogistic functions, C_n and $F_n(2,1)$, $F_n(1,1)$, $F_n(2,2)$, $F_n(1,2)$ the matrices representing those correspondences. If the schema R'^2_{n-1} derives from R_n^2 by a simple contraction with $R_j' = R_j \vee R_m$ ($j < m$), then the elements c'_{ik} of C'_{n-1} (representing the g-converse of R'^2_{n-1}) are determined as follows. The mth column of C_n is added to its jth, and the mth row to the jth row; in the matrix thus obtained column number k ($k = m+1, m+2, ..., n$) replaces column number $k-1$, and row number k ($k = m+1, m+2, ..., n$) replaces row number $k-1$. Each of the elements of the matrices C_n and C_{n-1} is either 0 or 1, and the elements are added in accordance with the rules: $1+1 = 1+0 = 0+1 = 1+1 = 1$, $0+0 = 0$.

This way of deducing C'_{n-1} depends upon the fact that C_n is linear: $C_n(R_j \vee R_m) = C_n(R_j) \vee C_n(R_m)$. As $F_n(2,1)$, $F_n(1,1)$, $F_n(2,2)$ and $F_n(1,2)$ are also linear, similar considerations apply to them. That is, the matrices $F'_{n-1}(2,1)$, $F'_{n-1}(1,1)$, $F'_{n-1}(2,2)$, and $F'_{n-1}(1,2)$ for the contracted schemata are obtained from the matrices $F_n(2,1)$, $F_n(1,1)$, $F_n(2,2)$, and $F_n(1,2)$, by applying an analogous procedure.

2. *Expansions and Regularisations of a Relation Schema*

A relation schema R_n^2 is an expansion of a schema R_m^2 ($m < n$) if, and only if, R_m^2 can be obtained from R_n^2 by a series of contractions. R_n^2 is a simple expansion of R_{n-1}^2 if, and only if, R_{n-1}^2 is a simple contraction of R_n^2. Relation schemata with widely differing features may be obtained by expanding a given schema, depending upon the way basic relations are split up into new basic relations. A simple expansion need not preserve the regular or stable character of a schema; but it will lead to an alternating schema if the original schema is an alternating one.

One important type of expansion leads from non-regular relation schemata, to regular ones which can be defined in a relatively simple way in terms of the original schemata. Let R_n^2 be a binary non-regular relation schema, C_n its g-converse and C_n the corresponding matrix. The basis of the schema must then contain at least one basic relation R_i such that $C_n(R_i)$ has at least two components: $C_n(R_i) = R_j \vee R_k \vee \ldots$. This enables us to define new relations R_{i_1}, R_{i_2}, ... as follows:

R_{i_1} (x,x') holds if, and only if, $R_i(x,x') \cdot R_j(x',x)$;
R_{i_2} (x,x') holds if, and only if, $R_i(x,x') \cdot R_k(x',x)$

and so on, using all the components of $C(R_i)$ (among which, of course, may be R_i itself). Obviously, whenever R_i applies to an ordered couple (x,x'), then one, and only one, of the relations R_{i_1}, R_{i_2}, ... holds for (x,x'). For the sake of having a convenient term, let me refer to the right hand side of the above expressions as the c-conjunctions (e.g. of R_i and R_j). To determine the order of the new basic relations into which R_i is split, we stipulate that the new basic relation equivalent to the c-conjunction of R_i and R_j should precede the one equivalent to the c-conjunction of R_i and R_m if, and only if, $j < m$. We assume that the same procedure is applied to all the other basic relations the converse relations of which involve two or more components. We then determine a new ordered set of basic relations, starting from the original basis $R_1, R_2, \ldots, R_n$: R_i $(i = 1,2, \ldots, n)$ is retained if, and only if, its converse relation (as represented by means of the old basis) has only one component; otherwise we replace it by the new relations R_{i_1}, R_{i_2}, ... in the agreed order. Let us now denote the relations thus constructed by $R_1', R_2', \ldots, R_p'$. It can easily be shown that the ordered set of these relations is again a basis over the domain X associated with R_n^2 and that the resulting schema is a regular one. We shall refer to it as the C-regularised form of the schema R_n^2, and to the process by means of which it is derived as a C-regularisation.

Let the C-regularised form of a relation schema R_n^2 be a schema of dimension $p\,(p \geqslant n)$. We shall call the difference $p-n$ the degree of irregularity of R_n^2. The degree of irregularity of a schema of dimension n cannot be greater than $n^2 - n = n(n-1)$; and it assumes this value whenever the converse of each basic relation involves all basic relations as components, that is, when all the elements of the original matrix *C* are equal to 1.

There are other important ways of expanding relation schemata. In syllogistics for instance, a schema may be expanded with the help of what we may call the counterpositions K_1, K_2 and K_3. In order to define these, let us assume that X contains with every element x also its negative $\bar{x}$. K_1 associates with each basic relation R_i that relation R of the schema which satisfies the following two conditions: (1) Whenever $R_i(x,x')$, then $R(\bar{x},x')$; (2) each relation R' (of the schema) such that whenever $R_i(x,x')$, then $R'(\bar{x},x')$ contains all the components of R. K_2 and K_3 can be similarly defined. $K_2(R_i) = R$ holds if, and only if: (1) Whenever $R_i(x,x')$, then $R(x,\bar{x}')$; (2) each R' such that whenever $R_i(x,x')$, then $R'(x,\bar{x}')$ contains all the components of R. And $R_i = K_3(R_i)$ holds if, and only if: (1) Whenever $R_i(x,x')$, then $R(\bar{x},\bar{x}')$, (2) there is no relation besides R which satisfies (1) and implies R. K_1, K_2 and K_3 are 'linear' correspondences of the schema into or onto itself.

A counterposition K_i defined in a given schema may induce a one-one-correspondence of the basis onto itself; if this is the case K_i will be said to be regular, and the schema K_i-regular. If a relation scheme R_n^2 is not K_1-regular, we may expand it in the following way into a K_1-regular one. Let R_j be a basic relation of R_n^2 such that $K_1(R_j) = R_k \vee R_m \vee \ldots$. We define the new basic relations R_j, R_{j_2}, ...:

$$R_{j_1}(x,x') \text{ holds if, and only if, } R_j(x,x').\ R_k(\bar{x},x'),$$
$$R_{j_2}(x,x') \text{ holds if, and only if, } R_j(x,x').\ R_m(\bar{x},x'),$$

ans so on, making use of all the components of $K_1(R_j)$. Whenever R_j applies to an ordered pair (x,x'), then one, and only one, of these newly defined basic relations applies to it. All the other basic relations of R_n^2 whose images $K_1(R_i)$ involve more than one component are treated analogously. The order of the new basic relations is fixed in a similar manner as explained in section 2. We thus obtain a new, K_1-regular schema. Instead of using K_1 we could use K_2 or K_3 instead and construct a K_2 or K_3-regular schema. If a schema is C-regular and regular with respect to K_1 (or K_2), then it is also regular with regard to K_2 (or K_1) and K_3. For in this case we have

$$K_1 = CK_2C,\ K_2 = CK_1C,\ K_1K_2 = K_2K_1 = K_3.$$

3. *Three Extensions of Classic Syllogistics*

Let us now sketch out the way in which classic syllogistics can be enlarged and the three syllogistic systems α, β and γ be obtained. The three systems are based upon relation schemata of dimensions 3, 5 and 7 respectively. As is well known, classic syllogistics([2]) works

([2]) See e.g. O. Bird, *Syllogistic and its Extensions*, 1964.

with variables x, x′, x″ etc. for general referential names, with the constants A and I and their negations 0 and E, and with the propositional forms

Axx′:	All x are x′;
Ixx′:	Some x are x′;
Oxx′:	Some x are not x′;
Exx′:	No x are x′.

To obtain a binary relation schema of dimension 3 and the syllogistic *system* α, we introduce the following basic relations, defined over the set of general referential names:

R_1 all ... are ...,
R_2 some, but not all ... are ...,
R_3 no ... are ... .

If instead of R_1 and R_2 we use the traditional symbols A and I, and instead of R_2 the symbol U, then the constant I can be represented as the disjunction $A \vee U$, and 0 as $U \vee E$. R_1 is the negative of $R_2 \vee R_3$, R_3 the negative of $R_1 \vee R_2$. In addition we obtain a relation $R_1 \vee R_3$ which is the negative of R_2. We shall refer to the disjunction $R_1 \vee R_2 \vee R_3$ as the universal relation and denote it by R_u. For any general referential name x $R_1(x,x)$ must hold. Hence R_1 cannot be an asymmetric relation. We shall assume that there exist terms x,x′ such that $R_1(x,x') \cdot R_2(x',x)$ and terms x,x″ such that $R_2(x,x'') \cdot R_2(x'',x)$. R_3 is symmetric. We thus arrive at the conversion matrix

$$C = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

A schema of this kind can be C-regularised by introducing five new basic relations R'_1, R'_2, R'_3, R'_4, R'_5:

$R'_1(x,x')$ holds if, and only if, $R_1(x,x') \cdot R_1(x',x)$;
$R'_2(x,x')$ holds if, and only if, $R_1(x,x') \cdot R_2(x',x)$;
$R'_3(x,x')$ holds if, and only if, $R_2(x,x') \cdot R_1(x',x)$;
$R'_4(x,x')$ holds if, and only if, $R_2(x,x') \cdot R_2(x',x)$;
$R'_5(x,x')$ holds if, and only if, $R_3(x,x')$.

This is the basis upon which *system* β is built. The matrix for the g-converse is

$$C' = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix};$$

that is, we are concerned with a C′-regular schema. However, the first counterposition is represented by the matrix

$$K_l' = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix},$$

which may be used for a further regularisation. To this effect we introduce a new basis consisting of the following seven basic relations:

$R_1''(x,x')$ if, and only if, $R_1'(x,x')$;
$R_2''(x,x')$ if, and only if, $R_2'(x,x')$;
$R_3''(x,x')$ if, and only if, $R_3'(x,x')$;
$R_4''(x,x')$ if, and only if, $R_4'(x,x') \cdot R_2'(\bar{x},x')$;
$R_5''(x,x')$ if, and only if, $R_4'(x,x') \cdot R_4'(\bar{x},x')$;
$R_6''(x,x')$ if, and only if, $R_5'(x,x') \cdot R_3'(\bar{x},x')$;
$R_7''(x,x')$ if, and only if, $R_5'(x,x') \cdot R_1'(\bar{x},x')$.

This is the basis on which the third system, *system* γ, is founded. We find

$$C'' = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

and

$$K''_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

4. *The Syllogistic System* γ

As indicated this system is constructed by means of the basic relations R''_1, R''_2, R''_3, R''_4, R''_5, R''_6 and R''_7 which form the basis of a binary relation schema of dimension 7. Its g-converse as well as its three counterpositions are regular correspondences, and hence a most elegant treatment of the syllogistic functions can be given. We suppose that F(2,1)″, the syllogistic function determining the syllogisms of the first figure in γ, is given by the matrix([3])

$$F(2,1)'' = \begin{pmatrix} R''_1 & R''_2 & R''_3 & R''_4 & R''_5 & R''_6 & R''_7 \\ R''_2 & R''_2 & R''_1 \vee R''_2 \vee R''_3 \vee R''_5 \vee R''_7 & R''_2 \vee R''_4 \vee R''_5 \vee R''_6 \vee R''_6 & R''_2 \vee R''_5 \vee R''_6 & R''_6 & R''_7 \\ R''_3 & R''_1 \vee R''_2 \vee R''_3 \vee R''_4 \vee R''_5 & R''_3 & R''_4 & R''_3 \vee R''_4 \vee R''_5 & R''_3 \vee R''_4 \vee R''_5 \vee R''_6 \vee R''_6 & R''_4 \\ R''_4 & R''_4 & R''_3 \vee R''_4 \vee R''_5 \vee R''_6 \vee R''_7 & R''_1 \vee R''_2 \vee R''_3 \vee R''_5 & R''_3 \vee R''_4 \vee R''_5 & R''_3 & R''_3 \\ R''_5 & R''_2 \vee R''_4 \vee R''_5 & R''_3 \vee R''_5 \vee R''_6 & R''_2 \vee R''_4 \vee R''_5 & R''_4 & R''_3 \vee R''_5 \vee R''_6 & R''_5 \\ R''_6 & R''_2 \vee R''_4 \vee R''_5 \vee R''_6 \vee R''_7 & R''_6 & R''_2 & R''_2 \vee R''_5 \vee R''_6 & R''_1 \vee R''_2 \vee R''_3 \vee R''_5 \vee R''_6 & R''_2 \\ R''_7 & R''_4 & R''_6 & R''_2 & R''_5 & R''_3 & R''_1 \end{pmatrix}$$

Using the matrices C'', K''_1, K''_3 of system γ we can then easily determine the ordinary syllogisms of the second, third and fourth figures, also the contrapositional syllogisms of all four figures.

([3]) *F(2,1)″* may be logically deduced from a proper subset of the 48 basic syllogisms in conjunction with a number of existence assumptions and other assumptions about the basic relations and their properties. As my main objective is to draw attention to, and explain the connections between, the three syllogistic systems I shall not concern myself herewith providing such a deduction, nor with pointing out the various symmetries of the matrix. With the help of Euler diagrams the reader may confirm the validity of the matrix for the idealised (non-fuzzy) general referential names of a natural language.

Dropping from now on the double dashes in order to simplify the formulae we obtain:

$$CC = E \quad \text{and } K_1K_1 = E,$$

where E is the identity correspondence. From this and

$$K_2 = CK_1C$$

we derive that

$$K_2K_2 = CK_1CCK_1C = CC = E;$$

and from $K_1K_2 = K_2K_1 = K_3$ that

$$K_3K_3 = K_1K_2K_2K_1 = K_1K_1 = E.$$

Thus, the correspondences C, K_1, K_2, and K_3 are involutions on the set of relations constituting the relation schema. If E and the three 'linear' correspondences $L_1 = K_1C$, $L_2 = K_2C$, and $L_3 = K_3C$ are added, we obtain eight correspondences which form a group whose multiplication table is given below.

	E	C	K_1	K_2	K_3	L_1	L_2	L_3
E	E	C	K_1	K_2	K_3	L_1	L_2	L_3
C	C	E	L_2	L_1	L_3	K_2	K_1	K_3
K_1	K_1	L_1	E	K_3	K_2	C	L_3	L_2
K_2	K_2	L_2	K_3	E	K_1	L_3	C	L_1
K_3	K_3	L_3	K_2	K_1	E	L_2	L_1	C
L_1	L_1	K_1	L_3	C	L_2	K_3	E	K_2
L_2	L_2	K_2	C	L_3	L_1	E	K_3	K_1
L_3	L_3	K_3	L_1	L_2	C	K_1	K_2	E

Let us now assume we are given C, $F(2,1)$ and K_1. We can then determine all the syllogisms of the first figure starting with the products

$$R^{\mathrm{T}}F(2,1)\,R',$$

where R and R′ are any two relations of the schema other than the universal or the void relations, R' the 'column vector' representing R′, and R^T the transposed 'column vector' (i.e. the 'row vector') representing R. If the product differs from R_u a syllogism with the premises R and R′ is obtained. As

$$F(2,2) = F(2,1)C,$$
$$F(1,1) = CF(2,1),$$

and

$$F(1,2) = CF(2,1)$$

the syllogisms of the second, third and fourth figures can be determined in a similar way by forming the products

$R^T F(2,1)C\ R'$ (for the second figure),
$R^T CF(2,1)\ R'$ (third figure),
$R^T CF(2,1)C\ R'$ (fourth figure). (4)

Each of these four figures gives rise to seven sets of counterpositive syllogisms. The ordinary syllogisms of the first figure are of the form

for all x,x′,x″: if R(x,x′) · R′(x′,x″), then R″(x,x″),

whereas the corresponding (genuinely different) counterpositive syllogisms are of the forms

(1a) for all x,x′,x″: if R($\bar{x}$,x′) · R′(x′,x″), then R‴(x,x″),
(1b) for all x,x′,x″: if R(x,$\bar{x}$′) · R′(x′,x″), then R‴(x,x″),
(1c) for all x,x′,x″: if R($\bar{x}$,$\bar{x}$′) · R′(x′,x″), then R‴(x,x″),
(1d) for all x,x′,x″: if R(x,x′) · R′(x′,$\bar{x}$″), then R‴(x,x″),
(1e) for all x,x′,x″: if R(x,x′) · R′($\bar{x}$′,$\bar{x}$″), then R‴(x,x″),
(1f) for all x,x′,x″: if R($\bar{x}$,x′) · R′(x′,$\bar{x}$″), then R‴(x,x″),
(1g) for all x,x′,x″: if R($\bar{x}$,x′) · R′($\bar{x}$′,$\bar{x}$″), then R‴(x,x″).

These syllogisms are calculated by means of the matrix products:

(1a) $R^T\ K_1 F(2,1)\ R'$,
(1b) $R^T\ K_2 F(2,1)\ R'$,
(1c) $R^T\ K_3 F(2,1)\ R'$,

(4) The 'strong' syllogisms are obtained directly in this way. It is then a trivial step to derive the syllogisms with a weakened conclusion.

(1d) $R^{T}\ F(2,1)K_2\ R'$,
(1e) $R^{T}\ F(2,1)K_3\ R'$,
(1f) $R^{T}\ K_1F(2,1)K_2\ R'$,
(1g) $R^{T}\ K_1F(2,1)K_3\ R'$,

and by using the identities $(K_iR)^{T} = R^{T}K_i$ (i = 1,2,3). It can easily be confirmed that each of the counterpositions is identical to its transpose. That this is so for K_1 follows from the symmetry of the matrix K_1; on the other hand, $K_2 = CK_1C$, and hence

$K_2^{T} = C^{T}K_1^{T}C^{T} = CK_1C$, as C is also symmetric; and
$K_3^{T} = (K_1K_2)^{T} = K_2^{T}K_1^{T} = K_2K_1 = K_1K_2 = K_3$.

In the same manner we proceed with the syllogisms of the second, third and fourth figures. The ordinary ones are obtained by forming the products

$$R^{T}\ F(2,2)\ R' = R^{T}\ F(2,1)C\ R' \quad \text{(second figure)},$$
$$R^{T}\ F(1,1)\ R' = R^{T}\ CF(2,1)\ R' \quad \text{(third figure)},$$
$$R^{T}\ F(1,2)\ R' = R^{T}\ CF(2,1)C\ R' \quad \text{(fourth figure)}.$$

We then find the matrices of counterpositive syllogisms of these three figures, either by applying the matrices K_i to the above matrix products, or simply by using the expressions (1a) – (1g) and multiplying the matrix products between R^{T} and R' with C from the right (for the second figure), from the left (for the third figure), or from both the left and the right (for the fourth figure). For given that

for all x,x′,x″: R(x,x′) · R′(x′,x″) → R″(x,x″)

is a syllogism of the first figure, then

for all x,x′,x″: R(x,x′) · CR′(x″,x′) → R″(x,x″)

is a syllogism of the second figure,

for all x,x′,x″: CR(x′,x) · R′(x′,x″) → R″(x,x″)

one of the third, and

for all x,x′,x″: CR(x′,x) · CR′(x″,x′) → R″(x,x″)

one of the fourth figure; and this holds irrespective of whether the terms x,x′,x″ are negative or positive.

Taking into account that the transpose of CK_iR equals R^TK_iC we find the following sets of matrices for the counterpositive syllogisms of the second, third, and fourth figures respectively:

(2a) $K_1F(2,1)C$	(3a) $K_2CF(2,1)$	(4a) $K_1CF(2,1)C$
(2b) $K_2F(2,1)C$	(3b) $K_1CF(2,1)$	(4b) $K_1CF(2,1)C$
(2c) $K_3F(2,1)C$	(3c) $K_3CF(2,1)$	(4c) $K_3CF(2,1)C$
(2d) $F(2,1)CK_1$	(3d) $CF(2,1)K_2$	(4d) $CF(2,1)CK_1$
(2e) $F(2,1)CK_3$	(3e) $CF(2,1)K_3$	(4e) $CF(2,1)CK_3$
(2f) $K_1F(2,1)CK_1$	(3f) $K_2CF(2,1)K_2$	(4f) $K_2CF(2,1)CK_1$
(2g) $K_1F(2,1)CK_3$	(3g) $K_2CF(2,1)K_3$	(4g) $K_2CF(2,1)CK_3$

5. *The Syllogistic System* β

The basis for this system has been introduced above (section 3). We utilise a relation schema of dimension 5 which can be obtained by a contraction of the schema underlying system γ. This contraction collapses R_4'' and R_5'' into the new basic relation R_4', and R_6'' and R_7'' into R_5'. The contraction of C'' yields

$$C' = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

The contractions of K_1'', K_2'', and K_3'' provide us with the matrices of the counterpositions in β:

$$K_1' = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix}, \quad K_2' = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}, \quad K_3' = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

From this we deduce that

$K_1' = C'K_2'C'$, hence also $K_2' = C'K_1C'$;
$K_iK_i \neq \mathrm{E}$ (i = 1,2,3).

All three counterpositions are alternating correspondences:

$K_1'^2 = K_1'^4 = K_1'^6 = \ldots = K_1'^{2n} = \ldots;\ K_1'^3 = K_1'^5 = K_1'^7 = \ldots = K_1'^{2n+1} = \ldots.$

$K_2'^2 = K_2'^4 = K_1'^6 = \ldots = K_2'^{2n} = \ldots;\ K_2'^3 = K_2'^5 = K_2'^7 = \ldots = K_2'^{2n+1} = \ldots.$

$K_3'^2 = K_3'^4 = K_3'^6 = \ldots = K_3'^{2n} = \ldots;K_3' = K_3'^3 = K_3'^5 = \ldots = K_3'^{2n+1} = \ldots.$

By contracting *F(2,1)″* we obtain

$$F'(2,1) = \begin{pmatrix} R_1' & R_2' & R_3' & R_4' & R_5' \\ R_2' & R_2' & R_u' & R_2' \vee R_4' \vee R_5' & R_5' \\ R_3' & R_1' \vee R_2' \vee R_3' \vee R_4' & R_3' & R_3' \vee R_4' & R_3' \vee R_4' \vee R_5' \\ R_4' & R_2' \vee R_4' & R_3' \vee R_4' \vee R_5' & R_u' & R_3' \vee R_4' \vee R_5' \\ R_5' & R_2' \vee R_4' \vee R_5' & R_5' & R_2' \vee R_4' \vee R_5' & R_u' \end{pmatrix}$$

R_u' denotes the universal relation in system β. The relation schema is a C-regular one. We can proceed as before and derive the matrices for the second, third and fourth figures:

$$\begin{aligned} F'(2,2) &= F'(2,1)C', \\ F'(1,1) &= C'F'(2,1), \\ F'(1,2) &= C'F'(2,1)C'. \end{aligned}$$

As the correspondences K_1', K_2' and K_3' are not regular, the counterpositive syllogisms cannot be derived from the ordinary ones in the way this can be done in system γ. However, we can obtain them simply by contracting the matrices for the counterpositive syllogisms in system γ. Thus, contracting the matrices (1a) – (1g) provides us with the matrices determining the counterpositive syllogisms of the first figure in system β; contracting (2a) – (2g) yields those of the second figure, and so on, for the third and fourth figures.

6. *The Syllogistic System* α

Here we use a relation schema of dimension 3, with the basic relations $R_1 = R_1' \vee R_2'$, $R_2 = R_3' \vee R_4'$, $R_3 = R_5'$. The contractions of C', K_1', K_2' and K_3' provide us with the matrices

$$C = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$K_1 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad K_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad K_3 = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

The degree of irregularity of this schema equals 2. Among the above correspondences only K_2 is a regular one. We obtain *F(2,1)*, *F(2,2)*, *F(1,1)* and *F(1,2)* by contracting *F'(2,1)*, *F'(2,2)*, *F'(1,1)* and *F'(1,2)* respectively. (5)

Contracting *F'(2,1)* yields

$$F(2,1) = \begin{pmatrix} R_1 & R_u & R_3 \\ R_1 \vee R_2 & R_u & R_2 \vee R_3 \\ R_u & R_u & R_u \end{pmatrix}$$

And from *F'(2,2)* we find,

$$F(2,2) = \begin{pmatrix} R_u & R_u & R_3 \\ R_2 \vee R_3 & R_u & R_2 \vee R_3 \\ R_3 & R_u & R_u \end{pmatrix}$$

from *F'(1,1)*

$$F(1,1) = \begin{pmatrix} R_1 \vee R_2 & R_2 & R_2 \vee R_3 \\ R_1 \vee R_2 & R_u & R_2 \vee R_3 \\ R_u & R_u & R_u \end{pmatrix},$$

and finally from *F'(1,2)*

$$F(1,2) = \begin{pmatrix} R_1 \vee R_2 & R_1 \vee R_2 & R_2 \vee R_3 \\ R_u & R_u & R_2 \vee R_3 \\ R_3 & R_u & R_u \end{pmatrix}.$$

(5) There are 42 ordinary syllogisms in system α (11 in each of the first three figures and 9 in the fourth), 4052 (ie 1013 in each of the four figures) in system β, and 95692 (ie 23923 in each figure) in system γ. I would like to thank my colleague Mr H G Moring (Department of Computer Science, The City University) as well as Mr R H Moring for having computed these numbers. As Mr H G Moring pointed out to me, the increase in the number of syllogisms is mainly due to the rapid increase of the number of syllogisms with a weakened conclusion.

By a twofold contraction starting from the matrices (1a)-(1g) we obtain the seven matrices for the counterpositive syllogisms of the first figure in system α. They are

$$G_{11}(2,1) = \begin{pmatrix} R_2 \vee R_3 & R_u & R_1 \vee R_2 \\ R_u & R_u & R_u \\ R_1 \vee R_2 & R_2 & R_2 \vee R_3 \end{pmatrix}, \quad G_{12}(2,1) = \begin{pmatrix} R_u & R_u & R_u \\ R_1 \vee R_2 & R_u & R_2 \vee R_3 \\ R_1 & R_u & R_3 \end{pmatrix},$$

$$G_{13}(2,1) = \begin{pmatrix} R_1 \vee R_2 & R_2 & R_2 \vee R_3 \\ R_u & R_u & R_u \\ R_2 \vee R_3 & R_u & R_1 \vee R_2 \end{pmatrix}, \quad G_{14}(2,1) = \begin{pmatrix} R_3 & R_u & R_1 \\ R_2 \vee R_3 & R_u & R_1 \vee R_2 \\ R_u & R_u & R_u \end{pmatrix},$$

$$G_{15}(2,1) = \begin{pmatrix} R_u & R_u & R_u \\ R_2 \vee R_3 & R_u & R_1 \vee R_2 \\ R_3 & R_u & R_1 \end{pmatrix}, \quad G_{16}(2,1) = \begin{pmatrix} R_1 \vee R_2 & R_u & R_2 \vee R_3 \\ R_u & R_u & R_u \\ R_2 \vee R_3 & R_2 & R_1 \vee R_2 \end{pmatrix},$$

$$G_{17}(2,1) = \begin{pmatrix} R_2 \vee R_3 & R_2 & R_1 \vee R_2 \\ R_u & R_u & R_u \\ R_1 \vee R_2 & R_u & R_2 \vee R_3 \end{pmatrix}$$

In a similar manner the counterpositive syllogisms of the second, third and fourth figures can be derived, using the matrices (2a)-(2g), (3a)-(3g) and (4a)-(4g) respectively.

Thus, what is normally referred to as classic syllogistics is a relatively undifferentiated and formally clumsy sub-system of the systems α, β and γ. It may have been strongly suggested by certain features of ordinary language, but from a formal point of view it is nonetheless a poor system whose structure can be more satisfactorily analysed if it is embedded in a wider context. I hope to have shown that relational logic is ideally suited to provide such a context.

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