# THE RELATIONAL THEORY OF MEANING

Johan VAN BENTHEM

## 1 *Changes in the semantic framework*

The systematic approach advocated by Richard Montague (cf. Montague 1974) has become a paradigm in the semantics of natural language. At least in principle, the task of semantic description is nowadays seen to consist in producing explicit grammars and well-defined model classes, while spelling out the interpretative links connecting the former with the latter. Within this general framework, however, Montague's specific proposals for all three components have been challenged continuously. Thus, linguistically better-motivated grammars have been substituted for the original one (lexical-functional grammar in Halvorsen 1984, generalized phrase structure grammar in Gazdar et al. 1985). Also, the original use of standard possible worlds models, as well as that of full classical type theory, is currently under attack – with more 'partial' and 'flat' alternatives being offered, e.g., in Barwise and Perry 1983. Finally, the mechanism of interpretation itself has been widely debated, witness the long debate about Frege's principle of Compositionality (cf. Janssen 1983, Partee 1984), or recent proposals for adding a component of 'representation' to the Montagovian scheme, thus returning to a more Saussurian semiotic triangle (cf. Kamp 1981).

This paper is concerned with one theme in the latter area, that of variations on Montague's austere homomorphism account of linguistic interpretation, viewed as assigning suitable denotations to linguistic expressions by recursion on their grammatical construction. Now, one of the many new proposals in Barwise and Perry's recent enterprise of 'Situation Semantics' may be called a *relational theory of meaning*. Its core idea is that meanings of linguistic expressions, properly understood, are relations between a *situation of utterance* for the expression and a *situation described*. Traditional presentations have neglected the former parameter, it is claimed, concentrating on a 'right projection' of this binary relation, viz. classes of models described. But actually, the argument continues, both sides of the coin are crucial. Information may flow just as well from situation described to situation of utterance (e.g., when an utterance about a situation that we know tells us something about the speaker's knowledge or attitude) as vice versa: the more traditional direction of attention. Incidentally, this connectedness does not imply total symmetry, of course. For instance, we have direct deictic access to the utterance situation, but not necessarily to a unique situation described.

Another, though related aspect of the above perspective or 'format' for semantic explication is a general preference for a *relational*, rather than a *functional* account of the 'combinatorics' of meaning. For instance, intransitive verbs will stand for simple properties applying to individuals. Their Montagovian treatment as denoting functions from entities to truth values seems an artifact of an obsession with the above direction from linguistic expressions to semantic values. (All this is even more true for transitive verbs.) Another notable example is the relational treatment of determiner expressions; a perspective whose technical virtues had already been explored independently by several authors (cf. van Benthem 1983a). More concrete examples will be found in the next section.

The purpose of this paper is to compare the relational approach with already existing ones in logical semantics, in both of its aspects. We shall find 'relational' features right in the heart of the area, viz. in Tarski's semantics for predicate logic – with the Montague and the Barwise and Perry formats, in a sense, being (too) uniform extrapolations from there, in different directions. Even so, the latter two turn out to be intertranslatable; and one ends up with two equally powerful ways of viewing the mechanics of interpretation. This conclusion does not imply that both may not have their particular merits for particular purposes, of course. Moreover, Situation Semantics also comes with proposals for a quite unorthodox model class; but, that is a topic beyond our present scope.

The above comparison presupposes familiarity with the theories involved. Fortunately, Montague Semantics is widely accessible by now, having reached the textbook stage in, e.g., Dowty, Wall and Peters 1981 – and this is also the case with Tarski Semantics. Hence, familiarity with these ideas will be presupposed in what follows. As

Situation Semantics is rather less-known and more tentative at this stage, a sketch of its relevant features will be included below.

Many of the points to be made here are part of current semantic folklore. Accordingly, this paper, though short on references, is full of common sense.

## 2 *Samples of Situation Semantics*

To begin with, here is a sketch of the semantic format in 'Situations and Attitudes', as presented in the two 'Aliass' appendices of Barwise and Perry 1983. Some later, unpublished modifications will be used as well. Throughout, we shall concentrate on general ideas. Situation Semantics is still in flux, and excessive attention to formal details would be unproductive at this stage.

Our grammar has lexical items and syntactic construction rules. Moreover, such items come with certain 'features', which may undergo modification when a rule is applied. Thus, in line with current linguistic wisdom, the grammar produces analysis trees with a variety of feature indications at each node: morphological, but also 'categorical', anaphoric, etcetera. As we are only concerned with the combinatorics of meaning, we shall disregard many of these here.

### *Lexical items and their interpretation*

Lexical entries look like this:

*Mary*, with one *individual indeterminate* x,
*girl*, likewise
*walk*, likewise,
*find*, with two individual indeterminates x,y,
*a*, with one individual indeterminate x and one *role indeterminate* X,
*every*, with two role indeterminates X,Y.

Indeterminates are 'place holders' for individuals or predicates (roles), whose combinatorial function will become clear as we proceed. In Situation Semantics, they have an independent ontological status; but the agnostic reader can get a long way by thinking of *variables*, first-order or higher-order.

Of course, assignment of indeterminates is uniform within linguistic categories.

In more elaborate settings, *polarity indeterminates* p,q may appear with verbs, common nouns, etcetera. These create room for a tripartition: definitely true (polarity value 1), definitely false (value 0) or undefined (no polarity value). And of course, dependence on (space-)time locations must eventually be accounted for, leading to the introduction of *location indeterminates* with verbs and tense markers.

We now turn to the interpretation of lexical items. Models M are 'structures of situations', satisfying certain conditions which need not concern us here. Against such a background, the basic schema of interpretation for a linguistic expression $\in$ is this (in our notation):

*in* M,I: $_{d,c}[\![\in]\!]_{s,a}$.

Here, I is the more permanent part of interpretation, assigning, e.g., verbs their standard denotations. (Once learnt, such assignments tend to be stable across many utterances.) The more volatile parts are taken care of by the *discourse connection* c, sending, e.g., deictic pronouns to their objects, as appropriate to this particular *discourse situation* d, in which the utterance takes place. A more recent move has been to gather d and c into one 'utterance situation'. This is not to be recommended, however, since c rather connects d with the *described situation* s. The latter will be thought of, typically, as a small corner of the universe M, with *partial* information about the behaviour of a certain number of individuals. Finally, the 'setting' or *anchor* a attaches appropriate denotations in e to indeterminates in $\in$. This auxiliary notion may be compared (with some reservations) with the *variable assignments* of standard logical semantics, oiling the wheels of recursion.

Here are some examples, with notation suppressed whenever convenient:

$_{d,c}[\![Mary_x]\!]_{s,a}$ iff a(x) = I(*Mary*)
$_{d,c}[\![walk_x]\!]_{s,a}$ iff $<$I(*walk*), a(x)$>$ *is a true fact in* s
$_{d,c}[\![every_{XY}]\!]_{s,a}$ iff a(X) is contained in a(Y)

This is the standard meaning for the determiner *every*, in a relational setting. But, there is a special treatment for the indefinite article, which carried a 'referential' indeterminate X:

$_{d,c}[a_x\, x]_{s,a}$ iff $a(x) \in a(X)$

The latter may be contrasted with a possible reading for its related determiner

$_{d,c}[some_{XY}]_{s,a}$ iff $a(X)$ *overlaps* $a(Y)$,

stating that there are individuals playing both roles.

*The interpretation of grammatical rules*

Next, the behaviour of grammatical rules is to be considered. In principle, each particular construction can have its own semantic instruction, involving manipulating of indeterminates. In practice, a small family of 'steps' will suffice – usually: *joining* all conditions from the immediate components of the rule, while making suitable *identifications* of indeterminates.

Roughly speaking, there are two strategies for describing the latter process. One is to let complex expressions resulting from applications of grammatical rules freely gain or lose indeterminates. For instance, $[Mary_x]\,[walks_y]$ might count as having lost the original individual indeterminates x,y; because semantically, this sentence is a closed statement. Hence, a semantic account of the rule combining a proper name and a verb to a sentence might read as follows:

$_{d,c}[\,[Mary_x]\,[walks_y]\,]_{s,a}$ iff *there exists some extension* a′ *of the anchor* a *such that* $a'(x) = a'(y)$ *and both* $_{d,c}[Mary_x]_{s,a'}$ *and* $_{d,c}[walks_y]_{s,a'}$.

Here, a need not be defined on x,y, and an *extension* is invoked to handle the latter. This is a recurrent pattern in Barwise and Perry 1983.

Another strategy is to always let indeterminates percolate higher up, while creating suitable identifications. (Not to prejudge issues, then, one has to start with different indeterminates for different parts of a sentence. Having the same indeterminate displayed already encodes obligatory identifications.) Thus, the fully analyzed sentence $[\,[Mary_x]\;[walks_y]\,]$ would have both indeterminates x,y, with an added equation '$x = y$'. Then, there is no need for appealing to extension of settings. For instance, it suffices to state

$_{d,c}[\![ [Mary_x] \; [walks_y] ]\!]_{s,a}$ iff a *is defined on all indeterminates in this expression, while satisfying all their accompanying equations, and* $_{d,c}[\![Mary_x]\!]_{s,a}$ *and* $_{d,c}[\![walks]\!]_{s,a}$.

A choice between the two strategies is largely one of taste. We shall presuppose the latter henceforth.

Evidently, all this bookkeeping compares not very favourably with the compact Montagovian account of these sentences – being, essentially, the application 'denotation (*walks*) (denotation (*Mary*))'. But, the principles involved here are very simple, and computationally perspicuous.

Instead of developing a general framework for semantic clauses here, we shall now present a few examples illustrating some key principles for interpreting complex expressions. First, here is the rule combining determiners and common nouns to noun phrases:

$_{d,c}[\![every_{XY} girl_x]\!]_{s,a}$ iff $_{d,c}[\![every_{XY}]\!]_{s,a}$ *and* $a(X) = \hat{x}. \; _{d,c}[\![girl_x]\!]_{s,a}$.

What happens here is that a role is *extracted* from the second clause (viz. that of being a girl in the situation described), which is *identified* with the first argument of the determiner. (On a more conservative reading, a 'lambda abstraction' is performed, and the result is 'substituted' for X.)

Actually, to get this to work out right, it is to be indicated which argument of the determiner comes 'first'. This encoding problem for argument order can be solved in various ways. 'Situations and Attitudes' has typographical indications for such purposes. A later suggestion has been to supply *every* with two *relativized roles*, whose respective functions can be told apart. We shall not pursue these intricacies here.

Regardless of precise implementations, what is becoming ever clearer as we proceed is that the items interpreted in our semantic scheme are not plain linguistic expressions $\in$ – but rather enriched linguistic analysis trees, carrying quite a bit of structure (including various equations) at each node. This is also the proper perspective to understand all subsequent examples.

Next, the rule combining noun phrases and verb phrases to sentences might work as follows:

$_{d,c}[\![ [every_{XY} girl_x] \, [walks_y] ]\!]_{s,a}$ iff $_{d,c}[\![ [every_{XY} girl_x] ]\!]_{s,a}$ *and* $a(Y) = \hat{y}. \, _{d,c}[\![ walks_y ]\!]_{s,a}$.

Thus, the remaining role indeterminate of the noun phrase is equated with the role abstracted from the verb phrase. (Again, there are possible variations here; such as providing verb phrases – as opposed to mere verbs – with role determinates in any case.) In all then, the usual reading results. Notice how the progressive identifications eliminate indeterminates, so to speak.

The next examples illustrate some further basic principles.

$_{d,c}[\![ Mary_x \, finds_{yz} \, a_{uX} \, girl_v ]\!]_{s,a}$ iff $_{d,c}[\![ Mary_x ]\!]_{s,a}$ (i.e., $a(x) = I(Mary)$) *and* $_{d,c}[\![ finds_{yz} \, a_{uX} \, girl_v ]\!]_{s,a}$ *and* $a(x) = a(y)$.
The middle phrase decomposes into
$_{d,c}[\![ finds_{yz} ]\!]_{s,a}$ *and* $_{d,c}[\![ a_{uX} \, girl_v ]\!]_{s,a}$ *and* $a(z) = a(u)$. Finally, the latter becomes $_{d,c}[\![ a_{uX} ]\!]_{s,a}$ *and* $a(X) = \hat{v}. \, _{d,c}[\![ girl_v ]\!]_{s,a}$.
Likewise,
$_{d,c}[\![ finds_{yz} \, every_{XY} \, girl_v ]\!]_{s,a}$ iff $\hat{z}. \, _{d,c}[\![ finds_{yz} ]\!]_{s,a} = a(Y)$ *and* $_{d,c}[\![ every_{XY} \, girl_v ]\!]_{s,a}$; where the latter is analyzed as above.

Notice again how, in all these cases, specific indeterminates are to be used for abstraction and identification. For instance, when combining a transitive verb with a direct object as in the last example, one needs to equate a(Y) with the role of 'being found by y', not that of 'finding z'. On current forms of the theory, even the simple expression *finds* would already come with such relativized roles, marked for their thematic behaviour.

Through these examples, the reader can form a fair impression of a Situation Semantics account of a modest fragment of English. The general idea is to proceed up the grammatical structure tree, forming conjunctions of conditions already obtained at daughter nodes, while performing (when needed) identifications and abstractions. The price for this attractive conjunctive picture is a certain proliferation of indeterminates. But, this is harmless for reasons explained above (and, of course, 'cosmetic' simplifications are always possible).

*Some further developments*

Evidently, the above fragment is still extremely poor, giving us only the basics of combining noun phrases and verb phrases. Some further topics will be reviewed now. These all raise important semantic questions, but do not change the above mechanism of interpretation in essential respects.

First, there is an issue of *negation*. Situation Semantics treats predicate negation (as in *Mary is not crying*), not as mere absence of the positive predicate, but as stating positive evidence to the contrary. Thus, a situation might validate Mary's crying, her non-crying, or neither (lacking sufficient information). This tri-valent view is reflected in the presence of polarity indeterminates p, which can be anchored to values a(p) in the set {0, 1} (or {FALSE, TRUE}). The clause for predicate negation will then make *does not cry* true (in a situation s) for all those individuals in s for which *cry* is false (not merely: 'not true').

Actually, there is some debate surrounding this issue, as other forms of negation in natural language seem rather bivalent. *Not every seagull cries* seems to be true if it is not true that every seagull cries, not necessarily demanding positive reasons for the latter failure. Accordingly, e.g., Fenstad et al. 1984 introduce *two* kinds of negation, with truth tables

| A | *not* A |
|---|---|
| 1 | 0 |
| 0 | 1 |
| – | 0 |

| A | NOT A |
|---|---|
| 1 | 0 |
| 0 | 1 |
| – | – |

The latter paper also has an extensive discussion of various alternatives for interpreting the traditional logical constants (connectives, quantifiers) in a partial perspective. The point is that there need not be a single uniquely preferred analysis for natural language items – and this realization may actually be a virtue.

Next, of course, a general procedure is needed for handling cases of *anaphora*. Here is where indeterminates come into play again. For instance, consider the sentence '*A unicorn* finds **Mary**, and puts *its* head in **her** lap'; the situation according to medieval legend (which Montague seems to have misunderstood). Anaphoric constraints may be encoded here quite simply by adding identities between the associated indeterminates of corresponding expressions. This idea can be implemented in many ways; such as the 'AF-store' in Situations and Attitudes, or according to any one of a number of such techniques. The equational approach has the advantages of being uniform – and already available from the earlier discussion.

It should be noted that here, for the first time, the discourse connection c will play a role. For instance, in linking *Mary* to *her*, the two associated individual indeterminates will become identified, with the basic clause for the pronoun working out as

$$_{d,c}[\![ her_x ]\!]_{s,a} \quad \text{iff} \quad a(x) = c(her).$$

Thus, in the absence of anaphoric 'capture', pronouns will be read deictically, according to c.

With anaphora, the other main concern in logical semantics must be in the vicinity too, viz. the phenomenon of *scope* for various operators. Actually, Situation Semantics is not well-disposed towards this notion – but it does have certain substitutes. For instance, in the standard example *every unicorn finds a girl*, 'Situations and Attitudes' would not postulate a wide scope reading for *a girl* (as provided by Montague's well-known 'quantifying in' rule): being committed to a principle of 'radical interpretation' for expressions (from left to right). Still, variation in meaning can occur, because of differences in 'loading' – a feature not encountered before. Up till now, we have considered evaluation of expressions within a fixed described situation s. But in many cases, further situations may already be available; most obviously, the original discourse situation. Moreover, one often assumes a 'resource situation', embodying the preceding context. Accordingly, there will be options for anchoring indeterminates. For instance, anchoring *a girl* in the discourse or resource situation gives rise to a 'specific' reading for the girl; without having to postulate different derivational histories for the sentence. Regardless of its eventual merit, this observation at least shows some of the additional sensitivity of this approach.

Finally, another pervasive phenomenon in natural language remains to be mentioned, viz. the occurrence of *tense* (and temporal indicators in general). For instance, *walks* really carries a present tense marker, say *n*, which again gives access to the context of utterance:

$_{d,c}[n_l]_{s,a}$ iff c(n) *temporally overlaps the location of the discourse situation* d, *and* a(l) = c(n).

Here, l is a location indeterminate. Then, the full account for the tensed verb becomes

$_{d,c}[walk_{x\,l1}\ n_{l2}]_{s,a}$ iff $_{d,c}[n_{l2}]_{s,a}$ *and*
$_{d,c}[walk_{x\,l1}]_{s,a}$ *and* a(l1) = a(l2).

Here, the middle entry now says that <I(*walk*), a(x)> is a true fact *in* s *at* a(l). Thus, we shift from a picture of described situations as small photographs to extended 'courses of events', containing various facts at various locations.

Observe the difference with more traditional treatments of tense in so-called 'tense logic'. This becomes even more noticeable with the past tense, whose marker refers deictically to some location temporally preceding the discourse location:

$_{d,c}[ed_l]_{s,a}$ iff c(*ed*) = a(l) *temporally precedes the location of the discourse situation,*
$_{d,c}[walk_{x\,l1}\ ed_{l2}]_{s,a}$ iff $_{d,c}[ed_{l2}]_{s,a}$ *and*
$_{d,c}[walk_{x\,l1}]_{s,a}$ *and* a(l1) = a(l2).

In ordinary tense logic, this 'deictic' approach is replaced by an existential quantification over past location:

x *walked* at t if x *walk* at t′ *for some* t′ *temporally preceding* t.

The latter is an instance of the 'projective' approach mentioned in the introduction, losing too much information about the specific past occasion involved. Interestingly, the latter approach may be better-suited for *temporal auxiliaries*, such as *have walked*, *will walk* (cf. Fenstad et al. 1984).

Perhaps more spectacular cases of contextual c,d-dependence occur with traditional indexicals, such as *I*, *you*. But the tense example already illustrates the kind of interplay involved between utterance situation and situation described.

With this very brief account, we end our sketch of the mechanics of interpretation in Situation Semantics. The points to be made below seem to generalize smoothly to its more elaborate versions.

Even this sketch already raises various questions of a more general import. For instance, the whole apparatus of indeterminates and role formation invites systematic comparison with more traditional Montagovian modes of using variable application and lambda abstraction. Some aspects of this analogy will return in section 5 below. Very detailed comparisons seem a bit premature, however, as Situation Semantics proposals keep changing. One question which will eventually be of interest here concerns possible *constraints* on the enterprise. As is well-known, Montague Semantics suffers from an overly accomodating syntax and model theory: the requirement of Fregean compositional interpretation hardly constrains the multitude of possible proposals. It would be pleasant if the present equational presentation were to suggest further constraints, giving the business of constructing a formal semantics more descriptive 'bite'. But also, new questions arise out of the variety of (partial) situations encountered on the above account, when evaluating a single expression. For instance, information contained in an utterance should be *persistent*, in the sense of remaining valid, even when a more detailed picture is obtained of the situation described. For the logic of this notion, cf. again Fenstad et al. 1984, which studies the question just when persistence occurs.

## 3 *Earlier schemes of interpretation*

### *Tarski's truth definition as a relational scheme*

The paradigm for modern formal semantics has been Tarski's *truth definition*, being a systematic account of the notion 'sentence $\in$ is true in model M' for certain formal languages. Analyzing a bit further, the earlier-mentioned three-partite perspective emerges: sentence $\in$ is true in model D under interpretation I. (In many text book accounts, the latter two components are lumped together, for technical convenience.) Moreover, an additional feature is needed, viz. an *assignment* a, correlating variables ('locally') with actual objects in the domain of discourse. Originally, the latter move may have been no more than a marginal, though clever trick to get recursion going on the structure of $\in$. For a natural language semanticist, however, the emergence of the assignment is a crucial symptom of context-dependent aspects of interpretation.

By way of illustration, here is a formal statement of some Tarski clauses:

| | | |
|---|---|---|
| $D,I \models Rxy\ [a]$ | iff | $I(R)$ *holds of* $a(x), a(y)$ *in that order* |
| $D,I \models \neg \in [a]$ | iff | *not* $D,I \models \in [a]$ |
| $D,I \models \in_1 \wedge \in_2 [a]$ | iff | $D,I \models \in_1 [a]$ *and* $D,I \models \in_2 [a]$ |
| $D,I \models \exists x \in [a]$ | iff | *for some* d *in* D, $D,I \models \in [a_d^x]$; |

where $a_d^x$ is a with x (re-)set to d. This innocent quantifier clause already makes an important anaphoric decision: there is to be 'local scoping', with variables governed by the innermost quantifier variable in whose syntactic domain they occur.

Actually, the Tarski scheme, and its subsequent uses, already exhibits many of the central features of the situation semantics approach. To begin with, it is obviously relational in form and spirit. (Curiously, this obvious feature has escaped many philosophers, who continue their discussions of truth-as-an-absolute-property in cheerful ignorance.) Then, as was observed just now, contextual aspects are present, in addition to more permanent interpretation functions: as with the earlier division of labour between I and c,a. And there are many further points of contact. For instance, at least in more philosophical, or practical expositions of Tarski Semantics, models are usually presented as 'universe of discourse'; i.e., presumably varying small situations described, being parts of some total background Reality (Tarski's original setting). Thus, a distinction enters between 'large models' M, constant for longer periods, and 'small models' m, of a more transient nature (cf. Westerståhl 1984). And, of course, there is essentially the earlier division of labour between the structure of situations M and the described situation s.

Also, some further uses of the Tarski scheme are relevant here. In mathematical logic, it underlies the development of a technical logical subdiscipline of 'Model Theory' (cf. Chang and Keisler 1973). And at least there, the multi-directionality of semantics has always been a central theme. Given a sentence $\in$, one studies its model class MOD($\in$), i.e., all situations to which it applies. But also conversely, given a model M, one studies its theory Th(M), being all sentences true of M. Thus, 'direct' and 'inverse' information are intertwined. In all this, some interpretation function I is held fixed. Varying the latter also, for given D and $\in$, is proposed in van Benthem 1984, as a reflection of the semantic concerns of Bernard Bolzano, a century before Tarski. More practically, of course, this is often the case one finds oneself in when learning a foreign language.

Another relevant theme from Model Theory is the study of various connections between different models (such as 'substructure', 'homomorphic image'), with the corresponding behaviour of sentences under the relevant transitions. The mathematical motivation behind this has been an interest in exploring various possible models for given mathematical theories. But, as more often, the mathematical consequences of Tarski's semantics acquire different features when re-interpreted in natural language semantics. In the earlier-mentioned perspective of 'small models', the latter may change, expand or contract as the discourse continues (as does, indeed, the local interpretation function). So, in order to account for the dynamics of interpretation, the above technical model relations become important. (Cf. Muskens 1983, for the interplay of 'relativization' in discourse and 'restriction' on models.) And then, an obvious analogy arises with the situation semantics concerns of persistence of information with changing situations.

Incidentally, many of the preceding observations also apply to actual mathematical discourse, as it occurs in ordinary proofs. But, owing to a preoccupation with formalized proofs, and often even rather global theories, semantics for mathematical discourse has been a marginal topic in mathematical logic. Likewise, philosophers have seldom used mathematical argument as a source of semantic examples. (One interesting exception is the 'Hilbert View' of *definite descriptions*, presented in Kneale and Kneale 1962 as a discourse-based alternative to the sentence-bound accounts of Russell and Strawson.)

Summarizing, then, there are analogies between the spirit of the Situation Semantics account of meaning and quite central features of standard Tarski Semantics. In fact, it now seems that the latter *is* a relational theory of meaning, with the former presenting orthodox dogma as the latest heresy. But, there is a complicating factor.

## *The role of functional expressions*

In setting up the truth condition for atomic formulas in the Tarski Scheme, one has to assign denotations to individual *terms* of the language, i.e., variables, constants, or functional compounds out of these. And here, the natural format changes:

from $D,I \models \in[a]$ to $value(t,D,I) = \ldots$.

For, such terms are most naturally thought of as denoting objects in the universe of discourse, given enough interpretative environment. Some formal clauses illustrating this move are

$$value(x,D,I,a) = a(x)$$
$$value(c,D,I,a) = I(c)$$
$$value(ft_1 \ldots t_k,D,I,a) = I(f)\ (value(t_1,D,I,a), \ldots, value(t_k,D,I,a)).$$

Thus, depending on the kind of linguistic expression, there are different favoured formats of interpretation in ordinary logic already. Of course, it is technically possible to move toward a uniform treatment, either by construing formulas as Boolean-valued terms, or by construing terms as formulas, adding suitable indeterminates (as in the Situation Semantics of pronouns and proper names). But either way, some naturalness is lost. For technical purposes, this may be no problem – for descriptive ones, more sensitivity may be essential.

All these points remain valid for the larger area of philosophical logic and logical semantics in general, where many further types of expression have been studied. For instance, in tense logic, one still finds the above relational scheme; as in

$D,I,t_0 \models [a,t]$:
'$\in$ is true at time point t, relative to utterance point $t_0$'.

Here, both a moment described and a moment of utterance have entered the picture; as temporal operators may be explicitly indexical. One example is the classic paper Kamp 1971 on predicate tense logic with a notion of 'now':

$$D,I,t_0 \models NOW \in [a,t] \quad \text{iff} \quad D,I,t_0 \models \in [a,t_0].$$

Thus, step by step, all components of the relational format of section 2 had already entered formal semantics by the early seventies. Explicit recognition of the role of utterance, and context generally, is even an article of faith in Creswell 1973. In many of these studies, the non-uniformity of the predicate-logical interpretation scheme is repeated. Indeed, in principle, every linguistic category may have its own peculiarities in this respect.

*The Montague format*

Now, when Montague combined and extended much of this work to obtain full and systematic coverage of complete natural language fragments, he chose one particular format, viz. the functional one; treating all expressions as terms. (In some ways, this choice echoes pre-modern logical attitudes, favouring terms over propositions.) Thus, the basic Montagovian schema assigns denotations throughout:

$value(\in, D, I, w, t, \ldots, w_0, t_0, \ldots, a) = \ldots$.

Here, w,t (and perhaps other parameters) refer to world-times described, while $w_0, t_0$ (and perhaps a much larger indexical package) represent the context of utterance. The latter is absent in the case study 'Formal Treatment' (cf. Montague 1974); but, of course, Montague himself was a pioneer of indexical studies, in his work on pragmatics.

Now, in one sense, the above scheme is just a notation, making no claim whatsoever about the actual process of interpretation. But, it has certainly drawn people's attention in one direction: from linguistic items to, often very baroque semantic entities.

One explanation for this unidirectional emphasis is a common concern with Compositionality of interpretation, both for formal reasons and as a natural account of our infinite semantic competence. Despite the undeniable merits of this principle, however, it should not be held too sacred. After all, already in predicate logic itself, some strains occur in its application. In the most common approach, formulas are interpreted as sets of their verifying assignments – a set-up which satisfies the letter of the recursion requirement. But surely, there is something very unattractive about having 'semantic' entities which themselves incorporate interpretative links from language to 'real' objects. (This objection may be met by re-working the whole situation into so-called 'cylindric algebras'. But, the latter are not very well-understood model structures, at least at present.) Likewise, compositional interpretation in tense logic sends formulas to sets of times where (when) they hold: a not unreasonable, but also not very intuitive choice of semantic denotations. These observations point in the same direction. Any semantic scheme with some recursion somewhere can probably be given some compositional reformulation. But, whether such a move is enlightening, remains to be seen from case to case. Judicious adherence to Compositionality is an art, rather than a science.

## 4 *Back and forth between Montague Semantics and Situation Semantics*

Given the above general picture, a global comparison is easily made between the interpretative mechanisms of Situation Semantics and its predecessors, notably Montague Semantics. Both schemes recognize essentially the same components as being essential in interpretation. After all, most of these have long been accepted common sense for flexible users of the Tarski scheme. Still, the situation semantics notation forces one to bring out more explicitly the dynamical aspects of interpretation of natural language, with its different division of labour between the parameters of evaluation.

In fact, all this is so obvious that the only urgent question remaining seems to be why the Montague scheme, with its emphasis on semantic values (and hence, to some extent, described situations) has been so adequate for so long a period. One practical reason is that natural language itself seems to have a similar bias: the majority of lexical items is descriptively, rather than indexically oriented. Moreover, perhaps, many researchers tend to interpret their schemes as being about *written* linguistic expressions $\in$, rather than *spoken* ones. And the former type of discourse is already somewhat more de-indexicalized than the latter. (The notion of 'de-indexicalization' has been emphasized by Bar-Hillel, in the philosophy of language and science.) 'Situations and Attitudes' is full of references to speakers and listeners, rather than writers and readers – an interesting change of cultural climate.

More concrete conclusions may be drawn when the above two frameworks are compared as to their combinatorics of meanings. We shall discuss this issue by means of examples – from which general translations can be extracted by the industrious reader.

*From relational to functional format*

Given a relational scheme for a linguistic expression $\in$, as presented in section 2:

${}_{d,c}[\![\in]\!]_{s,a}$,

it is easy to extract a functional denotation, Montague-style, by using suitable abstractions. In fact, this process already occurred in the Situation Semantics itself, when forming 'roles'. At least extensionally, this is just a case of varying the anchoring (partly or wholly), as in the following example:

in D,I: ${}_{d,c}[\![\in_x x]\!]_{s,a}$ becomes
*value*$(\in_x x_{,D,I,s,d,c,a}) = \{<u,U> \mid u,U$ in s such that
${}_{d,c}[\![\in_x x]\!]_{s,a(x/u)(X/U)}\}$

Thus, the relational format is easily recast in functional terms: if the account of the meaning for $\in$ involves essentially the indeterminates $x_1,\ldots, x_i, X_{i+1},\ldots, X_n$, then a functional account will assign sets of n-tuples of suitable anchoring values. And the latter may themselves be made functional, using characteristic functions in the standard manner.

In specific examples, this procedure may or may not produce the original Montagovian denotation. For instance, $walk_x$ will indeed denote the set of walkers in D. But, e.g., $Mary_x$ will get the singleton set {I(*Mary*)} (or, the role of being called 'Mary'), rather than I(*Mary*) herself. Nevertheless, the process itself is clear – and the reasons for possible deviations from standard practice are transparent. In fact, there may even be a slight advantage to the latter. For instance, *Mary* may indeed have *two* associated semantic entities, even classically. One is the 'bare denotation' itself. The other is a kind of 'co-operative denotation', indicating handles for links with the linguistic environment. In any case, the rich type hierarchy of Montague Semantics can handle all combinatorial possibilities arising in this way.

*From functional to relational format*

In the converse direction, from Montague Semantics to Situation Semantics, a complete transfer is possible too, turning a functional treatment of an expression into a relational one. Some examples will guide its formulation (compare section 2):

– Intransitive verbs (Montague type (e,t)) get one individual indeterminate, obviously corresponding to the argument type e.

Note that only *extensional* types are being considered here. Catering for Montague's intensional extravaganzas would only complicate our points – besides being more debatable to a situation semanticist.
– Likewise, transitive verbs (type (e,(e,t))) get two individual indeterminates, representing their two arguments.

Of course, according to Montagovian orthodoxy, the transitive verb type is rather (((e,t),t),(e,t)). But, this is now widely regarded as an unfortunate error, due to an overly rigid use of fixed types in his categorial grammar. (Cf. van Benthem 1983b; van Eyck 1984. A natural type-change calculus will let transitive verbs accept noun phrase objects without the above sacrifice.)
– Determiner expressions (type ((e,t),((e,t),t))) get two set (or role) indeterminates, again in accordance with their type arguments.

The general pattern here is simply this. Many Montagovian types have the form

$(a_1,(a_2,\ldots(a_n,t)\ldots))$;

and thus may also be read as expressing relations between objects of types $a_1,\ldots,a_n$.

Even so, Situation Semantics makes an interesting claim, viz. that, of all the possible types $a_1,\ldots,a_n$ in the above positions, we shall find only the need for individuals (e) and sets of these ((e,t)). (Actually, the situation is not quite this austere, because of the facility of 'parametrized' individuals and roles, carrying open slots. But in any case, the latter only increase the combinatorial chances for accomodating Montagovian types.)

Exceptions to this first transfer scheme would have to be sought, then, with types not of this 'final t' form. And indeed, for instance, proper names (type e) were treated differently in section 2, with an individual indeterminate – standing for, if one wishes, their value, rather than any argument. (Compare also the discussion of functional expressions in section 3.)

Another difficult example for the above scheme are the propositional connectives (*not*, *and*, etcetera). Even though these have types with final t, their meaning is deeply functional, rather than relational. Indeed, it is often claimed that Situation Semantics cannot handle such expressions satisfactorily at all. (But, cf. Kamp 1983, Cooper

1984.) For instance, on the above scheme, one might expect a relational account of negation to run somewhat as follows:

$_{d,c}[\![not_p]\!]_{s,a}$ iff a(p) *has value* 0.

But, this will produce unacceptable results, such as the equivalence of double with single negation. The only smooth way-out here is to introduce *two* polarity indeterminates:

$_{d,c}[\![not_{p1\ p2}]\!]_{s,a}$ iff a(p1) *has the complementary value of* a(p2).

Then, p2 is to be set equal to the leading polarity indeterminate of the following expression.

As a more concrete example, e.g., *propositional logic* may be set up in this fashion. For this purpose, polarity indeterminates are distributed as follows. Proposition letters q each get a special indeterminate $p_q$, negation gets two, conjunction and other binary operators get three associated polarity indeterminates – all with the obvious semantic clauses for their interpretation. Formulas φ can then be constructed, each with a 'leading polarity indeterminate', as follows:
- proposition letters q have leading indeterminate $p_q$,
- if φ has leading indeterminate p, and *not* has p1, p2 (with p1 leading), then *not* φ has p1 leading, with the stipulation that p2 = p (and all conditions stemming from the two components still in force)
- if $\varphi_1$ has leading p1, $\varphi_2$ leading p2, and *and* has p1′, p2′, p3′ (p1′ leading), then ($\varphi_1$ *and* $\varphi_2$) has p1′ leading, with added equations p2′ = p1, p3′ = p2.

Then, formulas can be interpreted in situations with anchors defined on all their polarity indeterminates:

$_{d,c}[\![q]\!]_{s,a}$ iff $a(p_q) = c(q)$
$_{d,c}[\![not\ \varphi]\!]_{s,a}$ iff $_{d,c}[\![not]\!]_{s,a}$ *and* $_{d,c}[\![\varphi]\!]_{s,a}$ *and* $a(p2) = a(p_\varphi)$.
$_{d,c}[\![\varphi_1\ and\ \varphi_2]\!]_{s,a}$ iff $_{d,c}[\![\varphi_1]\!]_{s,a}$ *and*
$_{d,c}[\![\varphi_2]\!]_{s,a}$ *and* $_{d,c}[\![and]\!]_{s,a}$ *and* $a(p2') = a(p_{\varphi_1})$, $a(p3') = a(p_{\varphi_2})$.

Evidently, shifting to a viewpoint where c is a *valuation* on proposition letters in the standard sense, the following equivalence is forthcoming:

$_{d,c}[\![\varphi]\!]_{s,a}$ if and only if $c(\varphi) = a(p_\varphi)$.

Whether there is any logical advantage to this perspective remains to be seen.

In the above, expressions got indeterminates, not just for their argument types, but also for their value type. To turn this into a general transfer scheme, this process will have to be performed uniformly. At least extensionally, the only ingredients required for this purpose are individual and polarity indeterminates – together with a realization that *all* Montague types are of one of the following forms:

basic types e,t, or complex types
$(a_1,(a_2,\ldots(a_n,t)\ldots))$ or $(a_1,(a_2,\ldots(a_n,e)\ldots))$.

Finally, applicative links in Montague Semantics are mimicked by suitable identifications of indeterminates.

One advantage which is sometimes claimed for such a transition is its replacement of ('subjunctive') application by *conjunction* of semantic conditions. And it is true that this view-point has proved useful for many types of linguistic expression, witness the earlier-mentioned case of verbs or determiners. On the other hand, a conjunctive view seems inappropriate or misleading for certain very 'operational' categories of expression. The above connectives form a case in point. Another example are adverbs or adjectives, whose meanings are not necessarily conjunctive (or 'intersective'). Still, their semantic type is ((e,t),(e,t)); and hence a relational treatment ought to be possible, on the above transfer schemes. For instance, on the first scheme, one might set:

$_{d,c}[\![ walk_x\ slowly_y\, x ]\!]_{s,a}$ iff $_{d,c}[\![ walk_x ]\!]_{s,a}$ *and*
$\hat{x}.\ _{d,c}[\![ walk_x ]\!]_{s,a} = a(X)$ *and* $_{d,c}[\![ slowly_y\, x ]\!]_{s,a}$
(i.e., 'a(y) has the property I (*slowly*) (= a(X)) in s').

Any more conjunctive account than this would run into the danger of producing 'x walks and x is slow', which is inadequate. But observe how an inherently functional account of *slowly* has been smuggled into the semantic clause (as was the case already with the above account of negation).

Thus, the relational format can handle all functional types; be it at the peril of its soul.

This conclusion does not exhaust all possible points of comparison. Notably, Montagovian types also encode an argument *ordering*: a feature often exploited in their linguistic application. The indeterminate approach either has to let this slip, or impose some additional constraints (witness the discussion of section 2). In addition, there remains the expectation, mentioned above, that simple indeterminates will turn out to suffice for natural language – a conjecture which has also been formulated within the setting of Montague Semantics. As an empirical claim about language, this claim stands a good chance of being true – especially when the earlier-mentioned calculus of type change is available to dispose of unduly 'inflated' counter-examples. (See van Benthem 1986.)

This completes our technical observations on the connection between functional and relational accounts of meaning.

## 5 *Conclusion*

The relational theory of meaning and earlier functional approaches seem equally powerful in principle. In retrospect, this outcome is not too surprising, given the existence of broad equivalences between more functional and more relational *type theories* in logic (cf. van Benthem and Doets 1983). Both frameworks are uniformizations of a semantic reality for natural language which seems to have non – uniform preferences with different types of linguistic expression. Even so, both have their own heuristic and intuitive virtues. In particular, they carry different suggestions as to the most appropriate *model classes* to be attached. Specifically, the relational account does not encourage the prevalent Montagovian picture of full function hierarchies, suggesting instead a lower-level universe of (parametrized) individuals and properties, with a cross-traffic between the two. But then, we are passing on to another component of the semantic enterprise, namely, what are the most appropriate models for natural language – an issue which even the founding fathers of Situation Semantics are still struggling with. The modest aim of this paper with its severely limited scope has been to clear the way toward an objective discussion of alternative semantic world-views.

*Universiteit van Amsterdam* Johan VAN BENTHEM
Mathematisch Instituut
Roetersstraat 15
1018 WB Amsterdam

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