MODALITY DE RE AND VASILIEV'S IMAGINARY LOGICS

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The theme of my report is the reconstruction of the Vasiliev's logical systems and the analysis of the modality de re. Unfortunately, modal syllogistic of Aristotle has not been adequately reconstructed till now. Even Charles Peirce in this time remarked that at the foundation of all Aristotle's philosophy lies his teaching of modality. In my opinion, the key problem in the understanding Aristotelean modal syllogistic is given in ''imaginative'' (non-Aristotelean) logic of Vasiliev. Vasiliev has published his original works in years 1910-1914. I'll give the reconstruction of imaginary logic of Vasiliev – the logic without the law of contradiction. Then I shall show how to pass on (from it) to the modal syllogistic of Aristotle proper. But first of all I'll dwell on assertoric syllogistic.

In 1974 I proposed interpretation of syllogistics in terms of predicate calculus. I proceeded from the idea of Ockam, that positive sentences affirm non-emptiness of the subject. But negative sentences do not. Lewis Carrol essentialy uses this idea. But he did not analyse particular negative sentences, that is OSP. This operation is as follows:

$$(ASP)^{+} = \exists x\, Sx \,\&\, \forall x\,(Sx \supset Px)$$
$$(ESP)^{+} = \forall x\,(Sx \supset \neg Px)$$
$$(JSP)^{+} = \exists x\,(Sx \,\&\, Px)$$
$$(OSP)^{+} = \exists x\, Sx \supset \exists x\,(Sx \,\&\, \neg Px)$$
$$(\neg \alpha)^{+} = \neg(\alpha)^{+}$$
$$(\alpha \circ \beta)^{+} = (\alpha)^{+} \circ (\beta)^{\circ}, \text{ where } \circ \text{ is } \&, \vee, \supset.$$

Then I gave the following axiomatization of syllogistic C2:

1. ASM & AMP $\supset$ ASP
2. ASM & EMP $\supset$ ESP
3. ESP $\supset$ EPS
4. ASP $\supset$ JSP
5. JSP $\equiv \neg$ESP
6. OSP $\equiv \neg$ASP
7. JSP $\supset$ ASS

I proved the following theorem: α is provable in C2 iff α^+ is provable in one-place predicate culculus.

The proof of this theorem is not trivial.

I think that system C2 is a very natural one. I could not find laws of identity ASS and ISS in Aristotle's works. These laws are introduced by Leibnitz.

The system C″ – without identity laws may be extended to get a system definitionally equivalent to Boolean-algebra. It was stated by Bocharov. I changed his axiomatization as follows

C2D = C2 +

8. $ASP \supset ESP'$
9. $ASS \;\&\; ESP' \supset ASP$
10. $E(S \cap P)M \supset E(M \cap S)P$
11. $EM(S \cup P) \equiv EMS \;\&\; EMP$
12. $EM(S \cap P)' = EMS' \;\&\; EMP'$

C2D is a definitionally equivalent to Boolean algebra of classes, that is $C2D + S \subseteq P \equiv ESP'$ is equivalent to $B + ASP \equiv \neg(S \subseteq S') \;\&\; (S \subseteq P) + ESP \equiv S \subseteq P' + JSP \equiv \neg(S \subseteq P') + OSP \equiv \neg(S \subseteq S') \supset \neg(S \subseteq P)$.

Let us go back to Vasiliev. In his first work "About Particular Judgements, about the Triangle of Opposites, about the Law of the excluded Fourth" Vasiliev constructs syllogistics without the "Law of the excluded Third". It is necessary to emphasize that Vasiliev distinguishes between two levels of logical Laws. He refers to the laws regulating activities of cognizing subjects as the laws of the first level. He calls these laws metalogical laws. Vasiliev accepts the principle that no statement can be both true and false (The Law of non-selfcontradiction). It is also accepted that every statement must be true or false. The other level is the level of ontological character. It is connected with ontological commitments: "It is not true that a is P and a is not P". This is just the Law of Contradiction. According to Vassiliev it is possible to think of the logic without the ontological laws of contradiction and the tertium non datur.

I shall proceede from the presupposition that the laws of metalogic are the laws of the logic of propositions. And, according to Vasiliev, this logic is always a classical logic.

Non classical elements are expressed by the fact that different types of predications are possible. Or, if we regard the logical system as a combined calculus of propositions and classes then the Logic of propositions may be expanded by different theories of classes.

I do not agree with Mal'cev and Kline that Vasiliev had formulated the idea of manyvalued logics. Of course Vasiliev's ideas prompt to us the propositional Logics, that differ from classical Logics. This approach was very well realized by Aida Aruda. But I think that if we identify Vasiliev's metalogics with propositional logics as I supposed – then the propositional logic is a unique and it is classical logic.

Vasiliev in his first work "About particular statements..." takes as basic three type of statements:

ASP – general positive
ESP – general negative
TSP – accidental (particular) – "Only some S are P".

And statements of each of these types relate to the whole extension of S. He gives two interpretations to the statement TSP: disjunctive and accidental.

Some words about the disjunctive interpretation. Sentences "all S are P or Q" Vasiliev interprets as "some S are P and all others are Q"; in sympolic notation of predicate calculus we have: $\exists x(Sx \& Px) \& \exists x(Sx \& Qx) \& \forall x(Sx \supset Px \vee Qx)$. That is why, the sentence TSP must be interpreted as $\exists x(Sx \& Px) \& \exists x(Sx \& \neg Px) \& \forall x(Sx \supset Px \vee \neg Px)$, that is as $\exists x(Sx \& Px) \& \exists x(Sx \& \neg Px)$.

It is important for us that ASP, ESP and TSP are pairwaise inconsistent in pairs and their disjunction is true (The Law of the excluded Fourth). Now it is easy to propose the axiomatic of this systems

1. ASM & AMP $\supset$ ASP
2. ASM & EMP $\supset$ ESP
3. ESP $\supset$ EPS
4. $\neg$(ASP & ESP)
5. $\neg$(ASP & TSP)
6. $\neg$(ESP & TSP)
7. ASP $\vee$ ESP $\vee$ TSP
8. ESP $\vee$ ASS

It is easy to show that this Vasiliev's system is definitially equivalent to C2. We add to Vasiliev's system:

$$JSP \equiv ASP \vee TSP$$
$$OSP \equiv ESP \vee TSP$$

and add to C2

$$TSP \equiv JSP \,\&\, OSP$$

Formally, Vasiliev's syllogistic system is given in his paper "About particular statements" is a definitional equivalent to standard syllogistic, though he himself gives a different interpretation to his system.

The *accidental interpretation* gives us another system of syllogistic. I propose the next translation of accidental sentences into $S5\pi$:

$$\varphi(A^{\Box} SP) = \exists x\, Sx \,\&\, \forall x \Box (Sx \supset \Box Px)$$
$$\varphi(E^{\Box} SP) = \forall x \Box (Sx \supset \Box \neg Px)$$
$$\varphi(T^{\nabla} SP) = \exists x \Diamond(Sx \,\&\, \Diamond Px) \,\&\, \exists x \Diamond(Sx \,\&\, \Diamond \neg Px)$$

In this translation $T^{\nabla} SP$ follows from JSP & OSP, but not vice versa and axiom 8 $E^{\Box} SP \vee A^{\Box} SS$ is not valid. Let CVA is a result of replacing of Axiom 8 for $E^{\Box} SS \supset E^{\Box} SP$. CVA is consistent for this translation. The question about completeness have not been discovered.

Another system, given in the work "Imaginative (non Aristotelean) Logic" is of greater interest to us. In this system the Law of Contradiction is non-valid. Vasiliev supposes that positive atomic statements of the kind "a is P" and only these, are the foundation of the usual standard Logics. Negative sentences are not atomic sentences. They are sentences about the falsity of atomic positive sentences. All negative statements are the result of deduction from atomic positive statements and statements about the incompatibility of properties. For example "a is not red" is the conclusion from "a is green" and "green is incompatible with red". In an imaginary situation is possible that both positive and negative statements are atomic. In this case, it is possible that positive and negative statements may be compatible. It is possible that sentences "a is and is not P" is true. We can regard the following statements as atomar statements:

a is P (and it is not true, that a is not P)
a is not P (and it is not true, that a is P)
a is and is not P

There are three kinds of atomic singular sentences in imaginary logic: positive, negative and indifferent.

I suggest that these singular sentences could been interpreted in topological terms, correspondently, as $a \in P^{\circ}$, $a \in P'^{\circ}$ and $a \in P^{F}$, where $^{\circ}$ is operation interior, $+$ is closure, ^{F} is frontier and $'$ is complement. Naturally, these sentences cannot be paarvise true

$$(\neg(a \in P^{\circ} \;\&\; a \in P'^{\circ}),\ \neg(a \in P^{\circ} \;\&\; a \in P^{F}),\ \neg(a \in P'^{\circ} \;\&\; a \in P^{F}))$$

and their disjunction is true ($a \in P^{\circ} \vee a \in P'^{\circ} \vee a \in P^{F}$).

Vasiliev constructs "sentences about classes" on the basis of a singular sentences. There are three kinds of general sentences and four kinds of particular or, as Vasiliev prefers to say, accidental sentences. This is a basic system of syllogistic in sence, that: every pair of them cannot be simultaneously true and disjuction of all seven sentences is true. Vasiliev considers general positive and general negative sentences to be apodectic. Let us denote them $A_{\Box}SP$ and $E_{\Box}SP$. I denote general indefferent sentences "All S are P and non-P" as $A_{\nabla}SP$. Four kinds of accidental sentences are the followings: (1) Some S are P and all others are non-P, (2) Some S are P and all others are P and non-P; (3) Some S are non-P and all others are P and non-P; (4) Some S are P, some S are non-P, and all others are P and non-P. Let us denote correspondingly $T^{p}_{n}SP$, $T^{p}_{i}SP$, $T^{n}_{i}SP$, TdSP. Following Vasiliev it is possible to propose the following translation of these sentences into predicate calculus:

$$\psi(A_{\Box}SP) = \exists x\, Sx \;\&\; \forall x(Sx \supset P^{\circ}x)$$
$$\psi(E_{\Box}SP) = \forall x(Sx \supset P'^{\circ}x)$$
$$\psi(A_{\nabla}SP) = \exists x\, Sx \;\&\; \forall x(Sx \supset P^{F}x)$$
$$\psi(T^{p}_{n}SP) = \exists x(Sx \;\&\; P^{\circ}x) \;\&\; \exists x(Sx \;\&\; P'^{\circ}x) \;\&\; \forall x(Sx \supset P^{\circ}x \vee P'^{\circ}x)$$
$$\psi(T^{p}_{i}SP) = \exists x(Sx \;\&\; P^{\circ}x) \;\&\; \exists x(Sx \;\&\; P^{F}x) \;\&\; \forall x(Sx \supset P^{\circ}x \vee P^{F}x)$$
$$\psi(T^{n}_{i}SP) = \exists x(Sx \;\&\; P'^{\circ}x) \;\&\; \exists x(Sx \;\&\; P^{F}x) \;\&\; \forall x(Sx \supset P'^{\circ}x \vee P^{F}x)$$
$$\psi(TdSP) = \exists x(Sx \;\&\; P^{\circ}x) \;\&\; \exists x(Sx \;\&\; P'^{\circ}x) \;\&\; \exists x(Sx \;\&\; P^{F}x)$$

The Proper axiomatization of imaginary syllogistic of Vasiliev is the following: These seven statements are paarwise inconsistent

($\neg(A_\Box SP \& E_\Box SP)$, $\neg(A_\Box SP \& A_\nabla SP)$, $\neg(A_\Box SP \& T^p_n SP)$,...), their disjunction is true ($A_\Box SP \vee \vee E_\Box SP \vee A_\nabla SP \vee T^p_n SP \vee \ T^p_i SP \vee T^n_i SP \vee TdSP$) and

$$A_\Box SM \ \& \ A_\Box MP \supset A_\Box SP$$
$$A_\Box SM \ \& \ E_\Box MP \supset E_\Box SP$$
$$A_\Box SM \ \& \ A_\nabla MP \circ A_\nabla SP$$

are valid.

In the above mentioned translation all axioms are valid formulas of predicate calculus with operation + and ° ($P^\circ \subseteq P$, $P \subseteq P^+$, $P'^{\circ\prime} = P^+$). Until now I have not checked up whether the operation ψ is an imbedding operation.

The system of syllogistics of Vasiliev possesses specific properties. General negative sentences are not conversible. From $E_\Box SP$ does not follow $E_\Box PS$ (From $\forall x(Sx \supset P'^\circ)$ does not follow $\forall x(Px \supset S'^\circ x)$). This inconversibility $E_\Box$ is emphasized by Vasiliev himself.

In the terms of basic statements there may be defined other statements too, for example usual particulars:

$$J_\Box SP \equiv A_\Box SP \vee T^p_n SP \vee T^p_i SP \vee TdSP$$
$$O_\Box SP \equiv E_\Box SP \vee T^p_n SP \vee T^n_i SP \vee TdSP$$
$$J_\nabla SP \equiv A_\nabla SP \vee T^p_i SP \vee T^n_i SP \vee TdSP$$

Their translations will be correspondingly:

$$\psi(J_\Box SP) \equiv \exists x(Sx \ \& \ P^\circ x)$$
$$\psi(O_\Box SP) \equiv \exists x(Sx \ \& \ P'^\circ x)$$
$$\psi(J_\nabla SP) \equiv \exists x(Sx \ \& \ P^F x)$$

$J_\Box$ is not conversible, too.

It is possible to introduce the statements of possibility. They are the duals of necessary sentences:

$$A_\Diamond SP \equiv \neg O_\Box SP$$
$$E_\Diamond SP \equiv \neg J_\Box SP$$
$$J_\Diamond SP \equiv \neg E_\Box SP$$
$$O_\Diamond SP \equiv \neg A_\Box SP$$

We must stress that in Vasiliev's system there are no assertoric statements proper. It seems to me that Vasiliev's syllogistic system is a key to understanding Aristotelean modal syllogistic.

In order $E_\square$ be conversible, it is enough to substitute S for S^+ in all translations. Assertoric statements are interpreted in the same way as in C2. $A_\square$, $E_\square$, $J_\square$, $O_\square$ are necessary statments. A_∇ SP and J_∇ SP are accidental (bilateral possibility).

This interpretation has moods of the first figure with one assertoric and other necessary premises. This system is the first approximation of Aristotalean syllogistics. There is the following discrepancy: $J_\square$ SP is not conversible without additional topological presupposition. Aristotle rejects mood $BARO_\square CO_\square$, but his translation is provable. However, mood $BO_\square CARDO_\square$ rejected by Aristotle, is rejected also in the interpretation, proposed by mine.

I don't assert that our interpretation of modal syllogistic is fully adequate to Aristotaelean texts. At present Georgian logicians put forward different interpretations of Aristotle's modal syllogistics. In any case, Vasiliev's ideas about the possibilities of contradictory statements become transparent and quite acceptable in the above mentioned topologicial interpretation.

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