EXACT MODELS OF DIALECTICAL SYNTHESIS

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The synthesis of dialectical opposites yields meaningful contradictions, but attempts to study them in the framework of an exact model are confronted by serious difficulties. There is a lot of misunderstanding about the strategy of conjoining the thesis and the corresponding antithesis in a unity of opposites where extremes meet and interpenetrate. Instances of coinciding opposites are most readily accepted as poetic inventions which have nothing to do with the rigorous requirements of logic. The only important thing about them, as far as verse is concerned, is that they should be expressed in a non-trivial way. The same holds good for some logics, as N.C.A. da Costa has observed. [1] Certain non-trivial formal systems admit of true contradictions and, as regards the syntax, allow for the inference of theorems which are the negation of some other theorems of the same system. Along these lines a substantiation is sought here for what I call 'dialectical synthesis' – a heuristic move or an inference-like structure that enables one to pass over from negative premises (the parallel refutal of a thesis and its antithesis) to a positive consequence. The latter is a specific unity of two opposites which have formerly been incompatible, but no longer fall under the notion of incompatibility. The conjunction of the thesis and the antithesis, then, stands for this 'coincidentia oppositorum' which is the result of dialectical synthesis.

Too ardent adherents of classical formal logic have been somewhat suspicious, if not hostile, to the very idea of letting dialectics in the realm of deductive sciences. As for proponents of dialectical logic, some of them have been rather reluctant to accept the method of formalization, so their views often do lack precision. This is to blame for most of the shortcomings of the position they stick to. Their position, however, should not be identified with the dialectical outlook in general. The latter, unfortunately, has been too often profaned by far-fetched confirmations and short-sighted 'counterexamples'. For centuries on end it has been regarded as incoherent philosophy or merely as poetry, utterly alien to the exact methods of science. Those who did not deny its scientific character, considered the state-of-the-art as "still-waking sleep that is not what it is", to put it in the words of Shakespeare. [cf. "Romeo and Juliet", Act 1, scene 1]

More than a hundred years ago Hegel stirred up the very stronghold of logic by declaring that "contradictio est regula veri, non regula falsi". This claim provoked strong opposition – it sounded heretical to all who took for granted the absolute invulnerability of the Law of Contradiction. As Wilhelm Dilthey has pointed out, "It was to Hegel's credit that in his logic he tried to express the restless stream of events. But he was mistaken when he tought that this could not be reconciled with the principle of contradiction..."[2] It is equally misleading however, to maintain that logics and theories, based on this principle cannot be synthesized with the assumption of true or meaningful contradictions. Along these lines Hegel's contributions have been appreciated by a number of outstanding thinkers – to begin with the classics of dialectical materialism and proceed to contemporary logicians like G. H. von Wright and the paraconsistentists. Here I shall try to add some arguments in favour of this synthesis, arguing that in the long run formal and dialectical logics can in principle be unified in their capacity of dialectical opposites.

The latter, generally speaking, are characterized by their individual development and by the link (bond, connection) between them. Due to it the individual changes of the two are brought together: they influence each other and leads to some relative changes concerning the systematic unity as a whole. This shapes the process of development, its continuity being interrupted in definite points. Up to a certain point or moment the relation of these changing opposites remains invariant. But beyond this limiting point the very relation between dialectical opposites is converted, i.e. undergoes a transition into an opposite one: from incompatibility to interpenetration, or vice versa. Thus formal and dialectical logic have been incompatible for quite a long time. Meanwhile changes have been taking place within each of them in the process of their individual developments. They have brought about a relative change, in the course of which the two logical traditions have begun to interpenetrate. The so called paraconsistent logics (the term is due to F. Miro-Quessada) are a manifestation of this tendency. It is in the main stream of these logics that I have tried to delineate an approach to dialectical synthesis. [3] Here I am not going to dwell on it in detail; I shall rather set forth some new ideas of mine and elaborate them in comparison with G. H. von Wright's illuminating approach.

As Georg Henrik von Wright has put it, "From Dialectical Logic an operation is known which goes under the name of Dialectical Synthesis leading to what is known as the Unity of Opposites (coincidentia oppositorum). It can be described as follows:

Let there be a proposition θ. We call it 'thesis'. Its negation not-θ is then called 'antithesis'. By some means or other, we disprove the thesis. The thesis is thus not true, $\sim T\theta$. By some means or other, we also refute the antithesis. Thus it too is not true, $\sim T \sim \theta$. From these findings, viz. that the thesis is neither true, nor false (= the antithesis true) we now 'conclude' that the thesis is both true and false. This move is called Dialectical Synthesis.

Is the arrow in Zeno's famous aporia moving or at rest at a given moment of time? Arguments may be produced to show that it is not moving – but also to show that it is not at rest. Therefore: the arrow is both moving and at rest." [4]

Obviously in the premise of this inferential move the thesis and the antithesis are regarded as incompatible, whereas in the consequence they are no longer such. This conversion markes a transition of the initial relation between the opposites ('thesis' and 'antithesis') into an opposite one. i.e. from incompatibility to compatibility. The transformation implies conceptual change which reflects either objective development in reality, or an alteration of viewpoints. [5]

There is an ancient Indian myth about Indra fighting the demon Vrta. Indra had sworn to attack the monster neither by day, nor by night, so he engaged in a battle at dawn, when it was already not night, and still not day. Evidently, it makes no difference to say that it was already day and still night at dawn. But this interpretation would imply, that Indra had broken his oath, whereas under the former he could not be accused of it. There are no other grounds for preference of the negative interpretation to the positive one. The choice of the former which usually takes place is a matter of deeply rooted logical prejudice, typical of Western thought. To overcome it takes staunch determination in favour of dialectics, as well as profound studies in the history of logics.

Looking back to the wellsprings of ancient science, one finds instances of invincible contradictions that rise from the ashes of severe criticism, misunderstanding or neglect. Such a methodological phoenix is the system of arguments, known as Zeno's paradoxes. In the contest of these arguments Diogenes Laertius (IX, 72) refers to a statement of Zeno's to the effect that the moving objects moves neither in the place it is occupying, nor in the one where it is not to be found. Hegel and Engels regard this as the premise of dialectical synthesis sui generis. In my opinion, it involves a distinction between the negation of predicates and – on the other hand – the negation of copula. Aristotle himself carries out such a distinction and this is the reason why in his "Organon" some strange predicates are to be found, e.g. 'neither finite, nor infinite..." (cf. "Prior Analytics", Chapter 46). Such predicates are common with the Stoic thinker Chrysippus – the forerunner of propositional logic. He is said to have written some fourteen books on the Liar paradox, their message being that this antinomic proposition is neither true, nor false.

Differentiating between types of negation, call them 'internal' and 'external', makes it possible for us to parallelly refute both the thesis and the antithesis, so that the negation of the latter does not take us back to the former. Kant, for example, layed special accent on statements with negative predicates in his famous "Critique of pure reason". As regards antinomies, the philosopher pointed out that if he declared the world to be either finite, or infinite, both might not be true. It is important to note that negative-dialectical phrases of this type seemed less dangerous to traditionally minded logicians. As for the positive-dialectical ones (viz. Heraclitus-style statements) they were systematically averted for fear of paradoxes. Hegelian logic was the first to get rid of this horror of contradictions, and to recognize in them the driving force of development. Later dialectical materialism took special interest in contradictions and particularly in the heuristic functions of dialectical synthesis. Marx has implemented it in "Capital" (volume 1, chapter IV) when analysing the genesis of surplus value. Naturally, he has put it in terms of political economy, but the underlying logic is, no doubt, dialectical.

A report on "Truth, Negation and Contradiction", delivered by G.H. von Wright at the 3-rd Soviet-Finnish Logic Symposium, conveys the idea that "Dialectical Synthesis is a logically legitimate inference in certain cases – but it involves a shift in the concept of truth from a stricter to a laxer notion, both of which, however, answer to common and natural uses of the words 'true' and 'false' when applied to propositions. This shift fits the facts particularly in situations when we are concerned with becoming or process, two ideas which are prominent in Hegelian and Dialectical Logic." [6] In his new book "Truth, Knowledge and Modality", in the chapter on Truth-Logic and 'Dialectical Synthesis' von Wright has shown that the two notions are interdefinable. "To be true in the laxer sense simply means not to be false in the strict sense – and to be true in the strict sense means not to be false in the more liberal sense of 'true' and 'false'. Thus we have

$$T'p =_{df} \sim T \sim p$$

and

$$Tp =_{df} \sim T' \sim p$$

As we know, for the notion of strict truth the Law of Contradiction holds good: no proposition is both strictly true and strictly false. But the Law of Excluded Middle does not hold: a proposition may be neither strictly true, nor strictly false.

For the relaxed notion of truth the situation is the opposite." [7] Thus, by way of a shift in the sense of 'true', Dialectical Synthesis makes good sense, in keeping with the main thrust of conceptual change, which is essential in dialectics.

As regards conceptual change, von Wright's approach implies a shift in the sense. But concept in general and particularly the notion of truth may undergo changes of meaning, too. In my opinion Dialectical Synthesis implies a shift either in the sense, or in the meaning of concepts, more specifically – of the concept of truth. A shift from one designated truth-value to another (also designated) may represent conceptual change with respect to different meanings of 'true'. Dialectical Synthesis, then, stands for a kind of non-tautological inference. It is truth-preserving and yet it changes the truth-value of the premise, so that the consequence gets a new designated truth-value. It is only natural that the underlying logic in this case should be a many valued one which, in addition, ought to be tolerant of some non-trivial contradictions.

By non-trivial contradictions I mean ones which, although being provable in a certain formal system S, do not render S overcomplete, i.e. they do not allow for the inference of an arbitrary formula of the calculus. So the class of its theorems is strictly included in (but not equivalent to) the class of all its well formed formulas. Thus the system itself does not explode into triviality, never mind that the negations of some theorems in it are also theorems, i.e. they are proved in the same system.

My approach to preventing a formal system from being trivialized by provable contradictions has been outlined in [8] and [9]. Here I am going to summarize it as follows. Dependence characteristics are ascribed to the theorems, as well as to the premises of the very inference rules. These characteristics indicate the list of axioms, from which the theorems (resp. the premises) have been deduced. For example, in the case of modus ponens we should write:

R* A "ax.1 – ax.n", $(A \rightarrow B)$ "ax.1 – ax.n" $\vdash$ B "ax.1 – ax.n"

instead of the usual A, $(A \rightarrow B) \vdash B$ which is equivalent to the former, provided the dependence characteristics "ax.1 – ax.n" exhaust the list of all the axioms in the system. If there are more axioms, say ax.n + 1 – ax.n + m, they and the theorems derived with their help cannot be used as premises of R*. Even if among them a genuine contradiction is to be found, it cannot be spread by means of this inference rule to spoil the system as a whole. (Instead of a deductive relativization of theorems and inference rules we might introduce a temporal one; then the dependence characteristics would indicate intervals of time.) Various parametres of relativization could be held out – the important thing about it is that by means of such intrinsic coordination of the inference machinery any consistent axiomatic formal system can in principle be made paraconsistent. Thus we could extend it in a direction which had previously been thought forbidden by postulating the negations of some of its theorems.

I wish the laws that determine classical negation to be such theorems which can peacefully coexist with their negations. To put it in a more precise way, I wish the formulas, the negations of which are known as the Law of Contradiction and the Law of Excluded Middle, to be provable in a non-trivial formal system that contains all the theorems of classical propositional logic. To my mind, even against the background of this calculus, including the notorious Duns Scotus

Law $((A \& \sim A) \rightarrow B)$ the principle of contradiction can be reconsiled with some contradictions that are provable in the syntax and true in the related semantics. Here is a way of doing this; it seems at first glance quite ad hoc, but it is simple to apply to a number of systems. The dependence characteristics "ax.1 – ax.n" can range over various logics, standing for the list of their axioms (resp. axiom-schemata).

Let ax.1 – ax.n be the axiom-schemata of classical propositional logic, with modus ponens as the only inference rule. It is well known that the combination of Duns Scotus Law with a provable contradiction yields the inference of an arbitrary well formed formula: $(A \& \sim A), ((A \& \sim A) \rightarrow B) \vdash B$. By way of defining modus ponens in the form of R* we can keep the two premises of the detachment rule apart, provided the contradiction itself is not proved with the help of axioms, included in the dependence characteristics "ax.1 – ax.n". Thus the inference of an arbitrary formula through modus ponens is rendered impossible. If one insists on having the whole system closed under modus ponens, one might introduce yet another detachment rule, the dependence characteristics of which include the additional axioms viz. ax.n + 1 – ax.n + m, but exclude the ones, from which Duns Scotus Law can be derived.

Now, in order to differentiate between internal and external negation, let us introduce a new monadic operator $\urcorner$ adding it to the list of propositional symbols. The latter also includes conjunction (&), classical negation ($\sim$); technical symbols and a list of propositional variables, some of them being designated. the formation rules state that:

1. Propositional variables are well formed formulas (w.f.f.).
2. If β is designated variable, then $\urcorner\beta$ is a w.f.f.
3. If A is a w.f.f., then so is $\sim A$.
4. If A and B are w.f.f., then so is $(A \& B)$.
5. Nothing else is a w.f.f.

Disjunction ($\vee$), material implication ($\rightarrow$) and the corresponding equivalence ($\leftrightarrow$) are introduced as usual, in their capacity of abbreviations, using the primitive symbols of conjunction (&) and external negation ($\sim$). Then internal negation is defined as follows:

D1 $\quad -A =_{df} \begin{cases} \urcorner A \text{ if A is } \beta \text{, i.e. a designated variable;} \\ \sim A \text{ otherwise} \end{cases}$

The transformation rules include:

ax.1 – ax.n the tautologies of classical propositional logic in the form of axiom-schemata,

ax.n + 1 $\sim(A \,\&\, — A)$

ax.n + 2 $\sim(\sim A \,\&\, \sim — A)$

ax.n + 3 $(\sim\beta \,\&\, \sim — \beta)$

ax.n + 4 $(\beta \,\&\, — \beta)$

and the following rules of inference:

R* A "ax.1 – ax.n + 2", (A → B) "ax.1 – ax.n + 2" ⊢ B "ax.1 – ax.n + 2"

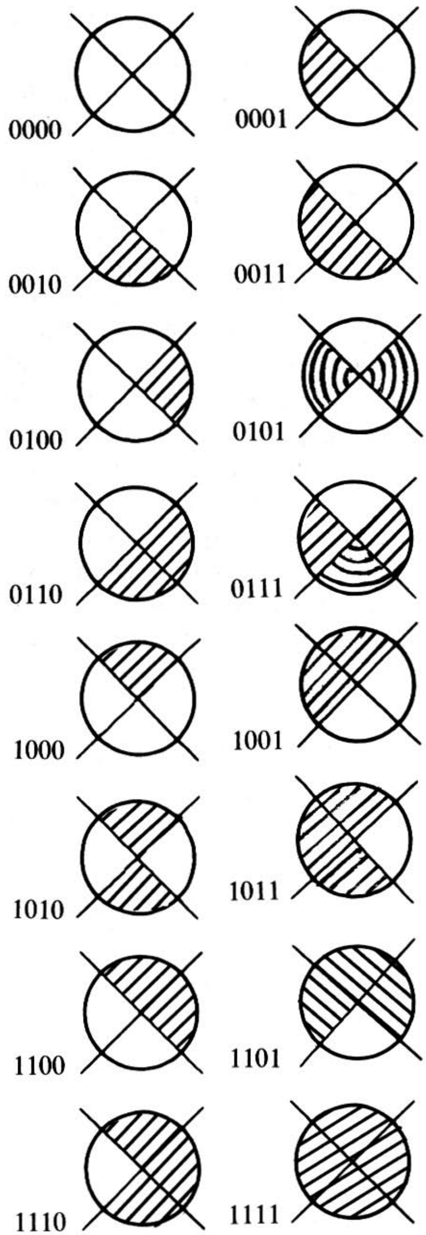
R** (A & B) "ax.1 – ax.n + 4" ⊢ A "ax.1 – ax.n + 4"
(A & B) "ax.1 – ax.n + 4" ⊢ B "ax.1 – ax.n + 4"

where A, B are metavariables, ranging over all w.f.f., whereas β ranges over designated propositional variables. The notions of proof and provable formula are as usual, bearing in mind the role of dependence characteristics. A theorem deduced with the help of axioms, not included in the dependence characteristics of a given rule cannot be used as its premise.

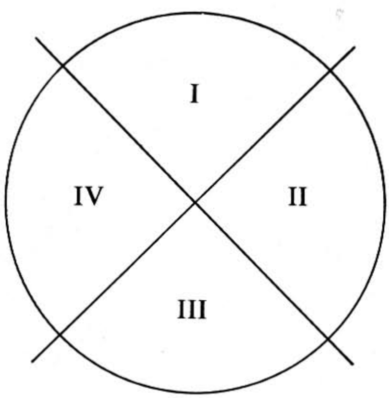
For the sake of interpreting this formal system I choose a table of 16 ordered quadruplets (ordered sequences of 0 or/and 1). Each of the quadruplets stands for a single truth-value. (The four columns of the Table, from left to right, correspond to the four sectors: I, II, III, IV of Diagram.) Let the evaluations of a formula A be the quadruplet $\langle a_1, a_2, a_3, a_4 \rangle$ or shorty W(A); the evaluation of a formula B, i.e. W(B) is the quadruplet $\langle b_1, b_2, b_3, b_4 \rangle$. Then the evaluation of their conjunction, viz. W(A & B) is a new quadruplet $\langle (a_1 \cdot b_1), (a_2 \cdot b_2), (a_3 \cdot b_3), (a_4 \cdot b_4) \rangle$, where a_i, b_j are either 0 or 1, and the point is a symbol for multiplication. W($\sim$ A) is a quadruplet $\langle \bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{a}_4 \rangle$ where $\bar{a}_i = 0$ if $a_i = 1$; $\bar{a}_i = 1$ if $a_i = 0$.

Table

0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1



Diagram



Among the sixteen different truth-values fifteen are designated ones. These are the quadruplets containing at least one token of '1' (resp. the diagrams with at least one sector filled in). The quadruplet <1111> stands for analytic truth: it is juxtaposed with the tautologies of classical logic, i.e. it serves to interpret ax.1 – ax.n, as well as ax.n + 1 and ax.n + 2, if in the latter A is not a designated propositional variable. These variables, viz. β stand for non-analytic truth, ranging over the quadruplets with at least one token of '1' and at least one token of '0', i.e. all quadruplets with the exception of <0000> and <1111>. To interpret $\urcorner\beta$, take the quadruplet corresponding to $\sim\beta$ and if it ends by 1, i.e. if the rightmost element is a token of '1', take the quadruplet immediately below the former: otherwise take the one immediately above the quadruplet which represents $W(\sim\beta)$.

It is quite easy to observe that under this interpretation β, $\urcorner\beta$, $(\beta \,\&\, \urcorner\beta)$, $(\sim\beta \,\&\, \sim\urcorner\beta)$ and their external negations are true in terms of non-analytic truth. Then the so called dialectical synthesis, e.g. $(\sim\beta \,\&\, \sim\urcorner\beta) \vdash (\beta \,\&\, \urcorner\beta)$ is not only a valid inference, but also an instance of a transformation rule which, although being truth-preserving, brings about a shift in the meaning of 'true' (resp. in the truth-value). I shall call it a non-tautological inference, arguing that such kind of inference is of considerable heuristic import in science.

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