DIALECTICAL DYNAMICS WITHIN FORMAL LOGICS.

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1. *Introduction*

A logic of the sort dealt with here was first described in my [1979] and studied more completely in my [1985b] – while I am writing these lines, the latter paper is still in print, viz. since 1981. I pointed to some applications in my [1985a] and [198+]. The present paper contains an essential improvement of the semantics and a closer look at the dialectical properties of the logics.

The motivation for these logics need not derive from dialectics. I stress this, because the term 'dialectics' causes disinterest or even enmity in many logicians. My paradigm case of an application is as follows. Consider a theory T which is based upon classical logic, and hence is intended to be consistent. If T turns out to be inconsistent, but is still taken to be interesting – examples are well-known – one will want to *improve upon* T by articulating some *related* T′ which is consistent, or at least is considered likely to be consistent. Unfortunately, the theory which one wants to improve upon is not easy to define. It is not, obviously, the *trivial* theory T, which contains *all* sentences as theorems. Nor is it the set of axioms of T, for one will usually want T′ to include certain theorems that are only derivable from some inconsistent subset of axioms of T. One might expect to find a solution by turning to the recently developed paraconsistent logics – a logic is paraconsistent iff some inconsistent theories which are based upon this logic are not trivial. Alas, the result obtained by substituting some paraconsistent logic for classical logic in T is too poor, because the paraconsistent logic restricts the rules of inference *globally* ; e.g., it takes $\sim A$, $A \vee B/B$ to be incorrect in general. In order to define the theory we want to improve upon, we need to *localize* inconsistencies and to apply the full classical logic, except where, because of specific inconsistencies, its application would lead to triviality. It is precisely this result which is arrived at by the logics studied below; they enable one to reason "as classical as possible" with respect to inconsistent theories. (1)

I use the name 'dynamic dialectical logics' with reference to 'static dialectical logics', a phrase which occurs in Routley and Meyer's [1976]. Another suitable name would be 'inconsistency-adaptive logics'. One of their typical properties is non-monotonicity: for some α, β and A, $\alpha \vdash A$ and $\alpha \cup \beta \nvdash A$. However, they are quite different, both in intended applications and in inferential properties, from all systems dealt with in the *Special Issue on Non-Monotonicity* of *Artificial Intelligence* (1980, vol. 13, nr. 1,2).

2. *Rules for constructing proofs*

In order to decide how the dynamic dialectical logic should behave with respect to some inconsistency, we may turn to some obvious paraconsistent logic, called *PI* in my [1980b], which *in some simple way* departs minimally from *PC* (the classical propositional calculus). Semantically *PI* is characterized by deleting the clause

(1) If $v(A) = 1$, then $v(\sim A) = 0$.

from the standard semantics for *PC*. Incidentally, I want to point out that the concept of truth remains completely classical; unlike, e.g., Professor von Wright in his contribution to the present volume, I do not consider truth-value overlaps or truth-value gaps. Syntactically (2) *PI* is the positive part of *PC*, including $((A \supset B) \supset A) \supset A$, together with the axiom-scheme

(2) $(A \supset \sim A) \supset \sim A$

There is a nice and for our purposes extremely important relation between *PC* and *PI*, viz. (3)

Th1 $\vdash_{PC} A$ iff there are (zero or finitely many) $C_1, \ldots, C_m$ such that $\vdash_{PI} (C_1 \& \sim C_1) \vee \ldots \vee (C_m \& \sim C_m) \vee A$.

(1) The phrase 'as classical as possible' is not unambiguous; I consider some alternatives in a subsequent section.

(2) The soundness and completeness proofs are in my [1980b]; they follow the method of my [1980a].

(3) If a theorem is listed without proof, the latter is either trivial or spelled out in my [1985b].

From now on I shall write $DK(C_1,\ldots, C_m)$ to denote $(C_1 \& \sim C_1) \vee \ldots \vee (C_m \& \sim C_m)$ or any formula obtained from the latter by commutativity. Both in *PC* and *PI* we have

Th2 $B_1, \ldots, B_n \vdash A$ iff $\vdash (B_1 \& \ldots \& B_n) \supset A$.

Consequently, to any correct *PC*-inference $B_1, \ldots, B_n \vdash A$ corresponds either

(3)

$$\vdash_{PI} (B_1 \& \ldots \& B_n) \supset A$$

or, for some or more $DK(C_1,\ldots, C_m)$,

(4)

$$\vdash_{PI} DK(C_1,\ldots, C_m) \vee ((B_1 \& \ldots \& B_n) \supset A)$$

The latter may be understood as follows: either one of the C_i *behaves inconsistently*, or else A is derivable from $B_1, \ldots, B_n$.

In order to define *DPI* from the previous results, one might try to proceed as follows: if (3) obtains and $B_1, \ldots, B_n$ are *DPI*-derivable from α, then A is *DPI*-derivable from α; if (4) obtains, $B_1, \ldots, B_n$ are *DPI*-derivable from α, and $DK(C_1,\ldots, C_m)$ is not *DPI*-derivable from α, then A is *DPI*-derivable from α. In doing so, however, we face two problems which I shall consider consecutively.

The first problem is that a definition of *DPI* according to the previous lines, would be flatly circular. I solved this problem by moving from the abstract level of the derivability-relation to the concrete level of proofs, and by applying the idea of the previous paragraph to the conditions under which a formula may be added to a *DPI*-proof at some time. The times may be identified with the stages of the proof: after the addition of a line, we are at a new time. A line of a *DPI*-proof will consist of (i) a line number, (ii) the formula, (iii) the set of line numbers referring to the formulas from which (ii) is derived, (iv) the *PC*-rule by which (ii) is derived, or 'premiss', and (v) the set of formulas the consistent behaviour of which is presupposed for the derivation. The *addition* of a line to a *DPI*-proof will proceed as follows: if (3) obtains and $B_1, \ldots, B_n$ occur in the proof at some time, then A may be added at that time (with no new formulas added to the fifth element); if (4) obtains, $B_1, \ldots, B_n$ occur in the proof at some time, and $DK(C_1,\ldots, C_m)$ does not occur in the proof at that time, then

A may be added to the proof at that time (on a line which has $C_1, \ldots, C_m$ in its fifth element). Moreover, lines will sometimes be *deleted*. Suppose that some line has been written by relying on the fact that $DK(C_1, \ldots, C_m)$ does not occur in the proof, whereas the latter is added to the proof at some later time; at this time, the line should be deleted, and so should be all lines depending on it (the third element of these lines contains the line number of the first). Intuitively, the aforementioned line is not derivable any more after $DK(C_1, \ldots, C_m)$ is added to the proof; if the line were deleted, it could not be added again (with only its line number adjusted). Incidentally, the fifth element of a line indicates clearly the conditions under which it should be deleted.

The meaning of 'A is *DPI*-derivable (at some time) from α' should be clear by now. If A is *DPI*-derivable from (a finite) α at some time in some proof, and cannot be deleted at any later time (unless by extending α), then A is *finally derivable* from α.

I previously announced a second problem. If proofs are constructed according to the procedure which was vaguely described before, A may be finally derivable from α in some proofs, but not be so in others. Moreover, whether or not A is finally derivable from α will depend on the accidental way in which the proof proceeds. This may be all right, e.g., if the way in which the proof proceeds may be given some specific sense, or if non-logical preferences enable us to want some consequences rather than others. However, I shall define *DPI* in such a way that A is finally derivable from α in all proofs, if it is in some. The matter may be clarified by the following example. If we do not pay any special attention to the present problem, then, relying on the absence of $q \& \sim q$, we may finally derive p from $\{p \vee q, \sim p, \sim q\}$, whereas, relying on the absence of $p \& \sim p$, we may finally derive q from the same set of premisses. However, once we derived p, we may also derive $p \& \sim p$, which prevents us from deriving q (or obliges us to delete it if it were already derived); and *vice versa*. The trouble is, of course, that $(p \& \sim p) \vee (q \& \sim q)$ is finally derivable from the set of premisses in any proof, but that the set of premisses does not provide us with sufficient information to decide either in favour of $p \& \sim p$ or in favour of $q \& \sim q$. Consequently, from a purely logical point of view – i.e. not taking extra-logical considerations into account – one can neither derive p nor derive q. (See also the end of section 4 of my [1985b].) I shall phrase *DPI* accordingly.

I shall now present a precise articulation of the rules governing the construction of *DPI*-proofs (from some set α). For simplicity's sake, I suppose that all correct expressions (3) and (4) are available. The specification of the first, third and fourth element of the added lines is obvious and will be omitted. The rule DEL *must* be applied whenever a line has been added.

PREM: If $A \in \alpha$, then add the following line:

... A $\emptyset$ premiss $\emptyset$

UNCOND: If (3) obtains and each of $B_1, \ldots, B_n$ occurs as the second element of some line – let the fifth elements of these lines be $\beta_1, \ldots, \beta_n$ respectively – then add the following line:

... A ... ... $\beta_1 \cup \ldots \cup \beta_n$

COND: If (4) obtains, each of $B_1, \ldots, B_n$ occurs as the second element of some line – let the fifth elements of these lines be $\beta_1, \ldots, \beta_n$ respectively – and, for all $D_1, \ldots, D_k$ either $DK(D_1, \ldots, D_k, C_1, \ldots, C_m)$ does not occur as the second element of some line or $DK(D_1, \ldots, D_k)$ occurs as the second element of some line the fifth element of which is $\emptyset$, then add the following line:

... A ... ... $\beta_1 \cup \ldots \cup \beta_n \cup \{C_1, \ldots, C_m\}$

DEL: Any line of the proof which, if it were deleted, could not be added (with only its line number adjusted), should be deleted.

The role of the β_i in COND and UNCOND may easily be understood as follows: if B_i is used to derive A, and B_i was itself derived on the supposition that C_j behaves consistently, then *this* derivation of A rests on the same presupposition. Incidentally, the specific condition in COND (as opposed to UNCOND) may be phrased in many different ways, and each formulation leads to different heuristics of the proofs. Also, other characterizations of *DPI*-proofs are possible, e.g., in terms of some finite set of rules of inference, or in terms of some axiomatic system.

Here is a simple *DPI*-proof.

(1)	$p \& \sim q$	–	premiss	$\emptyset$
(2)	$r \vee q$	–	premiss	$\emptyset$
(3)	$p \supset s$	–	premiss	$\emptyset$
(4)	$\sim r \vee \sim p$	–	premiss	$\emptyset$
(5)	$(r \vee s) \supset q$	–	premiss	$\emptyset$

(6)	p	*(1)*	A &B/A	∅
(7)	~ q	*(1)*	A & B/B	∅
[*(8)*	r	*(2)(7)*	A ∨ B, ~B/A	{q}] deleted at time *14*
(9)	s	*(3)(6)*	A ⊃ B, A/B	∅
[*(10)*	~ p	*(4)(8)*	~A ∨ B, A/B	{q,r}] deleted at time *14*
[*(11)*	p & ~ p	*(6)(10)*	A,B/A & B	{q,r}] deleted at time *14*
(12)	r ∨ s	*(9)*	B/A ∨ B	∅
(13)	q	*(5)(12)*	A ⊃ B, A/B	∅
(14)	q & ~ q	*(13)(7)*	A,B/A & B	∅
(15)	~ r	*(4)(6)*	A ∨ ~B,B/A	{p}

At time *7*, r is derivable in view of the absence of q & ~ q and of the *PI*-theorem (B & ~ B) ∨ (((A ∨ B) & ~ B) ⊃ A); at time *14*, r is not derivable any more. On the other hand ~ r is not derivable at time *11*, but becomes derivable after time *14*. At time *15*, the proof is essentially finished in the sense that no further lines will be deleted in any extension and that only trivial consequences of already derived formulas may be derived. In other words, the set of formulas that are finally *DPI*-derivable from the five premisses, is identical to the set of *PI*-consequences of {p, q, ~ q, ~ r, s}.

Some readers may think that it is possible to write down rather uninteresting *DPI*-proofs, and they are certainly correct. However, the situation is the same for all logical systems. In order to derive interesting consequences in an interesting way, it is necessary to have mastered the heuristics of the specific type of proofs.

Although the notion of a *DPI*-proof will be clear by now, some readers might not be convinced that such proofs define an interesting logic. This is why I at once move to the syntactic metatheory.

3. *Syntactic metatheory*

Th3 *PI* is decidable.

Def $\alpha \vdash^{*}_{DPI}$ A (A is finally* derivable from α) iff there is a *DPI*-proof from α, in which A occurs as the second element of a line which will not be deleted in any extension of the proof.

Th4 $B_1, \ldots, B_n \vdash^{*}_{DPI} A$ iff there are $C_1, \ldots, C_m (0 \leq m)$ such that (i) $B_1, \ldots, B_n \vdash_{PI} A \vee DK(C_1, \ldots, C_m)$ and (ii) for all $D_1, \ldots, D_k$, either $B_1, \ldots, B_n \nvdash_{PI} DK(D_1, \ldots, D_k, C_1, \ldots, C_m)$ or $B_1, \ldots, B_n \vdash_{PI} DK(D_1, \ldots, D_k)$.

In other words, final* *DPI*-derivability from finite sets of premisses is definable in terms of *PI*-derivability. In order to generalize this result to infinite sets of premisses, we need two more definitions.

Def An *intelligent extension* of a *DPI*-proof from α is an extension such that, if the result of dropping some disjunct from $DK(D_1, \ldots, D_k)$ is *PI*-derivable from α, then this longer formula does not occur as the second element of some line in the extension, unless the shorter formula occurs as the second element of some previous line the fifth element of which is empty.

Def $\alpha \vdash_{DPI} A$ (A is finally *DPI*-derivable from α) iff there is a *DPI*-proof from α, in which A occurs as the second element of some line which will not be deleted in any intelligent extension of the proof.

If the set of premisses is empty, then, in view of the decidability of *PI*, one might pick out a proof in which occur all *PI*-derivable disjunctions of contradictions which do not contain any redundant disjuncts. This may not be possible if the set of premisses is infinite. However, in the latter case it is possible, whenever one finds out that $DK(D_1, \ldots, D_k)$ is *PI*-derivable, to check whether some of the disjuncts is redundant.

Def $Cn_{DPI}(\alpha) = \{A | \alpha \vdash_{DPI} A\}$

Th5 $\alpha \vdash_{DPI} A$ iff there are $C_1, \ldots, C_m$ $(0 \leq m)$ such that (i) $\alpha \vdash_{PI} A \vee DK(C_1, \ldots, C_m)$ and (ii) for all $D_1, \ldots, D_k$, either $\alpha \nvdash_{PI} DK(D_1, \ldots, D_k, C_1, \ldots, C_m)$ or $\alpha \vdash_{PI} DK(D_1, \ldots, D_k)$.

Cor1 If, for some B, $A \vdash_{PI} B \& \sim B$, then $\alpha \vdash_{DPI} A$ iff $\alpha \vdash_{PI} A$.

In other words, final *DPI*-derivability is definable in terms of *PI*-derivability.

Th6 For all finite α, $\alpha \vdash_{DPI}^{*} A$ iff $\alpha \vdash_{DPI} A$.

Th7 If α is consistent, then $Cn_{DPI}(\alpha) = Cn_{PC}(\alpha)$.

Th8 If α is inconsistent but nontrivial, then $Cn_{PI}(\alpha) \subseteq Cn_{DPI}(\alpha) \subset Cn_{PC}(\alpha)$.

Th9 $Cn_{PI}(\alpha) \subset Cn_{DPI}(\alpha)$ iff there are $C_1, \ldots, C_m$ and an A such that $\alpha \vdash_{PI} A \vee DK(C_1, \ldots, C_m)$, $\alpha \nvdash_{PI} A$ and, for all $D_1, \ldots, D_k$, either $\alpha \nvdash_{PI} DK(C_1, \ldots, C_m, D_1, \ldots, D_k)$ or $\alpha \vdash_{PI} DK(D_1, \ldots, D_k)$.

Th10 If $Cn_{PI}(\alpha)$ is decidable and $\alpha \vdash_{DPI} A$, then any *DPI*-proof from

α may be extended in such a way that A is finally derived at some line in the extended proof.

Th11 If $Cn_{PI}(\alpha)$ is decidable, then all members of any finite $\beta \in Cn_{DPI}(\alpha)$ are simultaneously finally derivable from α (and may be simultaneously finally derived in an extension of any *DPI*-proof from α).

Th12 For all finite α, $Cn_{DPI}(\alpha)$ is decidable.

I mention two theorems about (non-final) *DPI*-derivability.

Th13 If α is consistent, then $A_1, \ldots, A_n$ are simultaneously *DPI*-derivable from α iff they are all *PC*-derivable from it (iff they are finally *DPI*-derivable from α).

Th14 If α is inconsistent, $\beta \subset \alpha$, $\gamma \in Cn_{PC}(\beta)$, γ is finite, and neither β nor γ contain some formula of the form $DK(C_1, \ldots, C_m)$ $(1 \leq n)$, then all members of γ are simultaneously *DPI*-derivable from α.

The reader may easily verify that these theorems justify the claims I made before. Most importantly, *DPI* is a decent formal logic, it is paraconsistent and behaves as *PC* with respect to any inference in which no specific inconsistency is involved. It is also easy to see that *DPI* is non-monotonic; e.g., to take a simple example, we have in *DPI* that $p \vee q, \sim p \vdash q$, whereas $p \vee q, \sim p, p \nvdash q$.

4. *Dialectics*

There are several senses in which *DPI* may be called a dynamic dialectical logic. The most obvious is this: contradictions do occur in proofs and, in contradistinction to what is the case for paraconsistent or static dialectical logics, they do form a problem and lead to a change of the rules of inference. E.g., in the proof of section 2, r ceases to be derivable from $r \vee q$ and $\sim q$ after $q \& \sim q$ has been derived.

It is quite obvious that *DPI* does not lead in general to the "resolution of contradictions". The derivation of one contradiction may lead to the elimination of another contradiction; e.g., in the proof of section 2, $p \& \sim p$ is eliminated by the derivation of $q \& \sim q$. However, this is clearly not the "strong resolution" which dialecticians have in mind and which requires the introduction of new concepts. It seems to me that this strong resolution cannot be the result of the mere application of a formal logic, but requires extra-logical means. (4) Yet, the dynamics of *DPI*-proofs displays a number of structural properties which are typical for the strong resolution of inconsistencies. In order to show this, I shall consider the conditions set forth by Leo Apostel on pp. 83-84 of his [1979].

The strong resolution is described by Leo Apostel in terms of the relations between consecutive theories. His eight conditions may be summarized as follows.

(a) The strongest contradiction(s) of the theory T_i is (are) eliminated.
(b) The inconsistent T_i is replaced by an inconsistent T_{i+1}.
(c) The relation between T_i and T_{i+1} must be analogous to the relation between T_{i-1} and T_i (pragmatic notion dependent on time).
(d) T_{i+1} is not a subtheory of T_i but of some extension of T_i.
(e) T_{i+1} contains statements and proofs that are on the average less distant from the statements and proofs of T_i than are the statements and proofs of all alternative T'_{i+1} which also eliminate the strongest contradiction(s) of T_i. (This distance should be measured in a way independent of any specific axiomatizations of the theories).
(f) More specifically, T_{i+1} should contain (in comparison to T'_{i+1}) as much as possible of the proofs that lead in T_i to "parts" of the strongest contradiction(s) or in which such "parts" are employed as premisses.
(g) T_{i+1} should be weakly maximal with respect to the extension of T_i mentioned in (d). This means that, if any nontheorem of T_{i+1} which is a theorem of this extension is added to T_{i+1}, then the result contains a contradiction stronger than any contradiction in T_{i+1}.
(h) T_{i+1} should neither be the union nor the intersection of the consequence sets of all weakly maximal subtheories of the extension (see d) of T_i. This is an implicit rejection of a Rescher-like solution of the problem (see Nicholas Rescher's [1964], viz. his "weak" and "strong" consequence).

(4) As suggested in section 6 of my [198+], the approach taken by Thomas Nickles in his [1980] may be clarifying in this respect.

I shall not discuss these conditions here, but merely show that most of them are fulfilled by *DPI*, if interpreted in a specific way. To any stage of a *DPI*-proof I shall associate a theory, viz. the *PI*-consequence set of the formulas that occur in the stage of the proof. Incidentally, the subsequent reasoning would also work if we considered *DPI*-consequence sets, but I prefer the more classical approach and define theories in terms of a static logic. First consider an application of a rule of inference which is *PI*-valid. If no previous line is deleted, the new theory is *identical* to its predecessor. If previous lines are deleted, the new theory is a *subtheory* of its predecessor, and possibly eliminates some of its inconsistencies. Next consider the application of a rule of inference which is not *PI*-valid but depends on the consistent behaviour of some formula. If no lines are deleted, we face an *enrichment* of the previous theory (the conclusion of this application is not a *PI*-consequence of the previous theory). ([5]) If some lines are deleted, we move at once to a *subtheory of an enrichment* of the previous theory. The reader may easily verify that, under this interpretation, *DPI*-proofs fail to satisfy conditions (a) and (c). However, each of (b) and (d) – (h) are exemplified in some *DPI*-proofs, although they do not obtain in general between any two consecutive theories.

As a final remark I repeat that the dynamical character of *DPI* is displayed at the concrete level of proofs and to some extent at the abstract level of (nonfinal) derivability, but not at the (abstract) level of final derivability. The fact that the *DPI*-consequence set of some set of premisses is determined before any actual proof is carried out, is probably not puzzling from a dialectical point of view. It is puzzling, however, that, as *DPI* is decidable (on the propositional level that is), a handy person may avoid proofs in which any lines are deleted. Nevertheless, even such proofs display dynamic dialectical properties if theories are defined in terms of *DPI*-consequence sets.

([5]) There is an exception: the conclusion of this application might be derivable in another way by *PI*-valid means. Such cases would be taken care of if theories were defined as sets of actually derived formulas.

5. *Semantics, soundness, completeness*

The handiest approach is in terms of model sets (to any valuation v corresponds a $\gamma = \{A|v(A) = 1\}$). Γ, the set of *PI*-model sets, contains the sets γ of formulas such that

$A \supset B \in \gamma$ iff $A \notin \gamma$ or $B \in \gamma$.
$A \& B \in \gamma$ iff $A \in \gamma$ and $B \in \gamma$.
$A \vee B \in \gamma$ iff $A \in \gamma$ or $B \in \gamma$.
If $A \notin \gamma$, then $\sim A \in \gamma$.(6)

Def $\Gamma_\alpha = \{\gamma|\gamma \in \Gamma$ and $\alpha \subseteq \gamma\}$
Def $\alpha \models_{PI} A$ iff, for all $\gamma \in \Gamma_\alpha$, $A \in \gamma$.
Th15 $\alpha \models_{PI} A$ iff $\alpha \vdash_{PI} A$.
Def $K(\gamma) = \{A|A \& \sim A \in \gamma\}$
Def $\Delta_\alpha = \Gamma_\alpha - \{\gamma|\gamma \in \Gamma_\alpha$ and, for some $\delta \in \Gamma_\alpha$, $K(\delta) \subset K(\gamma)\}$
Def $\alpha \models_{DPI} A$ iff, for all $\gamma \in \Delta_\alpha$, $A \in \gamma$.
Def $\Lambda_\alpha = \{\lambda|\alpha \vdash_{PI} DK(\lambda)$ and, for all $\mu \subset \lambda$, $\alpha \nvdash_{PI} DK(\mu)\}$
L1 $\gamma \in \Delta_\alpha$ iff $\gamma \in \Gamma_\alpha$ and $K(\gamma)$ contains *exactly one* member of each $\lambda \in \Lambda_\alpha$.
L2 For all $\gamma \in \Delta_\alpha$, $DK(C_1, ..., C_m) \in \gamma$, iff, for all $\gamma \in \Gamma_\alpha$, $DK(C_1, ..., C_m) \in \gamma$.
Th16 If $\alpha \vdash_{DPI} A$, then $\alpha \models_{DPI} A$.

Proof. Suppose first that $\alpha \vdash_{DPI} A$. It follows (by Th5) that there is a (possible empty) ε such that $\alpha \vdash_{PI} A \vee DK(\varepsilon)$ and, for any ζ, $\alpha \nvdash_{PI} DK(\zeta \cup \varepsilon)$ or $\alpha \vdash_{PI} DK(\zeta)$.
Case 1. $Cn_{PI}(\alpha)$ is trivial. Then $\alpha \models_{DPI} A$.
Case 2. $A \vdash_{PI} B \& \sim B$ for some B. Then $\alpha \vdash_{PI} A$ by Cor1, and hence $\alpha \models_{DPI} A$.
Case 3. $\alpha \vdash_{DPI} \sim A$. Then $\alpha \vdash_{PI} A \& \sim A$ (by Cor1) and hence $\alpha \models_{DPI} A$.
Case 4. $Cn_{PI}(\alpha)$ is non-trivial, $A \nvdash_{PI} B \& \sim B$ for all B, and $\alpha \nvdash_{DPI} \sim A$.
Second supposition: $\alpha \not\models_{DPI} A$. Then there is a $\delta \in \Delta_\alpha$ such that $A \notin \delta$. It follows that, for some $C \in \varepsilon$, $C \in K(\delta)$, and hence that there is a $\lambda \in \Lambda_\alpha$ such that $C \in \lambda$. Consequently, there is a ζ such that

(6) The *PC*-model sets are those which moreover fulfill the condition 'If $A \in \gamma$, then $\sim A \notin \gamma$.'

$\alpha \vdash_{PI}$ DK(ζ) and, for some $\lambda \in \Lambda_\alpha$, $\lambda \subset \zeta \cup \varepsilon$ and hence also $\alpha \vdash_{PI}$ DK($\zeta \cup \varepsilon$). This contradicts the first supposition. □

Th17 If $\alpha \models_{DPI}$ A, then $\alpha \vdash_{DPI}$ A.

Proof. Suppose that $\alpha \models_{DPI}$ A. It follows that $A \in \gamma$ for all $\gamma \in \Delta_\alpha$.
Case 1. $\alpha \models_{DPI} \sim A$ or, for some B, $A \models_{PI} B \& \sim B$. Then $\alpha \models_{PI}$ A by L2, and hence $\alpha \vdash_{DPI}$ A.
Case 2. $\acute{a} / \not\models_{DPI} \sim A$ and, for no B, $A \models_{PI} B \& \sim B$. Then either $\Gamma_\alpha - \Delta_\alpha = \emptyset$, or there is an ε such that $A \vee DK(\varepsilon) \in \cap \Gamma_\alpha$ and $DK(\varepsilon) \notin \cup \Delta_\alpha$; in the latter case we have, for all ζ, $DK(\zeta \cup \varepsilon) \notin \cap \Gamma_\alpha$ or $DK(\zeta) \in \cap \Gamma_\alpha$. In both cases it is easily shown that $\alpha \vdash_{DPI}$ A. □

Reading '$A \in \gamma$' as 'A is true in model γ' we obtain: A is a semantic consequence of α iff A is true in all models in which all members of α are true. For *PI* we consider all models (in Γ); for *PC* we consider all *consistent* models (of Γ); for *DPI* we consider all models (of Γ) which are *as consistent as possible with respect to* α.

6. *Alternative logics.*

There are (infinitely many) extensions of *PI* which are still paraconsistent and strictly weaker than *PC*. Any such logic determines a dynamic dialectical logic. One might expect that the latter lead in general to richer consequence sets than *DPI*, but the opposite obtains (see my [1985b]); in general, all these logics lead to less suitable results than *DPI*. The matter may be understood as follows: precisely because *PI* is so weak, it is possible for *DPI* to maximally adapt to the specific inconsistencies of some theory.

It goes without saying that a large number of dynamic dialectical logics may be devised in a way different from the one in which *DPI* is devised, e.g., by taking external preferences into account. It is even possible to do so by introducing preferential procedures which are based only on formal properties of formulas (e.g., their strength). Finally, there are numerous open problems concerning dynamic dialectical logics based on paraconsistent logics that are not extensions of the implicational fragment of *PC*.

7. *Final remark.*

Apart from all I said before, the logics dealt with here are interesting because they open new perspectives on the nature of logic. Incidentally, they are in this respect more interesting than the usual non-monotonic logics, because they lead both to non-monotonicity and to dynamic proofs in the absence of any special logical constants.

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