DIFFERENCES IN THE UNITY

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1. In his paper on "Logic Unified" [1], G.H. von Wright says that there is not one logic which is *the true one*, and that the variety of logics forms a systematic unity. I fully agree with his thesis but, at the same time, I would like to point out that there are some general methodological principles which, I think, must be followed in any system of logic, in any process of logical thinking and which, therefore, must be taken into consideration when we talk about the unity of logic. These principles were already stated by Aristotle. Although several of his formulations of the principle of non-contradiction can be interpreted as ontological theses there is no doubt that Aristotle also took that principle to be the precondition of any scientific inquiry, and thus a methodological principle, one concerning the method of correct reasoning.

In the "De Interpretatione", he defines the concept of contradiction with reference to that of denial:

"... every affirmation has an opposite denial and, similarly, every denial an opposite affirmation. We will call such a pair of propositions a pair of contradictories." [2] 17a33-

Taking Aristotle's definition of contradiction as our point of departure we can reformulate the General Methodological Principle of Non-Contradiction (GMPC) as follows:

It is a logical mistake both to assign a truth-value to a proposition and to deny that the proposition has that truth-value.

This phrasing of the Principle of Non-Contradiction carries the important point that it applies to propositions already asserted. It requires that, once a proposition is assigned a truth-value, it should not be denied that truth-value. It does not say, however, what truth-values a proposition may take within a logical system. Nor does it require that every proposition (within any logical system) take a truth-value or only take one truth-value at a time, etc. Therefore, GMPC is compatible with the existence of many-valued logic, and logics permitting a truth-value-gap or glut. It does not even presup-

pose that the meta-logic of those kinds of logic be necessarily two-valued.

Following the model of GMPC, we can formulate the general methodological principle of the excluded middle (GMPE) like this:

If we first assign a truth-value to a proposition and then deny that the proposition has that truth-value, then we must choose between assigning or not the truth-value in question.

GMPE permits us not to assign any truth-value to a proposition. It only requires that *once we have decided* that a proposition has a certain truth-value and *once we have also asserted* that the same proposition has not that truth-value, then we must decide upon one of those two assertions. Put in this form, GMPE is no less general a principle than GMPC.

Each of the Truth-Logics construed by G.H. von Wright seems to me in accordance with GMPC and GMPE. If, in any Truth Logic, we assign a truth value to p then, according to GMPC, we cannot assert at the same time that p has not that truth-value. Also, according to GMPE, we either assign a truth-value to p or we do not assign that truth-value to it.

In his "On the Logic of Negation" [3], G.H. von Wright points out that Aristotle makes a distinction between the proposition in the form "it is *not* the case that x is P" and "it *is* the case that x is not-P". Aristotle, who only considered the first as negation, wrote this:

"In establishing or refuting, it makes some difference whether we suppose the expressions 'not to be this' and 'to be not-this' are identical or different in meaning, e.g. 'not to be white' and 'to be not-white'." [4] 51a5-

"The expressions, 'it is a not-white log' and 'it is not a white log' do not imply one another's truth. For if 'it is a not-white log' it must be a log; but that which is not a white log need not be a log at all." [4] 51a28-

Aristotle concedes that there may be things of which neither 'it is a white log' nor 'it is a not-white log' is true. However, if by predicating those two expressions we got assertions denying each other, one of them should be true.

Even though Aristotle generally made a distinction between the meanings of the expressions 'not to be this' and 'to be not-this', he did not speak about the difference between 'not to be true' and 'to be

not-true'. If we distinguish between the latter two in the same way as between 'not to be this' and 'to be not-this' we get the distinction which is made in TL between 'not true' and 'false'. *False* means *not-true* in TL, and is related to *true* in the same way as 'to be not-this' is related to 'to be this' / as 'not-white log' is related to 'white log' /.

In classical two-valued symbolic logic, asserting that a proposition is not true is equivalent to asserting that it is not-true /or false/. That logic only covers situations in which for every x, either P or *not-P* is true, while the other is *not-true* /or false/. Thus all those objects are excluded from the possible values of x of which neither P nor *not-P* is true.([1])

Because of this interpretation of a proposition, classical symbolic logic holds "*It is true to call it not-white*" and "*It is not true to call it white*" to be different linguistic expressions of the same assertion, and they both correspond to the negation of "*It is true to call it white*". But this is not Aristotle's view: "For the negation of 'it is true to call it white' is not 'it is true to call it not-white' but 'it is not true to call it white'. [4] 52a30

It is a logical mistake not to realize that the same words may be the linguistic expressions of different concepts in different systems. I find typically such a fault with claims that paraconsistent logics have managed to get rid of the constraint of the principle of non-contradiction, referring to the fact that such logics permit the simultaneous truth of contradictory propositions. In a similar fashion, I find it misguided to criticise paraconsistent logics, arguing that any deviation from the conceptual framework of classical two-valued logic is futile and that one logic can be considered the "true" one. My point is that although paraconsistent logics do not conform to the conceptual frameworks of logics of the Frege/Russell type, they are in accordance with the general methodological principles mentioned earlier.

I think, and I would like to show, that the negations which appear in the paraconsistent systems of Da Costa, Rescher, Routley, and others are similar to Aristotle's affirmation of the form "x is not-P", which is

([1]) B. RUSSELL clearly states this: "... given any propositional function, say fx there is a certain range of values of x for which this function is 'significant' – i.e. either true or false. If a is in this range, then fa is a proposition which is either true or false." [5] p. 81

why the contradictions in those systems are not contradictions in the Aristotelian sense.

Paraconsistent logics are known to permit worlds (situations) in which both p and its negation hold, that is, so it is said, worlds which are inconsistent. However, the meaning of the expression "inconsistent world" (IW) is not sufficiently clear. At least the following variants of meaning can be identified:

IW is (1) the real world in itself
(2) the image of a situation which cannot be described consistently
(3) the world of notions.

Beside the problems mentioned earlier, I wish to submit these interpretations of IW to critical analysis.

2. In several of his articles on this topic, R. Routley's aim seems to have been to construct a paraconsistent logic presupposing the first meaning variant mentioned. He says that the world presupposed by dialectical ontology is an inconsistent world.

He defines the negation requirement in the following form:

$\sim p$ is in H iff p is not in H*

where H* is the image of H under a one-one function. Unfortunately, Routley does not say more about how we are to understand that H* is the image of H. We are only told that "the operation * is a reversal operation which takes a situation into its reverse, and hence incompleteness in a situation into inconsistency, and inconsistency into incompleteness, i.e. the reverse of a situation b where both p and $\sim p$ hold is a situation b* where neither p nor $\sim p$ hold." [6] p. 309

According to Aristotle, either affirmation or negation has to be chosen and there is no neutral area (situation, world). But for Routley, a neutral situation is possible, one in which neither the affirmation nor the negation is true. We can conclude that negation as used by Routley (from now on: Rn) is not negation in the Aristotelian sense. Furthermore, since Rn is not negation in the Aristotelian sense, Routley's notion of contradiction is *not* contradiction in the Aristotelian sense either. With Routley, it is possible that $p \in H$ and $\sim p \in H$, but it is not possible for both $p \in H$ and $p \notin H$ to obtain simultaneously. Thus in fact he preserves Aristotle's relation of negation: the

Aristotelian negation of $p \in H$ is $p \notin H$. These two are contradictory, for it is impossible for both to obtain and one of the two must obtain. This is what Routley implies when recognizing the Aristotelian principle of non-contradiction as correct.

3. In the introduction of their joint work "The Logic of Inconsistency" [7], N. Rescher and R. Brandom state that they are dealing with something else than epistemological inconsistency. They think it could happen ontologically in one and the same world *w*, that p and its negation, $\sim p$ both obtain in the strict "really-and-truly" sense (at the same time, in the same context, etc.). Nevertheless, I think, in spite of that declaration it is *not* "really-and-truly" inconsistency which their system permits.

N. Rescher and R. Brandom reject the following rules, which they call rules of classical logic:

$$t_w(\sim p) \Rightarrow \neg t_w(p)$$
$$\neg t_w(p) \Rightarrow t_w(\sim p)$$

(Where "$\Rightarrow$", "$\neg$" meta-level connectives stand for "if-then", "not", respectively. On the other hand, "$\sim$" is a connective in classical predicate logic. [7] p. x)
Later it turns out that their logic is only inconsistent for "$\sim$", but it is not inconsistent for its meta-logic i.e. for the negation: $\neg$.

For every world w,

$$\text{(a)}\ \neg(t_w(p) \wedge \neg t_w(p))$$
$$\text{(b)}\ t_w(p) \vee \neg t_w(p)$$

(where "$\wedge$", "$\vee$" are meta-level conjunction and disjunction, respectively). It follows that the principles called GMPC and GMPE earlier are valid as methodological principles underlying this system.

The world w in which $t(p)$ and $t(\sim p)$ occur is always, in the above authors' view, a world-disjunction composed of at least two consistent worlds (e.g. w_1, w_2). A disjunction-world w is inconsistent if, and only if, there is one among its component worlds in which p obtains, and there is also at least one in which $\sim p$ obtains. If e.g. p obtains in w_1 and $\sim p$ obtains in w_2, then in the disjunction-world $w_1\ w_2$, p obtains because $t_{w_1}(p)$ and $\sim p$ also obtains because $t_{w_2}(\sim p)$.

However, in this disjunction-world w, p, $\sim$p obtain based always on *different sources* (the authors' expression) and, therefore, *not* in the same context. Thus their occurrence in w does not imply that p & $\sim$p also occurs in w (where "&" is a truth-functional connective). That could only obtain in w if either t_{w_1} (p & $\sim$p) or t_{w_2} (p & $\sim$p). But, for our authors, the disjunction world w is composed of consistent worlds and, consequently, p & $\sim$p can neither obtain in w_1 nor in w_2. Thus a world w in which p & $\sim$p obtains is not possible.

Interpreted in a certain way, however, Rescher and Brandom's inconsistent disjunction-worlds appear to me really possible worlds. More than being possible, the actual world in which we are living is in fact composed of different situations, relations and of things behaving in these in diverse ways. It is a world-disjunction which consists of infinitely many components. An individual *a* may have the property *P* in a situation (relation, time point) w_1, and the same *a* may have *not-P* in a situation (relation, time point) w_2. Therefore, from some source (w_1) it is true that "*a* is *P*" (t_{w_1} (p)) and, from another source (w_2), it is true that "*a* is *not-P*" ($t_{w_2}(\sim p)$). Thus, both p, $\sim$p may obtain in a disjunction-world w_1 w_2, i.e. w. However, it is only at a certain phase of cognition, when world w has not yet been recognized to be a world-disjunction, that the conjunction p & $\sim$p may mistakenly arise. The emergence of such a contradiction, p & $\sim$p, must warn us that propositions p, $\sim$p have not been recognized as coming from different sources. Once this is recognized, p, $\sim$p obtaining in a world-disjunction will neither undermine the general validity of GMPC and GMPE nor conflict with classical two-valued logic for, in this case, "$\sim$" is by no means indentical with the classical operation of negation.

4. Unlike that of R. Routley, or N. Rescher and R. Brandom, the semantics of N.C.A. da Costa and R.G. Wolf's paraconsistent logic does not pretend to characterize a language-independent reality. It is rather meant to be about theories. Da Costa and Wolf suggest that, in certain cases, we may think about reality without falling into contradictions while in other cases, which they call fringe cases, contradictions will arise. "For example, they say, on a color spectrum, between two adjacent colors, say blue and green, there will be a 'fringe' consisting of color shades where one is unable to determine

which predicate (A /blue/ or -A /green/) actually applies and accordingly one is liable to apply both predicates." [8] p. 194

G. Priest puts forward a similar example: "I am in a room. As I walk through the door, am I in the room or not in (out of) it? ... Clearly, there is no reason for saying one rather than the other." [9] p. 252

Once more, the question arises whether, in these examples, the propositions formed through predicating A and not-A i.e. blue and not-blue (green) or being in the room and not being in it (being out of it) can be considered contradictory. The answer is that they cannot. Rather, they are similar to Aristotle's pair of expressions 'white log' and 'not-white log'. The two examples show that their authors conceive '*not-A*' in a *restricted* universe as complementary to A, in the same way as Aristotle distinguishes the species 'white log' and 'not-white log' whithin the genus 'log'. But there is another point to be noted in the above examples. Da Costa and Wolf use the expression 'not-blue' as equivalent to 'green' and, in Priest's usage, being 'not-in the room' has the same meaning as 'out of the room'. I think the root of the problem is here. Those equivalences only hold with certain restrictions. They only hold if the common genus of 'blue' and 'not-blue' has such a narrow extension that everything 'not-blue' in it has the differentia specifica 'green' or, in Priest's example, if the universe of discourse is so narrow that, within it, whatever is not in the room is *out of* the room. As soon as we have begun to consider that there are things which can neither take the predicate green nor the predicate blue (being in, being out of the room), we have extended the universe of discourse so that the equivalence of 'not-blue' with 'green' ('being not in the room' with 'being out of the room') no longer obtains. In that larger universe, other equivalences hold like 'not-blue' $\equiv$ def. 'green or greenish-blue or bluish-green', or like 'not being in the room' $\equiv$ def. 'being out of the room or coming in or leaving the room'. The *given* inconsistency is thus eliminated and we are again able to classify things under A or not-A. If we consider something definitely blue we shall predicate A of it, otherwise we shall hold not-A true of it.

This of course does not constitute a final solution to the "problem of classification". Nor do I claim that, to use da Costa and Wolf's word, our "liability" to inconsistent assertions no longer persists.

What I have tried to say is that inconsistencies of the kind discussed as well as our liability to them ought to be overcome through bringing the theory to greater precision.

It seems to me that N. da Costa and G. Priest's paradoxical examples display some sort of contradiction like in the well-known "Horned Man" paradox of Eubulides from the Megarian school:

> What you have not lost you still have.
> But you have not lost your horns.
> So, you still have horns.

It has been known for long that the fault in this reasoning comes from the changing scope of negation. In the first assertion, the genus to which our thinking is limited is a "thing in one's possession at some point of time". The classification into things lost and not-lost is valid within that limit. In the second assertion, however, a "thing not lost" is not restricted to the particular genus; it is negation in the Aristotelian sense.

5. "... Marx and Hegel did not deny the validity of classical logic for a large class of propositions (and processes and situations)", write N. da Costa and R. Wolf. [8] p. 198 According to the latter authors, dialectic requires that when a proposition (process, situation) is stable, then A and its negation cannot both be true. In such cases the laws of classical logic are valid. However, when it comes to examining processes or fringe cases the laws of classical logic no longer obtain. In this respect, da Costa and Wolf consider their own Dialectical Logic (hereafter DWL)([2]) to be in accordance with Hegel and Marx's views.

What does it mean that the laws of classical logic are only valid for certain propositions (processes, situations)?

The negation $\neg$ (herafter Dn) used in DWL does not differ from classical negation if the proposition is stable, i.e. in the case of A° (where $A^{\circ} \equiv \text{def } \neg(A \wedge \neg A)$. This alone is sufficient proof that the

([2]) The authors term their Dialectical Logic DL but I had to add one more letter since R. Routley and N. Rescher also use DL to refer to their dialectical logics.

point is not the refutation of theses of classical logic but the exploration of new areas. An unstable assertion (when P or not-P cannot be predicated of x in a stable manner) is not a proposition in classical logic. The fact that such 'non-propositions' follow different laws does not refute what is established in classical logic or implied by its presuppositions.

In this sense, DWL does not refute the laws of classical logic, i.e. it does not hold them to be false. Rather, it makes it apparent that through changing certain presuppositions of classical logic we arrive at a logical domain in which laws of classical logic have *no truth-value*. In that logical domain, those laws fall into a truth-value gap.

6. Was it really Hegel and Marx's opinion that, on the one hand, there are stable processes and situations which closely correspond to the mutually exclusive character of "x is P" and "x is not-P" and on the other hand, *apart* from these, there are cases in which both P and not-P can be truly predicated of one and the same object?

I do not think this is in line with their dialectic. In the rest of this paper, let me cite some of Hegel's ideas in support of my conviction.

We cannot, I think, reconcile with Hegel's standpoint a logical system in which the concepts of "true" and "false" only deviate from classical two-valued logic in that this system allows a proposition to have both those logical values at the same time.

Truth, for Hegel, is not a classical logical conjunction of the two classical truth values "true" and "false".

I wonder whether we get nearer to Hegel's idea if we think of a many-valued logic in which, besides the classical logical truth values, there is a third value "truth" taken in the Hegelian sense (which may correspond to the conjunction of von Wright's true in laxer sense and false in laxer sense?), and in that case this third one would be the only designated value.

Hegel distinguishes between formal truth or what he also calls correctness and Truth in a deeper sense, i.e. the Genuine Truth. Genuine Truth "lies in the coincidence of the object with itself, that is, with its notion". [10] § 172

It remains a question however, whether Hegel's Genuine Truth can be considered a logical value if we keep the sense of "logical value" as assigned to a *proposition* formulated within the frame of some

logical theory. Hegel's conception is different in that he holds a *notion* to be an element of the existence of the Genuine Truth and the scientific system of notions is the real form in which the Genuine Truth exists. According to Hegel, "... an immediate judgement in which an abstract quality is predicated of an immediately individual thing, however correct it may be, *cannot contain truth*. The subject and predicate of it do not stand to each other in the relation of reality and notion." [10] § 172 (italics mine K.G.H.)

If that is so, one may ask how Hegel's system can be connected to inconsistent logical systems at all. Is the idea really found in Hegel that both a proposition (judgement) and its negation may be true and, if so, which senses of "true" "false" and negation will allow for this?

Hegel writes: "If, for example, it is said that the rose is *not* red, it is only the *determinateness* of the predicate that is negated and separated from the universality which likewise belongs to it; the universal sphere, colour, is preserved; in saying that the rose is not red, it is assumed that it *has* a colour, but a different one. In respect of this universal sphere the judgement is still positive." [11] p. 640

Perhaps it needs no further explanation that here Hegel does not speak of negation in the Aristotelian sense. With him, a "negative judgement" is an affirmation similar to Aristotle's "it is a not-white log", '*x* is *not-P*'. Hegel's negative judgment has in common with Routley's and da Costa's negations (Rn, Dn) that they are interpreted in a restricted universe.

It must be remembered that Hegel distinguishes the positive and the negative types of judgment on the level of the "Qualitative Judgements", i.e. on the first level of the development of judgments. Understanding makes distinctions, separations, divisions, and isolations. Only when in the course of its progress, cognition has left the level of understanding and reason has accomplished its task of showing what is distinguished and isolated in the different judgments to be in fact interconnected – only then does it reach the Genuine Truth. On the level of understanding *P* and *not-P* are separated and their unity must be restored on the level of reason. But it is important to note that *P* and *not-P* are not the same in that unity as they were separately.

According to Hegel, it is not only "true" and "false" which change their meaning when they are considered in their unity, but also, e.g.

the terms in the expressions "the unity of subject and object", "the unity of finite and infinite", "the unity of being and thought" etc. (as we find in the Phenomenology of Mind). The words "subject", "object", "finite", "infinite", "being", "thought" etc. do not have the same meaning as when they are used separately. If we take *any* object (process, situation) in its truth i.e. being identical with its notion, it will not have the property P (what P means separately) or the property *not-P* (what not-P means separately).

According to the theory of dialectic, *every* concrete thing is changeable and *every* concrete thing is connected to other things through infinitely many, often contradictory, features. However, when we think of a concrete object, then

first, we separate it as having the property P from those not having it;

second, we ignore that the concrete things identified as having the property P may differ from each other in other respects;

third, we abstract from the fact that the concrete things identified as things having the property P may have the property P to different degrees.

This separation, ignoration or abstraction can always be made more precise but it must always take place to some extent. In his "From Dream to Discovery" J. Selye gives an enlightening account of this and then he adds: "No use worrying about these imperfections; we must learn to live with them. We must learn how to use our thought-units, no matter how imperfect they are, because no form of thinking is possible without them. The trouble is that we keep forgetting about these imperfections, and then, we make serious errors resulting in periodic ruts of insecurity and – especially if we are philosophers – pessimism about the power of thought." [12] p. 269

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