OUTLINES OF A LOGIC OF RELATIVE TRUTH

László HÁRSING

1. *Relative Truth as the Generalization of the Classical Truth Concept*

Scientific research is justly considered to be the most reliable cognitive method, for it is the best way of enriching human knowledge with new and true knowledge. Truth is the central value category of science. However, the concept of truth can only serve its axiological purpose in science if it also accounts for the truth status of hypotheses which are decisively important from the point of view of the progress of cognition.

If truth is interpreted in the classical sense as the congruence of thought and reality, then truth value can only be attributed to hypotheses in a definitional sense, at best, and, in most cases, not in a criterial sense, since comparison with reality cannot always be made. We could say that hypotheses have no truth value at all, as they are partly propositions referring to non-existent (past or future) phenomena. However, this regressive solution must be rejected unless we wish to challenge the validity of the truth multiplying function of science. We will therefore assume that the deterministic relationship of the present with the past and the future provides a sufficient basis for interpreting the concept of congruence.

Another suggested solution to the problem acknowledges that hypotheses have truth value in a classical sense but, without an adequate operative method, this cannot be defined. Instead, the introduction of the concept of *logical probability* is suggested, which measures the degree of grounding the hypotheses have. This proposition has a positive element, in that hypotheses, from the point of view of their cognitive status, can only be evaluated through comparison with other statements. If a hypothesis shows agreement with a background of knowledge which has already been accepted, then it is attributed a probability factor above minimum, if, on the other hand, it lacks agreement, a probability factor under minimum is attributed to it. Given the knowledge of logical probabilities, hypotheses can be compared, and, in addition, measure functions assuring quantification can also be defined. However, from an epistemological point of view, this solution is not satisfactory, either. Its primary inadequacy is that it considers truth to be an epistemological noumenon (thing in itself), the existence of which we presume, while denying its cognizability.

A possible egression can be found in the proposition which interprets logical probability as the probability of the truth of the hypotheses, i.e. it takes it as the degree to which the truth of the given hypothesis is grounded. We can argue as follows: let us suppose that previous knowledge has a relevant part which is true in the classical sense. This knowledge is generally acquired as an intellectual inheritance from former generations. The hypothesis set up as possible new knowledge must be compatible with this relevant part. This compatibility assures indirect correspondance between the hypothesis and the facts of reality, the degree of compatibility being denoted by logical probability. This epistemological form of indirection is of such great importance from the point of view of gathering knowledge that no truth concept of any philosophical significance can disregard it.

For all that, our suggestion here is to accept *relative truth* as the measure of value of the hypotheses, instead of logical probability. To support our proposition, we put forward the following arguments:

a) The concept of relative truth not only emphasizes the indirectness of the truth of hypotheses, but the historical character of this indirectness as well. It takes into account the fact that cognitive processes comprise a succession of situations and, in any of these situations, the relative truth value may alter.

b) Over and above the historical indirectness of that correspondence, the concept of relative truth also underlines the momentum of self-reflection. When we talk about the relative truth values of a certain statement, we are not thinking of some independently existing epistemological indirectness but, rather, we are stressing its coming to be known by some cognitive subject. This cognitive subject is considered to be an ideal individual who is in possession of all historically possible relevant knowledge and performs his cognitive activities as a representative of the human race.

c) Nevertheless, his relative truth value judgement may be false, because it is also possible that the hypothesis is false in the classical sense. This can occur in spite of the fact that it has been in agreement with relevant human knowledge so far, and it is attributed a high relative truth value.

d) Relative truth value, as indirect and historical knowledge composed of the classical truth value, does not supersede the concept of the classical truth value, but serves as a cognitive index for the latter.

e) When we determine the long-term goal of scientific research in the acquisition of new and true knowledge, we must necessarily think of relative truth values, since knowledge can be considered as new truth only in a historical sense.

f) Using the relative truth concept as a basis, it is possible to elaborate a logical system which will, in turn, enable us to develop a theory of reasoning for more differentiated than classical logic. Thus, it seems justified to regard relative truth as a generalization of the classical truth concept.

2. *The Structure of the Logic of Relative Truth (VL)*

Using as a basis the concept of relative truth, it is possible to elaborate a logical system which enables us to develop a theory of reasoning far more differentiated than classical logic. Thus, it seems justified to regard relative truth as a generalisation of the classical truth concept.

Synchronic axioms of the logic of relative truth specify one single cognitive situation, and, accordingly, truth values occuring in this are constant. On the other hand, diachronic axioms reckon with the fact that truth values in a new cognitive situation are different (due to the extension of the background-knowledge), and they thus indicate the relationship between two consecutive, cognitive situations. In most cases, the justification or refutation of the hypotheses are based on diachronic argumentation.

Let's take the following synchronic axioms containing V-formulas:

A1 $0 \leqq V(p) \leqq 1$

A2 $V(\sim p) = 1 - (V)p$

A3 $V(p \,\&\, q) = \mathrm{V}(p) - \mathrm{V}(\mathrm{p} \,\&\, {\sim}\mathrm{q})$

A4 $V(t) = 1$ if t is a tautology of classical logic.
A5 The indenties of classical logic are valid for the variables p, q, r etc. For the V-formulas the operational rules of the rational numbers are valid.

Diachronic axiom:

A6 $V_1(p \& q) = V_1(p)V_2(q)/(V_2(p)$ iff $V_1(p)V_2(q) \leq V_2(p)$ and $V_2(p) > V_1(p)$

Interpretation: The truth value of 'p and q' in a diachronic epistemic situation equals to a quotient in the numerator of which we multiply the truth value of proposition p in the present epistemic situation by the truth value of proposition q in a later epistemic situation, and then divide this result by the truth value of proposition p in the later epistemic situation, if and only if the truth value of proposition p in the later epistemic situation is not greater than the numerator of the quotient and greater then zero.

Let us transform the axiom A6 as follows:

A6′ $V_2(q) = V_1(p \& q)V_2(p)/V_1(p)$ iff $V_1(p \& q)V_2(p)/V_2(p) \leq V_1(p)$ and $V_2(p) > V_1(p) > 0$

Interpretation: The truth value of the proposition q in a given epistemic situation is the greater, the greater is the truth value of proposition 'p and q' in a former epistemic situation as well as the truth value of proposition p in the given epistemic situation; and it is the smaller, the greater is the truth value of proposition p in the former epistemic situation, if and only if the indicated suppositions are realised.

A7 Every synchronic formula is valid in a definite diachronic epistemic situation too, assumed that it does not contain double negations and provides opportunity to put the axiom A6 in operation.

Here are some synchronic and diachronic theorems

T1 $V(p \vee q) = V(p) - V(\sim p \& q)$
T2 $V(p \supset q) = V(\sim p) + V(p \& q)$
T3 $V(p \& p) = V(p)$

T4 $V(p \& \sim p) = 0$
T5 $V(p \vee \sim p) = 1$
T6 $V(p \& q) \leq V(p)$
T7 If $V(p \supset q) = 1$, then $V(p) = V(p \& q)$
T8 If $V(p \supset q) = 1$, then $V(p) \leqq V(q)$

Let us consider some diachronic theorems:

T9a $V_1(p \& \sim p) = V_1(p)\, V_2(\sim p)/V_2(p)$, if $0 < V_1(p) \leq 1/2$
$V_1(p \& \sim p) = 0$, if $V_1(p) = 0$ or $V_2(p) = 1$
$V_1(p \& \sim p) \approx 1$, if $V_1(p) = V_2(p) \approx 0$
T9b $V_1(\sim p \& p) = V_1(\sim p)\, V_2(p)/V_2(\sim p)$, if $0 < V_2(\sim p) \leq 1/2$
$V_1(\sim p \& p) = 0$, if $V_1(\sim p) = 0$ or $V_2(\sim p) = 1$
$V_1(\sim p \& p) \approx 1$, if $V_1(\sim p) = V_2(\sim p) \approx 0$
T10a $V_1(p \vee \sim p) = 1 - V_1(p \& \sim p)$, if $0 < V_2(p) \leq 1/2$
$V_1(p \vee \sim p) = 1$, if $V_1(p) = 0$ or $V_2(p) = 1$
$V_1(p \vee \sim p) \approx 0$, if $V_1(p) = V_2(p) \approx 0$
T10b $V_1(\sim p \vee p) = 1 - V_1(\sim p \& p)$, if $0 < V_2(\sim p) \leq 1/2$
$V_1(\sim p \vee p) = 1$, if $V_1(\sim p) = 0$ or $V_2(p) = 1$
$V_1(\sim p \vee p) \approx 0$, if $V_1(\sim p) = V_2(\sim p) \approx 0$.

These theorems say that a contradiction in a diachronic epistemic situation is not alway totally false and the principle of the excluded middle is not unlimitedly valid in the diachronic epistemic situations. If the truth value of the proposition p respectively $\sim p$ remains nearly zero in the former and later epistemic situations then the contradiction represents adequately the status of knowledge. The middle can not be excluded as a truth value, if the truth value of the proposition p respectively $\sim p$ are both nearly zero, in the above-mentioned situations.

In fact, the diachronic character becomes especially prominent when $V_1(p \& q)$, $V_1(\sim p \& q)$ and V-expressions similar to these occurring in the V-formulas are developed according to A6 or A6′. If none of the mentioned V-formulas appears in a synchronic V-formula, or if one occurs but is not developed according to A6 or A6′, then it can be easily transcribed into a diachronic form so that each V is provided with an identical index.

Very important diachronic theorems are the followings:

T11. If a) $V_1(p \supset q) = 1$ and b) $V_2(q) = 1$, then $V_2(p) = V_1(p)/V_1(q)$.

Proof: from T7 and a) consequently $V_1(p) = V_1(p \& q)$, b) and A6 enable us to conclude: $V_1(p) = V_1(q)V_2(p)$. Thus, the thesis can be derived directly.

T16. If a) $V_1(p \supset q) = 1$, b) $V_1(p) > V_1(q)V_1(r)$ and c) $V_2(r) = 1$, then $V_2(p) > V_1(q)$.

Proof: it follows from premise b) that 1) $V_1(p)/V_1(r) > V_1(q)$. According to a) and c), T11 is 2) $V_2(p) = V_1(p)/V_1(r)$. From (1) and 2), $V_2(p) > V_1(q)$.

3. *Examples of Synchronic and Diachronic Reasoning*

The analogue of every single inferential procedure dealt with in *CL* can be constituted in *VL*. From these, let us consider a variant of the so-called *destructive dilemma*:

S1. a) $V(p \supset q) = 1$
b) $V(p \supset r) = 1$
c) $V(q \& r) = 0$
$\overline{V(p) = 0}$

Proof: on the basis of a) and b) and T8, $V(p) \leqq V(q)$ and $V(p) \leqq V(r)$. According to c) and T6, from $V(q)$ and $V(r)$ at least one equals 0. Hence $V(p) = 0$.

However, it is possible to justify a good number of reasoning procedures, the analogues of which cannot be formulated in *CL*. Let us mention a weak version of modus ponens:

S2. a) $V(p \supset q) > 0$
b) $V(p) = 1$
$\overline{V(p) > 0.}$

Proof: from a) and T2, $V(\sim p) + V(p \& q) > 0$. Since following from b), $V(\sim p) = 0$ and $V(p \& q) \leqq V(q)$, thus $V(q) > 0$.

The diachronic version of *modus* ponens is important:

D1. a) $V_1(p \supset q) = 1$
b) $V_1(p) > 0$
c) $V_1(q) < V_2(p)$

$$\frac{}{V_1(q) < V_2(q)}$$

Proof: it follows from and T7 1) $V_1(p) = V_1(p \& q) = V_1(p)V_2(q)/V_2(p)$. Hence 2) $V_2(p) = V_2(q)$ according to b). From 2) and c) follows $V_1(q) < V_2(q)$.

Yet, from the point of view of those sciences where empirical information is also used as premise, and the justification and refutation of hypotheses is considered to be their primary task, it is diachronic reasoning which is really important. Below, we will deal with the so-called *inverse modus ponens* or confirmative reasoning and with analogical argumentation. These methods of reasoning are emphatically important in the field of factual sciences, and we advance the opinion that the only logic which will play a part in the methodology of these sciences is the one which can account for these methods.

D2. a) $V_1(p \supset q) = 1$
b) $V_1(p) > 0$
c) $V_1(q) < 1$
d) $V_2(q) = 1$

$$\frac{}{V_2(p) > V_1(p)}$$

Proof: by virtue of a) and T7, $V_1(p) = V_1(p \& q)$. According to d), and A6, $0 < V_1(p) = V_1(q)V_2(p)$. Since in compliance with b) and c), $0 < V_1(q) < 1$, thus $V_2(p) > V_1(p)$.

D3. a) $V_1(p \supset q) = 1$
b) $V_2(p \supset q) = 1$
c) $V_1(p \supset r) = 1$
d) $V_1(p) > V_1(q)V_1(r) > 0$
e) $V_1(r) < 1$
f) $V_2(r) = 1$

$$\frac{}{V_2(q) > V_1(q)}$$

Proof: According to a), d) and f) T12. is $V_2(p) > V_1(q)$. By virtue of b) and T8., $V_2(p) \leqq V_2(q)$. Hence $V_2(q) > V_1(q)$.

4. *Epistemic Utility of the Hypotheses*

When accepting or rejecting a hypothesis, we must consider its relative truth value and the *cognitive situation* in which our epistemic decision has been made. To begin with, let us consider the concept of cognitive situation.

We can distinguish two types of cognitive boundary situation: namely, the *revolutionary situation* which renews the given field of cognition, and the *process which only adds* to the given scope of experience. A certain boldness in the formation of hypotheses is characteristic of the former, while moderate advancement characterises the latter. Between these two extreme situations, all the other "mixed research situations" occupy an intermediate position.

Let λ signify the *factor qualifying the nature of the cognitive situation*. Let us postulate that regarding it, $0 \leqq \lambda \leqq 1$ is fulfilled, where $\lambda = 0$ denotes the moderate, and $\lambda = 1$ the bold cognitive situation. For lack of a better method, the value of λ must be determined using estimation.

As already stated, the relative truth value (V truth value) constitutes an indirect type of correspondence between the thought content of a statement and reality. It may happen, then, that a certain V truth value is attributed to a hypothesis in a given cognitive situation, although it does not correspond to reality; that is to say, it is false in the classical sense. (C-false). It is feasible that we may be entirely right in our reasoning and cognition still suffers a loss, if we wrongly accept a hypothesis with a V truth value higher than the minimum, but C-false. Naturally enough, we also cause a loss if we reject a C-true hypothesis on the basis of a given V truth value. Nevertheless, it is evident that epistemic utility can only be expected if we accept C-true hypotheses or reject C-false hypotheses in the function of the cognitive situation.

Let us introduce the following notations to measure *epistemic utility*:

$U^+(p^+)$ = the epistemic utility resulting from the acceptance of the C-true hypothesis p;
$U^+(p^-)$ = the epistemic utility resulting from the acceptance of the C-false hypothesis;
$U^-(p^+)$ = the epistemic utility resulting from the rejection of the C-true hypothesis;
$U^-(p^-)$ = the epistemic utility resulting from the rejection of the C-false hypothesis.

Let us start from the following intuitive considerations:

(1) The bolder the cognitive situation, and/or the higher the V truth value of the hypothesis is, the less advantageous its acceptance, and vice-versa, presuming that the hypothesis is C-true.
(2) The bolder the cognitive situation, and/or the higher the V truth value of the hypothesis is, the more destructive its acceptance, and vice-versa, presuming that the hypothesis is C-false.
(3) The bolder the cognitive situation, and/or the less the V truth value of the hypothesis is, the more destructive its rejection, and vice-versa, presuming that the hypothesis is C-true.
(4) The bolder the cognitive situation, and/or the higher the V truth value of the hypothesis is, the more advantageous its rejection, and vice-versa, presuming that the hypothesis is C-false.

The following equations fulfil conditions (1)-(4):

(i) $U^+(p^+) = 1 - \lambda V(p)$
(ii) $U^+(p^-) = -\lambda V(p)$
(iii) $U^-(p^+) = -\lambda V(\sim p)$
(iv) $U^-(p^-) = 1 - \lambda V(\sim p)$

Let us assume that $\lambda = 1$ and $V(p) = 1$, i.e. we have a V-true statement of a maximum degree in a bold research situation. In this case, the following epistemic gains are possible:

$$U^+(p^+) = 0;\ U^+(p^-) = -1;\ U^-(p^+) = 0;\ U^-(p^-) = 1$$

We believe that the obtained values correspond to our intuitive expectations.

Let us assume that $\lambda = 0$ and $V(p) = 0$, i.e. we have a statement of minimum V truth value in a precautious research situation. The following epistemic gains proceed:

$$U^+(p^+) = 1;\ U^+(p^-) = 0;\ U^-(p^+) = 0;\ U^-(p^-) = 1$$

These values call for some explanation. It may be surprising, but is still conceivable, that the acceptance of a V-false but classifically true statement in a cognitive situation demanding precaution is maximally advantageous. Just as advantageous is to reject a V-false and a C-false statement. It does not directly follow from (1)-(4), but is required by equalities (ii) and (iii), that the hypotheses of the value of $V(p) = 0$, if they are C-false, and their rejection if they are C-true, should be epistemologically indifferent. Every formalization has less obvious consequences.

Due to lack of space, we will not analyse the other two pair of possibilities.

5. *Epistemic Utility of Synchronic and Diachronic Reasoning*

Epistemic utility can not only be attached to the acceptance or rejection of certain statements, but to *inferences* as well. Here, however, we must introduce the concept of *expected epistemic utility*.

We start from the principle than an argumentation in a given cognitive situation is prospectively the more advantageous, the higher the V truth value of its conclusion, and the greater the epistemic utility of the conclusion when it is C-true and accepted: and also, the least its destructivity if it is C-false and yet accepted. This intuitive requirement is satisfied by the following formula:

$$\text{(i)}\ E_s(p) = V(p)\,U^+(p^+) + V(\sim p)\,U^+(p^-).$$

The appropriate substitutions and calculations give:

$$\text{(ii)}\ E_s(p) = V(p)(1-\lambda).$$

(Index s indicates that $E_s(p)$ measures the expected epistemic utility of the synchronic inferences.)

To be able to apply formula (ii), however, we must determine the minimum value of the expected epistemic utility which still enables us to speak of a plausible acceptance. This is called the *norm of* *acceptance*. E_s must be at least as large as $1-\lambda$, which can be interpreted as the degree of reliability or cautiousness, namely:

(iii) $E_s(p) = 1-\lambda$

Nevertheless, it can be seen from a comparison of (ii) and (iii) that $E_s(p)$ can only be, at the very best, equivalent to $1-\lambda$, that is, when $\lambda = 1$ or if $V(p) = 1$ and $\lambda < 1$. In every other case, the expected epistemic utility falls short of the norm of epistemic utility, i.e. types of reasoning like S2. do not provide acceptable epistemic utility.

The situation is entirely different in the case of diachronic argumentations. Here we must take the following premise as our starting point: the higher the V truth value of an argumentation measured in a later cognitive situation, and the more the epistemic utility gained by its acceptance, if it is C-true (and the less the loss resulting from its acceptance if it is C-false) then the higher its expected epistemic utility will be. This requirement is fulfilled by the following formula:

$$E_d(p) = V_2(p)\,U^+(p^+) + V_2(\sim p)\,U^+(p^-).$$

Substitutions and calculations lead to

(iv) $E_d(p) = V_2(p) - \lambda V_1(p)$.

Let the norm of the expected epistemic utility also be $1-\lambda$ in the case of diachronical inferences, and let it be required that

(v) $E_d(p) \geqq 1-\lambda$.

Let us consider reasoning D1. from the point of view of requirement (v):

Suppose that $\lambda = 1$. Then

$$E_d(p) = V_2(p) - \lambda V_1(p) > 0 \text{ and}$$
$$1-\lambda = 0.$$

Consequently $E_d(p) > 0$.

Let $\lambda = 0$. In this case

$$E_d(p) = V_2(p) > 0 \text{ and}$$
$$1-\lambda = 1.$$

Therefore, only the equality

$$E_d(p) = 1$$

may subsist, and only when $V_2(p) = 1$.

In other words, in a precautious cognitive situation, the only acceptable diachronic reasoning is that which offers a maximally V-true conclusion. This result does not contradict our intuitive expectations.

It is hoped that we have succeeded in demonstrating that VL, even in this roughly outlined form, can be a useful device in the philosophy of science. It is suitable in elucidating many problems which are beyond the reach of the majority of logical systems. It is a significant merit of VL that is clarifies the relationship between the relative and the classical truth concept and in many respects generalizes probabilistic logic.

Technical University, Miskolc
Institute of Philosophy, Budapest

László HÁRSING

REFERENCE

Bunge Mario, Treatise on Basic Philosophy. Vol. 6. D. Reidel Publ.

Hársing László, Scientific Reasoning and Epistemic Attitudes Akadémiai Kiadó, Budapest, 1982.

Hilpinen Risto, Rules of Acceptance and Inductive Logic. North-Holland Publ. Co., Amsterdam, 1968.

von Wright G.H., Truth, Negation and Contradiction. Manuscript, 1984.