PARADOXES SOLVED BY SIMPLE RELEVANCE CRITERIA

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ABSTRACT

In this paper we show that a number of paradoxes in the areas of deontic and epistemic logic, value theory, explanation, confirmation, lawlikeness and disposition predicates can be solved by applying two simple relevance criteria based on classical logic: the Aristotelean Criterion (A-relevance) and the Körner-Criterion (K-relevance). They are roughly as follows: An inference (or the corresponding valid implication) is A-relevant iff there is no propositional variable and no predicate which occurs in the conclusion but not in the premises. And an inference (or in general any valid formula) is K-relevant iff it contains no single occurrence of a subformula which can be replaced by its negation salva validitate.

The purpose of this paper is to show that a number of paradoxes in quite different areas can be easily solved by applying two very simple relevance criteria. The paradoxes in question are those in the areas of deontic logic, value theory, epistemic logic, explanation, confirmation, lawlikeness, and disposition predicates. (1) The paper is divided into two main parts: In the first one we apply the relevance criteria to Standard Propositional Logic and to an extension of it by allowing operators to be attached to propositional variables. In this part we describe the criteria and its properties and show how they rule out paradoxa of Standard Propositional Logic and of the usual systems of Deontic Logic, Value Theory, Epistemic Logic, and Logic of Volitions. In the second part we apply the relevance criteria to First Order Predicate Logic. Here we show how they rule out most of the well

(1) Notice that we use the term "paradox" in a wide sense. The kernel of a paradox is that some logical consequences of plausible and important axioms or definitions are counterintuitive. For our discussion we concentrate only on standard cases which are known as paradoxical from the philosophical and scientific literature.

known paradoxes of explanation and confirmation, and furthermore some paradoxes of laws of nature and of disposition predicates.

Let us make a few remarks in order to avoid misunderstanding. The purpose of our relevance criteria is different from that of the well-known logics of relevance and entailment developed by Ackermann, Anderson and Belnap, Routley and Meyer, Dunn and others. (2) Since the main aim of the latter seems to be to avoid typical paradoxes of implication the intention of constructing the systems is concentrated to alter implication of classical logic. However, the main aim of our relevance criteria is to avoid paradoxes not only in one area but in different areas as mentioned above. Therefore it is not sufficient to concentrate on implication because there are also paradoxes which are dependent on disjunction, conjunction and negation. For instance, the formula $p \rightarrow (p \vee q)$ which is relevant in all well-known systems of entailment and relevance-logic (3) is irrelevant according to both of our criteria. The reason is that this formula is responsible for a persistent series of paradoxes in different areas. – Furthermore, another important difference lies in the fact that whereas the usual systems of entailment and relevant logics are non-classical our criteria are based on classical logic.

For the reasons mentioned above our criteria are not to be understood as rivalizing with the mentioned known relevant logics. The independent significance of our criteria emerges from their capacity of solving different kinds of paradoxes in different areas of application.

1. *Two Simple Relevance Criteria*

1.1 *The Aristotelean Relevance Criterion*

A very simple and transparent relevance criterion goes back to Aristotle's Syllogistics. In an intuitive form it says that the conclusion must not contain predicates which do not already occur in one of the premises. More accurately, the conclusion contains only subject-term

(2) Cf. ACKERMANN, W. (1956), ANDERSON-BELNAP (1975), ROUTLEY-MEYER (1973), and MEYER-DUNN (1969).

(3) Cf. ANDERSON-BELNAP (1975), pp. 339-340.

and predicate-term, whereas the premises contain in addition the middle-term which is eliminated by the syllogistic inference. If we apply this idea to propositional logic we may say that an inference of propositional logic is relevant iff the conclusion does not contain propositional variables which do not occur already in the premises. (4)

The intuitive motivation of the Aristotelean Criterion is based on a well-known concept of inference (or implication), which is very close to Kant's notion of analyticity. (5) Put into a nutshell it says that there should not be anything essentially new in the conclusion (consequent) which was not already in the premise (antecedent).

1.2 *The Körner Relevance Criterion*

The other relevance criterion goes back to Stephan Körner and will be called Körner-Criterion. Körner writes: "A component of a valid logical implication (of the type considered here) is inessential if, and only if, it can salva validitate be replaced by its negation". (6) Körner gave different formulations of his criterion in different passages of his writings. They are however not unambiguous, since they do not make a clear distinction between a subformula of a given formula and the concrete occurrences of the same subformula in different places of a formula. (7) In 2.2. we will remove this ambiguity.

(4) That this criterion was used by Parry (1933) as a background for constructing his axiom system of "analytische Implikation", Weingartner learned from Prof. Belnap (in spring 1984). Cf. Anderson-Belnap (1975), pp. 430ff. However Parry's system differs from A_0-relevance since the valid A_0-relevant formulas are representable by finite matrices (as shown in Weingartner (1985)) whereas Parry's system is not. The criterion was also used by Gärdenfors (1976), pp. 425 and 430 for explanations and by Weingartner in Bellert-Weingartner (1982), p. 225 for relevant consequences. The definition of relevant consequence (in which the misprint "minimal" should be replaced by "maximal") given there is more restrictive than A-relevance as defined in this paper because the consequence class does not contain logically true propositions except in the case where the premises contain exclusively logically true propositions.

(5) We do not want to enter into questions of the history of philosophy. But Kant's view that logic is analytic (whereas mathematics is not) is understandable from his saying that logic didn't develop since Aristotle (being ignorant as he was of Stoic, Scholastic, and Leibnizian logic) and in Aristotle's Syllogistics we have a model for an A-relevant logic being analytic in Kant's sense.

(6) KÖRNER (1979), p. 378

(7) KÖRNER (1979), p. 378, and (1959), pp. 24f. and 66f. Cf. Schurz (1984), pp. 161-163.

To put it shortly, the intuitive motivation of the Körner-Criterion is based on the very clear idea that a relevant inference (implication, theorem) must not contain inessential components, i.e. components which can salva validitate be replaced by their negations.

2. *The Relevance Criteria Applied to Propositional Logic*

In this chapter we define the Relevance Criteria in such a way as to apply them to formulas of the standard (two valued) propositional logic. The set of all formulas of propositional logic is built up in the usual way from an infinite set of propositional variables (*p, q, r, ...* being different propositional variables) with the help of the connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$ (α, β, γ... represent formulas). The notion of subformula is defined in the usual way. ([8])

2.1 A_0-*Relevance*

Since the Aristotelean Relevance Criterion is concerned only with inferences it will be natural to restrict its applicability to so-called implicational formulas, i.e., formulas of the form $\alpha \rightarrow \beta$ where α and β are wffs of propositional logic. Since we also want to apply the relevance-criteria to propositions which are not logically valid (i.e., not valid in either Standard Propositional or Predicate Calculus of First Order), for instance to laws of Deontic Logic, of Action Theory or Value Theory, we shall not restrict the definition of A-relevance and K-relevance to logically valid formulas only. On the other hand we want to emphasize that an adequate application of A-relevance is rather dependent on validity, since the very idea of A-relevance demands that there should be nothing new in the conclusion in respect to the premises. Therefore, the application of A-relevance makes good sense only if the formula to which it is applied is valid at least in some relative sense, i.e., "valid in system *S*". (The same does not hold for the extension of K-relevance given in 2.2.)

Def 1: An implicational formula $\alpha \rightarrow \beta$ is A_0-relevant iff there is no propositional variable which occurs in β but not in α.

([8]) For a suitable definition cf. Leblanc (1968).

Since we also want to apply our relevance criteria to inferences we shall formulate a general method for transferring the respective relevance criterion from the valid implicational formula to its inferential analogon. This is done with the help of Def 2 and Def 2.1 by reducing relevance of valid inferences to relevance of logically true implicational formulas.

Def 2: A logically valid inference $\pi \vdash \beta$ is A_i- (or K_i-) relevant iff its logically true implicational counterpart is A_i- (or K_i-) relevant (where "i" stands for an index of our relevance-criteria).

Def 2.1: The logically true implicational counterpart of a logically valid inference $\pi \rightarrow \beta$ is

(1) $\pi \rightarrow \beta$ if π is a formula of propositional logic (in this case we write: $\alpha \rightarrow \beta$)

(2) $\wedge\pi \rightarrow \beta$ if π is a (finite) set of formulas of propositional logic (where "$\wedge\pi$" stands for the conjunction of all elements of π)(9)

(3) $\pi^* \rightarrow \beta$ if π is a formula of predicate logic, where π^* is the universal closure of π

(4) $(\wedge\pi)^* \rightarrow \beta$ if π is a (finite) set of formulas of predicate logic, where $(\wedge\pi)^*$ is the universal closure of the conjunction of the elements of π.

Remark 1: Notice that in predicate logic the Deduction Theorem $\alpha \vdash \beta$ iff $\vdash \alpha \rightarrow \beta$ holds only if α is closed. If α is not closed it holds that $\alpha \vdash \beta$ iff $\vdash \alpha^* \rightarrow \beta$, where α^* is the universal closure of α, i.e., the formula $\forall x_1 \ldots \forall x_n \alpha$ (where $x_1, \ldots, x_n$ are the variables which are free in α; trivially, $\alpha = \alpha^*$ if α is closed).

Remark 2: "Logically valid" means valid in the standard two-valued Propositional Logic or in the Standard Predicate Logic of First Order. Since we want to apply our relevance criteria also to deductive systems of various areas, say of deontic logic, value theory, epistemic logic etc. we have to use, in addition, a relative concept of validity in the sense of "valid in *S*", where *S* is a certain deductive system.

(9) If π has only one element α then $\wedge\pi = \alpha$; if $\pi = \emptyset$ then $\wedge\pi = \emptyset$.

2.1.1 *Examples*

Valid formulas which are A_0-relevant are: $p \to p$, $p \to \neg\neg p$, laws of commutation, association, distribution, DeMorgan, modus ponens, modus tollens, hypothetical syllogism in the form $[(p \to q) \wedge (q \to r)] \to (p \to r)$, disjunctive syllogism, $(p \wedge q) \to p$ (simplification), $(p \wedge p) \leftrightarrow p$, $[(p \vee q) \to r] \to (p \to r)$, $[p \vee (p \wedge q)] \to p$, $(p \to q) \leftrightarrow (\neg q \to \neg p)$, $[(p \to r) \wedge (q \to r)] \to [(p \vee q) \to r]$, $[(p \to q) \to p] \to p$, ... etc.

Valid formulas which are not A_0-relevant are: $(p \wedge \neg p) \to q$, $\neg p \to (p \to q)$, $p \to (p \vee q)$, $\neg p \to [p \to (q \wedge \neg q)]$, $(p \to q) \to [(p \wedge r) \to q]$, $(p \to q) \to [(p \wedge r) \to (q \wedge r)]$ ... etc.

2.1.2 *Properties of A_0-Relevance*

In two other papers(10) the following properties of A_0-relevance have been proved:

(1) A_0-relevant implication is transitive, i.e., if $\alpha \to \beta$ and $\beta \to \gamma$ are both A_0-relevant and valid then so is $\alpha \to \gamma$.

(2) The set of all valid implicational formulas satisfying A_0-relevance is closed under substitution, whereas for valid implicational formulas which are not A_0-relevant this is not the case.

(3) The set of all valid implicational formulas satisfying A_0-relevance is representable by finite matrices (truth-tables).

(4) The set of all valid implicational formulas satisfying A_0-relevance is not closed under modus ponens (in respect to A_0-relevance).

(5) A_0-relevant implication is not A_0-relevance preserving.

(6) If $\alpha \to \beta$ is A_0-relevant then all four cases are possible concerning the A_0-relevance of α and β (provided α and β are implicational formulas): Both, neither or one of both may be A_0-relevant.

(7) A_0-relevant implication is not closed under contraposition (for instance, $(p \wedge q) \to p$ is A_0-relevant but $\neg p \to \neg(p \wedge q)$ is not).

2.2 *K_0-Relevance*

Whereas A-relevance is restricted to implicational formulas and inferences, K-relevance is applicable to arbitrary wellformed formu-

(10) Cf. WRONSKI-WEINGARTNER (forthcoming) and WEINGARTNER (1985).

las. We now introduce an extension of Körner's original criterion from valid formulas to arbitrary formulas (due to Cleave): [11]

Def 3: A formula α is K_0-relevant iff no single occurrence of a subformula in α can be replaced by its own negation without changing the logical content of α. [12]

Def 3.1: α and β have the same logical content iff $\alpha \Vdash \beta$ and $\beta \Vdash \alpha$ (where $\Vdash$ is the semantic relation of consequence).

Remark: If α is a logically valid formula the expression "without changing the logical content of α" in Def 3 can be equivalently replaced by "salva validitate of α" (since all valid formulas have the same logical content according to Def 3.1). [13]

2.2.1 *Examples*

The valid formulas $p \to p$, $\neg(p \wedge \neg p)$, $p \vee \neg p$, $p \leftrightarrow \neg\neg p$, laws of commutation, association, distribution, DeMorgan, modus ponens, modus tollens, hypothetical syllogism, disjunctive syllogism ... etc. are K_0-relevant.

The following valid formulas are not K_0-relevant: $(p \wedge q) \to p$ (simplification), $p \to (p \vee q)$ (addition), $(p \wedge \neg p) \to q$ (ex falso quodlibet), $q \to (p \to q)$, $\neg p \to (p \to q)$, $(p \to q) \to [(p \wedge r) \to q]$, $(p \to q) \vee (q \to p)$, $[(p \vee q) \to r] \to (p \to r)$ ... etc.

2.2.2 *Properties of K_0-Relevance*

Cleave has already proved the following properties of K_0-relevance: [14]

(1) Every subformula of a K_0-relevant formula is K_0-relevant.

(11) Cf. CLEAVE (1973/74), p. 119.

(12) More formally: Let $\pi^{\alpha}_{\neg}$ be the set of all formulas which result from α by replacing some single occurence of some subformula of α by its own negation. Then Def 3 states: A formula α is K_0-relevant iff there exists no $\gamma \in \pi^{\alpha}_{\neg}$ such that $\alpha \dashv\Vdash \gamma$. A formal definition of "single occurence of a subformula" is given in Schurz (1984), pp. 162f.

(13) We are indebted to Georg Kreisel for suggesting an interesting modification of the Körner-Criterion: consider not the replacement of single occurrences of subformulas by their negations but by any other (arbitrary choosen) formula. While in classical propositional logic this modification would be equivalent to the Körner-Criterion (as can be shown simply), in weaker (e.g., intuitionistic) logic it could bring new results.

(14) CLEAVE (1973/74), pp. 120-122.

(2) If $\alpha \rightarrow \beta$ is K_0-relevant (for short: $\alpha \overset{K_0}{\rightarrow} \beta$) then none of α, β, $\neg\alpha$, $\neg\beta$ are valid. (15)

(3) $\alpha \wedge \beta$ is valid and K_0-relevant iff α and β are such.

(4) If $\alpha \overset{K_0}{\rightarrow} \beta$ and $\alpha \overset{K_0}{\rightarrow} \gamma$ then $\alpha \overset{K_0}{\rightarrow} (\beta \wedge \gamma)$. If $\alpha \overset{K_0}{\rightarrow} \beta$ and $\gamma \overset{K_0}{\rightarrow} \beta$ then $(\alpha \vee \gamma) \overset{K_0}{\rightarrow} \beta$. $\alpha \overset{K_0}{\rightarrow} \beta$ iff $\neg\beta \overset{K_0}{\rightarrow} \neg\alpha$.

(5) K_0-relevance is preserved under commutation, association, double negation, and DeMorgan's laws.

(6) K_0-relevant implication is not transitive.
Moreover the following can be proved:

(7) The set of valid formulas which are not K_0-relevant is closed under substitution, whereas for K_0-relevant valid formulas this is not the case.

(8) The set of valid formulas which are K_0-relevant cannot be represented by either finite or infinite matrices (truth-tables). On the other hand the set of valid not K_0-relevant formulas is representable by infinite matrices but not by finite ones.

(9) K_0-relevance does not imply A_0-relevance because $p \rightarrow [p \wedge (q \vee \neg q)]$ is K_0-relevant but not A_0-relevant. (16)

All these facts can be derived from a few theorems proved in Wronski-Weingartner (forthcoming).

3. *Paradoxes of Propositional Logic Excluded by the Relevance Criteria*

3.1 *Ex falso quodlibet*

$(p \wedge \neg p) \rightarrow q$
$(p \wedge \neg p) \rightarrow (q \wedge \neg q)$
$\neg p \rightarrow (p \rightarrow q)$

3.2 *Verum ex quodlibet*

$p \rightarrow (q \rightarrow q)$
$(p \rightarrow p) \rightarrow (q \rightarrow q)$

(15) From this property of K_0-relevance it is easily seen that the "Wright-Geach entailment" as it is defined by Geach in his book (1958), p. 187 – and was proposed already by von Wright (1957), p. 182 in a bit different way – is implied by K_0-relevance, but not by A_0-relevance, since the latter is transitive. On the other hand the "Wright-Geach entailment" does not imply A_0-relevance since for instance the principle of Addition is A_0-irrelevant.
Cf. further Wessel's system of strict implication, which combines A_0-relevance with the "Wright-Geach entailment" (Wessel (1979)).

(16) However, there exists a modification of K-relevance which implies A-relevance. Cf. Schurz (1984), p. 166.

3.3 *Redundant Element*

$p \to [p \lor (q \land \neg q)]$

3.4 *Absorption*

$p \to [p \lor (p \land q)]$
$p \to [p \land (p \lor q)]$

3.5 *Negation and Contradiction*

$\neg p \to [p \to (q \land \neg q)]$
$(p \to \neg p) \to [p \to (q \land \neg q)]$

3.6 *Adding Premises*

$q \to (p \to q)$
$q \to [(p \lor \neg p) \to q]$
$(p \to q) \to [(p \land r) \to q]$

All the above formulas are examples of logically valid propositions in Propositional Calculus which are excluded as not-relevant by both criteria. All these formulas have one important characteristic in common which makes them paradoxical in one way or other: they introduce an arbitrary new proposition in the conclusion (consequent of an implicational formula) which is not connected with the propositions in the premises (antecedent). Of the following examples of further theorems of Propositional Calculus, 3.7 are ruled out only by K_0-relevance, 3.8 only by A_0-relevance:

3.7 *Paradoxes of Implication*

$(p \land q) \to (p \to q)$
$p \lor (p \to q)$
$(p \to q) \lor (q \to p)$
$(p \to q) \lor (p \to \neg q)$
$(p \to q) \lor (\neg p \to q)$
$(p \to q) \lor (\neg p \to \neg q)$

3.8 *Redundant Elements*

$p \to [p \land (q \lor \neg q)]$
$p \to [(p \land q) \lor (p \land \neg q)]$
$p \to [q \to (p \land q)]$
$(p \to q) \to [(p \land r) \to (q \land r)]$
$(p \to q) \to [(p \lor r) \to (q \lor r)]$
$(p \to q) \to [(s \lor \neg s) \land [(q \to r) \to (p \to r)$

4. *The Relevance Criteria Applied to Extended Propositional Logic*

4.1 *Extension to Operators*

We extend Standard Propositional Logic by allowing operators to be attached to propositional variables or formulas in the usual way. These operators may be modal, deontic, epistemic, or any others. Writing P, P_1, P_2 ... for operators, Pp, $P\alpha$, $P_1p \to P_1q$, $P(\alpha \to \beta)$ ... etc. are examples of wellformed formulas (the well-known recursive definition of "formula" being extended respectively).

We will apply our criteria of A_0- and K_0-relevance also to formulas of extended propositional logic. But in addition to that, it is possible now to define a stronger criterion of A_1-relevance which refers not only to propositional variables but also to operators. (Note that it is not possible to strenghten K_0-relevance in an analogous way.)

4.2 *A_1*-Relevance

4.2.1 For the application of A-relevance to operators (as it is carried out in 4.2.2, Def. 4, condition (2)) we select only the following four types of operators:

(1) deontic operators ("obligatory", "permitted", "forbidden")
(2) value operators ("it is a value that")
(3) epistemic operators ("knows that", "believes that", "doubts that")
(4) volitive operators ("wishes that", "desires that", "feels that")

Of course the list may be extended for instance to include action operators ("acts that", "acts in such a way as to bring about that" ...) or speech operators ("says that", "writes that" ...). But it is important to notice that we do not include modal operators. One of the reasons for not including modal operators is that we do not want to treat $p \rightarrow \Diamond p$ as A_1-irrelevant.

Further, when applying A_1-relevance to predicate logic we do not include quantifiers as operators though the application of A_1-relevance to quantifiers might have some interesting effects on handling the problem of existential presuppositions. For instance the application of A_1-relevance to quantifiers would render Existential Generalization as irrelevant.

4.2.2

Def 4: An implicational formula $\alpha \rightarrow \beta$ is A_1-relevant iff the following conditions are satisfied:
(1) $\alpha \rightarrow \beta$ is A_0-relevant
(2) there is no operator (in the sense of the four types 4.2.1) which occurs in β but not in α (where operators which can be reduced to one another by definition do not count as different operators). [17]

[17] The addition "where ..." is important in order not to rule out such basic laws as $Op \rightarrow PEp$ in Deontic Logic whenever PE is definable in terms of O.

5. *Paradoxes of Deontic Logic and Value Theory*

That the following paradoxes of Deontic Logic listed in 5.1.1 to 5.1.7 are ruled out by the proposed relevance-criteria is seen easily: one just has to apply the definitions Def 1, Def 3 or Def 4. These paradoxes may also be formulated as inferences. In this case, Def 2 has to be used in addition. The formulations of the paradoxes we give are usually taken to be the most simple formulations. Since it is easy to apply A_0-, A_1- and K_0-relevance to other versions of these paradoxes we do not discuss other formulations.

For the following, *O, PE, F* are deontic operators, understood as operators of our extension of propositional logic (cf. 4.1) and to be read as "it is obligatory that", "it is permitted that", and "it is forbidden that".

5.1 *Paradoxes of Deontic Logic* ([18])

5.1.1 *Ross-Paradox* ([19])

$Op \rightarrow O(p \vee q)$

A_0-irrelevant (because q is not in α), K_0-irrelevant (because q can be replaced by non-q salva validitate (relative to standard axioms and rules of Deontic Logic)).

The proof of such a paradoxical statement with the help of standard logic (and standard systems of Deontic Logic) is obvious. It starts with the principle of addition which is neither A_0- nor K_0-relevant:

(1) $\vdash p \rightarrow (p \vee q)$ Principle of Addition: A_0- and K_0-irrelevant.
(2) $\vdash O[p \rightarrow (p \vee q)]$ by the principle: if $\vdash p$ then $\vdash Op$.
(3) $\vdash Op \rightarrow O(p \vee q)$ by the distribution of O concerning $\rightarrow$: $O(\alpha \rightarrow \beta) \rightarrow (O\alpha \rightarrow O\beta)$ – available in even the weakest systems of deontic logic – and by modus ponens from (2). Conditions (1) – (3) are sufficient, but whether they are necessary depends on the system.

([18]) For a surveay article concerning different paradoxa of Deontic Logic cf. Morscher (1974) and Stranzinger (1977)
([19]) Cf. Ross (1944), p. 38.

5.1.2 *Paradox of Free Choice* (20)

$PEp \rightarrow PE(p \vee q)$ — A_0- and K_0-irrelevant

5.1.3 *Paradox of Good Samaritan* (21)

(1a) $(p \dashv q) \rightarrow (Fq \rightarrow Fp)$ — A_1-irrelevant
(1b) $(p \dashv q) \rightarrow (Op \rightarrow Oq)$ — A_1-irrelevant
(2) $O \neg p \rightarrow O \neg (p \wedge q)$ — A_0- and K_0-irrelevant

5.1.4 *Paradox of Derived Obligation* (22)

$O \neg p \rightarrow O(p \rightarrow q)$ — A_0- and K_0-irrelevant

5.1.5 *Paradox of Commitment* (23)

$\neg p \rightarrow (p \rightarrow Oq)$ — A_0- and K_0-irrelevant

Of course, paradoxes 5.1.1 to 5.1.4 are also A_1-irrelevant according to condition (1) of Def 4. Note that paradox 5.1.5 is also A_1-irrelevant according to condition (2) of Def 4. (24)

5.1.6 *Paradox of Hintikka* (25)

$\neg \Diamond p \rightarrow Fp$ — A_1-irrelevant

The paradoxes 5.1.3 (1a, 1b) and 5.1.6 cannot be derived in the way shown in paradox 5.1.1 (which is also applicable to 5.1.2, 5.1.3 (2), 5.1.4). The additional principle presupposed here is usually the principle: $\Box p \rightarrow Op$. Replacing p by $p \rightarrow q$ and defining $p \dashv q$ as $\Box(p \rightarrow q)$ gives: $(p \dashv q) \rightarrow O(p \rightarrow q)$, which leads by transposition, by the mentioned distribution principle and by replacing $O \neg p$ by Fp to 5.1.3 (1a, 1b). 5.1.6 is obtained by replacing p by $\neg p$ in the above principle. It is plain that the presupposed principle $\Box p \rightarrow Op$ is already A_1-irrelevant.

(20) Cf. v. Wright (1967) and (1968), p. 22.

(21) Cf. Prior (1958). The first version is also called paradox of the robber or victim.

(22) Cf. Prior (1954), p. 64.

(23) Provided that "$p \rightarrow Oq$" is an adequate translation for commitment.

(24) Paradoxes like 5.1.5 can easily be constructed out of valid formulas of Propositional Logic which contain an unrelated component in the conclusion (consequent).

(25) Cf. Prior (1967), p. 511f.

5.1.7 *Ought-Can-Principle* (26)

$Op \rightarrow \Diamond p$ A_1-irrelevant if modal operators are included in 4.2.1

Note that the contraposition $\neg\Diamond p \rightarrow \neg Op$ follows from 5.1.6 (replace Fp by $O\neg p$ and use the law $Op \rightarrow \neg O\neg p$) and is A_1irrelevant in any case, i.e., even if modal operators are not included in 4.2.1 (cf. 2.1.2, (7)).

Note that the paradoxes listed in 5.1.1 to 5.1.5 (except 5.1.3 (1a, 1b) are all ruled out by A_0-relevance or by K_0-relevance; i.e., though deontic operators occur in the paradoxical statements we need not apply A_1-relevance. On the other hand, ruling out 5.1.3 (1a, 1b), 5.1.6 and 5.1.7 requires the extension of A_0-relevance to A_1-relevance.

5.2 *Paradoxes of Value Theory*

Value judgments have two different main forms:

(1) Those where a value-predicate (one or more-place) is applied to individual expressions forming an elementary value judgment like in "life is a high value" and in "knowledge is better than error".

(2) Those where a value operator (one or more-place) is applied to propositional variables forming again an elementary value judgment, like in "it is good that some actions are caused by charity", "that a person *a* lies to his neighbour *b* is morally worse than that *a* tells *b* an inconvenient truth". For the following paradoxes we are only concerned with elementary value judgments of the second kind and with respective compound statements built with the help of the connectives.

Using value operators one can construct analogous paradoxes to the deontic paradoxes of 5.1.

5.2.1 $WT(p) \rightarrow WT(p \vee q)$ A_0-, K_0-irrelevant

Example: If it is a high value that life on earth be preserved, then it is a high value that (either) life on earth be preserved or life on earth is extinguished.

These paradoxes arise basically with the same assumptions which lead to the deontic paradoxes: (1) Acceptance of all valid formulas of Standard Propositional Calculus. (2) Acceptance of the principle: If *p*

(26) We do not claim that the Ought-Can principle is paradoxical (though controversial in the literature).

is logically valid or provable it is a value that p. Like the analogous principle in Deontic Logic (what is logically valid should be the case), this principle can of course be questioned, even if logical validity is semantically interpreted as true in all possible worlds or necessarily true. (3) Acceptance of a distribution principle of the form $WT(p \to q) \to (WTp \to WTq)$. The latter seems hardly avoidable in a deductive value theory.

However, this is not the place to discuss the acceptability or defensibility of such principles. What we want to show here is that the arising paradoxes can be ruled out as non-relevant by our relevance-criteria. In addition, it holds that in the usual derivations of these paradoxical statements at least one premise (or one corresponding rule) is irrelevant.

5.2.2 $WT(\neg p) \to WT\neg(p \wedge q)$ A_0-, K_0-irrelevant

Example: If it is a high value that the earthquake does not take place, then it is a high value that it is not the case that the earthquake takes place and that the rich help the poor.

5.2.3 $WT(\neg p) \to WT(p \to q)$ A_0-, K_0-irrelevant

Example: If it is a high value that the earthquake does not occur, then it is a high value that if the eartquake occurs many people will die.

Similar paradoxes can be constructed along the lines of 5.1.5 and 5.1.6.

5.3 *Paradoxes of Epistemic Logic*

The usual systems of Epistemic Logic are reconstructed out of modal systems for instance by reinterpreting "□" as "knows that". If this is done, one has at once a semantics of Epistemic Logic because there is well-known possible world semantics for the different systems of modal logic (for instance S4, S4.2, S5 etc.). This leads, however, to serious paradoxes: Whereas it is very plausible that a statement which is logically valid (true in all possible world) is said to be necessarily true, it follows for the usual systems of epistemic logic that every logically valid statement is known. Again though it is defensible that all logical consequences of necessary statements are again necessary it follows for the usual systems of epistemic logic that all logical consequences of known statements are known, too. Both logical omniscience and deductive infallibility are not human properties and therefore the usual logics are not theories of human or of idealized human knowledge but of a partial (concerning logical truth and deduction) omniscient being.(27)

The two main principles coming from modal logics are these:

5.3.1 a) if $\mid\vdash p$, then $\mid\vdash Kp$ b) If $\vdash p$, then $\vdash Kp$

(If p is logically valid or provable then it is logically valid or provable that p is known.)

5.3.2 if $\mid\vdash (p \rightarrow q)$, then $\mid\vdash (Kp \rightarrow Kq)$ (analogously for $\vdash$)

This principle is obtained from 5.3.1 with the help of the distribution principle $K(p \rightarrow q) \rightarrow (Kp \rightarrow Kq)$.

Transferring A_1-relevance to informal implication makes 5.3.1 and 5.3.2 A_1-irrelevant. Moreover, because of the acceptance of 5.3.1 in the usual systems of Epistemic Logic one gets some similar paradoxes to that of Deontic Logic: "If it is known that arithmetic is undecidable then it is known that if arithmetic is decidable then Chomsky's theory of grammar is false." (cf. 5.1.4 above). Also these are ruled out by A_1-relevance (but also by A_0- and by K_0-relevance).

Though this is not the place to discuss the usual systems of Epistemic Logic and their semantics we want to stress the following: It is not our opinion that one should first adapt an inadequate semantics for constructing an Epistemic Logic, which consequently contains paradoxical theorems (logical omniscience and deductive infallibility), and then rule out these paradoxes by relevance criteria. A better policy would be to be more careful when choosing the semantics. But here relevance consideration could be a helpful guidance.

5.4 *Paradoxes with Volitive Operators*

Should a man who wills rationally will what is necessarily the case or what is true in all possible worlds? If this principle – i.e., if $\vdash p$, then $\vdash WI(p)$ – is accepted then analogous paradoxes to those of

(27) For a detailed criticism and a proposal for weaker and acceptable desiderata see Weingartner (1981).

Deontic Logic arise by replacing the operator '*O*' by the operator '*WI*' (for "wills that") in 5.1.1, 5.1.3, 5.1.4, and 5.1.5. Of course, one has also to presuppose – besides the logical truths of propositional calculus – the respective distribution principle: $WI(p \rightarrow q) \rightarrow (WI(p) \rightarrow WI(q))$. But such a principle seems necessary also in any logic of willing. An example of an analogue of 5.1.4 would be: If (somebody) wills that it is not the case that a war would occur then he wills that if a war occurs then all mankind is killed: $WI(\urcorner p) \rightarrow WI(p \rightarrow q)$.

Again all these paradoxes are ruled out by A_1-relevance, all of them except the analogue of 5.1.3 (1a, 1b) are also ruled out by A_0- and K_0-relevance.

6. *Application of the Relevance Criteria to First Order Predicate Logic*

Many of the central concepts in Philosophy of Science (like law of nature, explanation, confirmation, etc.) have to be formulated within predicate logic. Hence, for handling the paradoxes in the area of philosophy of science a suitable application of A- and K-relevance to predicate logic is necessary.

6.1 *A_2-Relevance*

This is the relevance originally coming from Aristotle's syllogistics: No predicate in the conclusion which does not already occur in the premises.

Def 5: An implicational formula $\alpha \rightarrow \beta$ of predicate logic is A_2-relevant iff there is no predicate which occurs in β but not in α. (Predicates which can be reduced to one another by definition don't count as different.)

6.1.1 *Examples*

(1) Substitution of formulas of predicate logic for propositional variables in an A_0-relevant formula of propositional logic yields an A_2-relevant formula (since A_0-relevance is closed under substitution, cf. 2.1.2).

(2) Substitution of formulas of predicate logic for propositional variables in an A_0-irrelevant formula of propositional logic may sometimes yield an A_2-relevant formula (since A_0-irrelevance is not closed under substitution, cf. 2.1.2, e.g., $p \to (p \vee q)$ is A_0-irrelevant, but $Fa \to (Fa \vee Fb)$ is A_2-relevant).
(3) Special examples of A_2-relevant valid formulas of predicate logic: $\forall xFx \to Fa$, $Fa \to \exists xFx$, $(a = b \wedge Fa) \to Fb$, $\forall x(Fx \wedge Gx) \to \forall xFx$, $Fa \to (Fa \vee Fb)$, etc. Special examples of A_2-irrelevant valid formulas of predicate logic: $\forall xFx \to \forall x(Fx \vee Gx)$, $\forall x(Fx \to Gx) \to \forall x((Fx \wedge Hx) \to Gx)$, $\forall x(Fx \to Gx) \to \forall x(Fx \to (Gx \vee Hx))$, etc.

6.2 *K_2Relevance* (28)

Def 6: A formula α of predicate logic is K_2-relevant iff no single occurrence of a subformula in α can be replaced by its own negation without changing the logical content of α.

Def 6.1: Like Def 3.1.

Remark: Like the remark to Def 3.

6.2.2 *Examples*

(1) Substitution of formulas of predicate logic for propositional variables of a K_0-relevant formula of propositional logic may sometimes yield a K_2-irrelevant formula (since K_0-relevance is not closed under substitution, cf. 2.2.2, e.g., modus ponens and its implicational counterpart are K_0-relevant but $(\forall xFx \wedge (\forall xFx \to \forall x(Fx \to Gx))) \to \forall x(Fx \to Gx)$ is K_2-irrelevant.
(2) Substitution of formulas of predicate logic for propositional variables of a K_0-irrelevant formula of propositional logic always yields a K_2-irrelevant formula (since K_0-irrelevance is closed under substitution, cf. 2.2.2).
(3) Special examples of K_2-relevant valid formulas of predicate logic: $\forall xFx \to Fa$, $Fa \to \exists xFx$, $(a = b \wedge Fa) \to Fb$, etc. Special examples of K_2-irrelevant valid formulas of predicate logic: $\forall x(Fx \wedge Gx) \to \forall xFx$, $Fa \to (Fa \vee Fb)$, $\forall xFx \to \forall x(Fx \vee Gx)$, $\forall x(Fx \to Gx) \to \forall x((Fx \wedge Hx) \to Gx)$, $\forall x(Fx \to Gx) \to \forall x(Fx \to (Gx \vee Hx))$, etc.

(28) For sake of parallel indices we continue here with K_2-relevance, since a K_1-counterpart of A_1-relevance does not exist.

7. *Strengthening of A_2- and K_2-relevance: A_3- and K_3-relevance*

7.1 *A_3-Relevance*

The essential characteristic of A_2-relevance is that there are no new predicates in the conclusion (or consequent of an implication); "new" in the sense that they do not occur in the premises and hence are in this sense "irrelevant". There are, however, two other paradoxical cases of explanation: (1) some predicates occuring in the premises may be "irrelevant" (or unrelated) in some respect or other – for example they are not related to those in the law statement or they are not needed for the deduction, etc. (2) The conclusion is weakened by introducing irrelevant individual constants or free individual variables via disjunction. An example for the first case is a total self-explanation of the form "$\forall x(Fx \rightarrow Gx), Ha / Ha$" (cf. 8.2) and one for the second case is an explanation of the sort "$\forall x(Fx \rightarrow Gx), Fa / Ga \vee Gb$" or "$\forall x(Fx \rightarrow Gx), Fa / Ga \vee Gy$", respectively (cf. 8.6.2).

7.1.1 To solve the first group of paradoxes the following idea for strengthening A_2-relevance suggests itself. Whereas the above mentioned characteristics of A_2-relevance applies only to the specific given form of an inference (or implication) we let it apply also to that inference (or implication) which is obtained from the first one by turning the non-lawlike premise into an antecedent of the conclusion with help of the Deduction-Theorem (or law of exportation). Thus we require that not only "$L, A/E$" is A_2-relevant but also "$L/A \rightarrow E$". [29] Applied to the above example it follows that "$\forall x(Fx \rightarrow Gx) / Ha \rightarrow Ha$" is not A_2-relevant.

7.1.2 To solve the second group of paradoxes it would seem that we simply have to extend A_2-relevance to individual constants and free individual variables: all individual constants and free individual variables occurring in the conclusion (consequent) of a relevant inference (implication) must also occur in the premises (antecedent). But it is easily seen that this requirement is too strong: universal instantiation would be ruled out as non-relevant. Hence we may

(29) L = law, A = antecedent, E = explanandum. Hempel (1965), p. 415, calls "$L/A \rightarrow E$" the elliptic formulation of "$L, A/E$".

extend A_2-relevance only to those individual constants and free individual variables of the conclusion which are not the result of applying universal instantiation to the premises or to some purely quantificational alteration of them by Herbrand's rules of passage. [30]

7.1.3 Both proposals described in 7.1.1 and 7.1.2 lead to the following definition of A_3-relevance:

Def 7: An implicational formula $\alpha \to \beta$ of predicate logic is A_3-relevant iff the following three conditions are satisfied:

(1) $\alpha \to \beta$ is A_2-relevant.

(2) If α has the form of a conjunction $\alpha_1 \wedge \alpha_2$, where α_1 is a law or a conjunction of laws[31] and α_2 is a singular sentence (not necessarily atomic), then: $\alpha_1 \to (\alpha_2 \to \beta)$ is also A_2-relevant.

(3) There is no individual constant or free individual variable which occurs in β but not in α, except as a result of applying universal instantiation and possibly some Herbrand's rules of passage to α.

7.2 K_3-Relevance

The essential characteristics of K_2-relevance is that it rules out inessential components in either the premises or the conclusion of valid inferences. However, there are paradoxical cases – called partial self explanations by Hempel (cf. 8.4) – which are not ruled out by K_2-relevance. An example is: "$\forall x(Fx \to Gx), Fa \wedge Ha \,/\, Ga \wedge Ha$". A solution for strengthening K_2-relevance in order to rule out these cases is easily obtained by requiring that K_2-relevance be applied to every conjunct of the conclusion (implication-consequent), separately. This leads to the following definition:

Def 8: A formula α of predicate logic is K_3-relevant iff the following two conditions are satisfied:

(1) α is K_2-relevant.

(2) If α has the form of an implication $\beta \to \gamma$, then it holds: for every conjunct C_γ of γ the implication $\beta \to C_\gamma$ is K_2-relevant.

[30] We are indebted to Paul Gochet for an important hint concerning this condition.

[31] Clearly a conjunction of laws is lawlike itself. We leave open the question however, whether a disjunction of laws can be called lawlike, too.

7.3 *Special Remarks*

7.3.1 Between the relevance criteria A_2, A_3, K_2, K_3 the following implications hold: $K_3 \to K_2$, $A_3 \to A_2$. Therefore, we will not mention an inference in the following as A_3- or K_3-irrelevant, if it is already stated that it is K_2- or A_2-irrelevant.

7.3.2 Of course, also $A_1 \to A_0$ holds. Furthermore, we can also apply A_0- and K_0-relevance to predicate logic if we define a formula of predicate logic as A_0- (K_0-) relevant iff its propositional counterpart is A_0- (K_0-) relevant. The propositional counterpart of a formula α of predicate logic is that (unique) formula of propositional logic from which α emerges by replacing (distinct) propositional variables by (distinct) elementary formulas, i.e., formulas which are either atomic or begin with a quantor (in polish notation). By this definition it can now easily be seen that the implications $K_2 \to K_0$ and $A_0 \to A_2$ hold (so A and K behave here quite differently). But notice that A_0-relevance is no adequate criterion for predicate logic since it is too strong for those valid implications of predicate logic which have no valid propositional counterpart; e.g., $(\forall x(Fx \to Gx) \wedge Fa) \to Ga$ *is valid and* A_2- (also A_3-) relevant, but its propositional counterpart $(p \wedge q) \to r$, which is invalid, is A_0-irrelevant. This shows that an application of A_0-relevance to predicate logic must be restricted to formulas with valid propositional counterparts only (cf. 8.9.4).

8. *Paradoxes of Explanation (and Prediction)*

8.1 *Definition of Deductive-Nomological (D-N) Explanation*

We will deal here only with deductive-nomological explanations, not with probabilistic explanations. Analogously, we will deal in the later chapters only with deductive confirmation, deterministic laws and deterministic dispositions but not with their probabilistic counterparts. ([32])

([32]) A way of applying Körner's criterion to probabilistic relations is suggested in Schurz (1984), pp. 175f.

The famous definition of D-N-explanation given by Hempel(33) runs as follows:

Def 9: A logical argument of the form "$L, A \,/\, E$" is a D-N-explanation iff (1) L (the law) is a lawlike sentence (i.e., a universal sentence; see Def 15) or a set or a conjunction of such; A (the antecedens) is a singular sentence (not necessarily atomic) or a set or a conjunction of such; and E (the explanandum) is a singular sentence; (2) L and A are true; (3) $L, A \vdash E$.

Notice that according to Hempel (1965), p. 234, Def 9 holds for D-N-explanations as well as for predictions; the difference between them lies only in the relation between the time of knowing the premises and the time of knowing the explanandum. As the literature on explanation has shown, Def 9 involves serious problems. Some of these problems are independent from our topic of relevance,(34) but those problems which concern Def 9 as a logical frame of explanation are typically problems of relevance. They result from the fact that Def 9 covers not only intuitively reasonable examples of explanation (like "$\forall x(Fx \to Gx), Fa \,/\, Ga$"), but also extremely counterintuitive examples which we call "explanation paradoxes" and which are eliminable by our relevance criteria, as we will show now.

8.2 *Paradox of Total Self-Explanation* (35)

L: $\forall x(Fx \to Gx)$	K_2-irrelevant
A: Ha	A_3-irrelevant
E: Ha	

Notice that this paradox is already K_0-irrelevant (cf. 7.3.2). Other cases of total self explanations are "$\forall x(Fx \to Gx), Ha \wedge Qb \,/\, Ha$" ($K_2$- ($K_0$-) irrelevant, A_3-irrelevant), "$\forall x(Fx \to Gx), Ha \,/\, Ha \vee Qb$" ($K_2$- ($K_0$-) irrelevant, A_2-irrelevant), and others.

(33) Cf. HEMPEL (1965), p. 232 (orig. Hempel 1942) and Hempel (1965), pp. 247-249 (orig. Hempel-Oppenheim 1948).

(34) Important problems which are independent from relevance problems are: a) the question of requiring truth for L at all and the question of replacing the semantic truth requirement for L and A by a pragmatic requirement of acceptability; b) the problem of further conditions concerning lawlikeness and causality; c) the problem of extending Def 9 to a general explanation scheme "$P_1, \ldots, P_n/E$" which includes also the explanation of laws, i.e., where every P_i and E may be universal sentences.

(35) Cf. HEMPEL (1965), p. 274.

8.2.1 *Remark:* We give no illustrative examples for the considered paradoxes, since we consider them obvious. The "proofs" of the A_i- and K_i-irrelevance properties of the considered paradoxes are always very simple; e.g., "$\forall x(Fx \rightarrow Gx)$, Ha / Ha" is A_3-irrelevant since "$\forall x(Fx \rightarrow Gx)$ / $Ha \rightarrow Ha$" is A_2-irrelevant and it is K_2-irrelevant since the subformula occurrence "$\forall x(Fx \rightarrow Gx)$" is replaceable by its negation salva validitate (i.e., "$\neg \forall x(Fx \rightarrow Gx)$, Ha / Ha" is valid, too). Because of their triviality we will omit such proofs in the following chapters.

8.3 *Paradox of Total Theoretical Explanation* (36)

L: $\forall x(Fx \rightarrow Gx)$	K_2-irrelevant
A: Ha	A_3-irrelevant
E: $Fb \rightarrow Gb$	

8.3.1 *Remark:* Whereas in 8.2 L is deductively superfluous, in 8.3 A is deductively superfluous. Notice that the case in which E is a tautology is a special case in which all premises (L and A) are superfluous, and which is hence an 8.2 and an 8.3 paradox.

8.4 *Paradox of Partial Self-Explanation* (37)

L: $\forall x(Fx \rightarrow Gx)$	K_3-irrelevant
A: $Fa \wedge Ha$	A_3-irrelevant
E: $Ga \wedge Ha$	

8.5 *Paradox of Partial Theoretical Explanation* (38)

L: $\forall x(Fx \rightarrow Gx)$	K_3-irrelevant
A: Fa	(A_3-relevant)
E: $Ga \wedge (Fb \rightarrow Gb)$	

(36) Cf. Ackermann, R. (1965), p. 163; Küttner (1976), p. 286.

(37) Cf. Hempel (1965), p. 275.

(38) Partial theoretical explanations are the counterpart to partial self explanations (in the latter an explanandum-conjunct follows from A alone, in the former from L alone). They are introduced in Schurz (1983), p. 252.

8.6 *Paradoxes with Irrelevant Explanandum-Components* ([39])

8.6.1 L: $\forall x(Fx \rightarrow Gx)$ — K_2-irrelevant
A: Fa — A_2-irrelevant

E: $Ga \vee Ha$

8.6.2 L: $\forall x(Fx \rightarrow Gx)$ — K_2-irrelevant
A: Fa — A_3-irrelevant

E: $Ga \vee Gb$

A case analogous to 8.6.1 is "$\forall x(Fx \rightarrow Gx), Fa \,/\, Ha \rightarrow Ga$" ($K_2$-, A_2-irrelevant).

8.6.3 *Remark*

Notice that the case in which the premises of the explanans are inconsistent can be regarded as a special case of 8.6., in which the whole explanandum is irrelevant since here it can be replaced by its own negation or by any other formula salva validitate. Furthermore notice that 8.5 is A_3-relevant, whereas 8.6.2 is A_3-irrelevant.

8.7 *Paradoxes with Superfluous Premise-Conjuncts* ([40])

8.7.1 L: $\forall x(Fx \rightarrow Gx)$ — K_2-irrelevant
A: $Fa \wedge Ha$ — A_3-irrelevant

E: Ga

8.7.2 L: $\forall x(Fx \rightarrow Gx) \wedge \forall x(Hx \rightarrow Qx)$ — K_2-irrelevant
A: Fa — (A_3-relevant)

W: Ga

8.7.3 *Remark:* Paradoxes 8.7.1 and 8.7.2 are much "weaker" paradoxes than, e.g., 8.2 and 8.3 and it has been doubted if one should count them as "really" paradoxical or only as "odd" cases of explanation. We don't discuss this question here.

([39]) Cf. ACKERMANN, R. (1965), p. 165; GÄRDENFORS (1976), p. 425.

([40]) These paradoxes are discussed in Omer (1970), p. 426 and Tuomela (1972), pp. 379f. without giving special examples. An example similar to 8.7.2 is mentioned in Hempel (1965), p. 273, (footn.). Paradox 8.7.1 is introduced in Schurz (1983), p. 252.

Notice that paradox 8.7.2 is A_3-relevant. A further strengthening of condition (2) of Def 7 would be to require that if the law L is a conjunction $L_1 \wedge L_2$, then also $L_1 \vdash L_2 \rightarrow (A \rightarrow E)$ must be A_2-relevant (which is not the case in 8.7.2). But this strengthening would be too strong since good candidates of explanation like "$\forall x(Fx \rightarrow Gx) \wedge \forall x(Gx \rightarrow Hx), Fa \mid Ha$" would then also become irrelevant.

8.8 *Other Paradoxes of Explanation*

All of the considered paradoxes of explanation can be ruled out by adding the following condition to Def 9 of D-N-explanation: (4) $T, A \vdash E$ is A-relevant and K-relevant (notice that K-relevance alone would be also strong enough). Of course, there exist more explanation paradoxes mentioned in the literature, but Schurz[41] has shown that they are reducible to a small number of basic paradoxes from which all others are constructable by logical combination; e.g., "$\forall x(Fx \rightarrow Gx) \wedge \forall x(Hx \rightarrow Qx), Fa \wedge Pa \wedge Rb \mid (Ga \vee Sb) \wedge Pa \wedge (Fc \rightarrow Gc)$" is a combination of 8.4, 8.5, 8.6.1, 8.7.1, and 8.7.2.

There exist two further explanation paradoxes which we have not yet considered and which are controversial, namely

8.8.1 "$\forall x(Fx \rightarrow Gx), (Fa \rightarrow Ga) \rightarrow Hb \mid Hb$" ($A_3$-irrelevant, K_3-relevant), and

8.8.2 "$\forall x(Fx \rightarrow Gx), \neg Fa \rightarrow Ga \mid Ga$" ($A_3$-relevant, K_3-relevant).[42]

Some authors have shown that the argument forms 8.8.1 and 8.8.2 are paradoxical only in some interpretations of its descriptive signs (predicates and individual constants), in other interpretations they can serve as very reasonable explanations.[43] In short, these paradoxes are context-dependent. If this argumentation is true, these paradoxes should not be excluded by formal (i.e., context-independent) relevance criteria.

[41] Cf. SCHURZ (1982) and SCHURZ (1983), pp. 254-260.

[42] For 8.8.1 cf. Hempel (1965), p. 276; for 8.8.2 cf. Eberle-Kaplan-Montague (1961), pp. 419f.

[43] Cf. STEGMULLER (1969), p. 769; SCHURZ (1982), pp. 326-328; SCHURZ (1983), pp. 264-318.

8.9 *Special Remarks and Modifications*

8.9.1 In the literature on explanation it has often been presupossed that every argument which results from 8.2 – 8.7 by uniform substitution of predicates and individual constants has to be considered paradoxical as well. Now, K-irrelevance is closed under substitution whereas A-irrelevance is not (cf. 2.1.2 and 2.2.2; this holds also for the criteria applied to predicate logic). Hence, the stated K-irrelevance properties of the mentioned paradoxes hold for all uniform substitutions, whereas the A-irrelevance properties do not. E.g., there exist total self-explanations like "$\forall x(Fx \rightarrow Gx)$, Ga / Ga" or total theoretical explanations like "$\forall x(Fx \rightarrow Gx)$, $Fa \vee Ga$ / $Fa \rightarrow Ga$" which are K_2- and also K_0-irrelevant, but A_3- and also A_0-relevant. Hence, if one considers explanation paradoxes closed under uniform substitution, then K-relevance is the more suitable relevance criterion than A-relevance.

8.9.2 In the literature on D-N-explanation it is often required that the adequacy of a D-N-explanation should be invariant in respect to logical equivalent transformations of the premises and the explanandum. ([44]) But our relevance criteria are not invariant in respect to such transformations. Now, this problem can easily be solved by introducing a suitable standard-form of a D-N-explanation, which is unique for all logically equivalent transformations. Then a D-N-argument is defined as relevant iff its standard-form is relevant. ([45])

8.9.3 Our criterion of K_3-relevance makes all D-N-explanations irrelevant which contain superfluous law-components. Now it has been argued that a law with deductively superfluous components must be accepted iff the law has "global" character. ([46]) This problem can be solved by (a) decomposing the law statement in its globality-preserving conjuncts and (b) applying a weakened version of K_2- (or K_3-) relevance, called K_2'- (K_3'-) relevance, in which the replacements in the premises are restricted to closed subformulas: ([47])

([44]) F.i., cf. STEGMULLER (1969), p. 89.

([45]) More detailed in Schurz (1983), pp. 237, 292-297.

([46]) Cf. MORGAN (1976), p. 523; TUOMELA (1972), p. 380; SCHURZ (1983), pp. 309-315.

([47]) More detailed in Schurz (1983), pp. 309-315.

Def 10: A formula α of predicate logic K_3'-relevant iff ... (like Def 8, only substitute K_2'-relevance for K_2-relevance).

Def 11: A formula α of predicate logic is K_2'-relevant iff the following conditions are satisfied:

(1) If α has the form of an implication $\beta \to \gamma$, then there is no single occurrence of a closed subformula in β and no single occurrence of a subformula in γ which can be replaced by its own negation without changing the logical content of α.

(2) If α does not have the form of an implication, α is K_2-relevant.

E.g., "$\forall x(Fx \vee Gx) \to Hx)$, Fa / Ha" is K_2-irrelevant, but K_2'- (also K_3'-) relevant, which is desired, since "$\forall x((Fx \vee Gx) \to Hx)$" is global. On the other hand, paradox 8.7.2 is K_2'-irrelevant as well. Notice further that $K_3' \to K_2'$, $K_3 \to K_3'$ and $K_2 \to K_2'$ hold.

8.9.4 As already mentioned in 7.3.2, A_3-relevance can be strengthened in the following way, called A_3'-relevance:

Def 12: An implicational formula α is A_3'-relevant iff (1) α is A_3-relevant and (2) if α is valid and its propositional counterpart is valid too, then α is also A_0-relevant.

Of course, $A_3' \to A_3$ holds. This strengthening is useful for some paradoxical explanation cases, e.g., "$\forall x(Fx \to Gx)$, Fa / $Fa \vee Ga$", which is A_3-relevant, but A_3'-irrelevant.

9. *Paradoxes of Confirmation*

9.1 *A Simple Definition of Deductive Theory Confirmation*

The notion of deductive theory-confirmation is based on the idea that a theory is confirmed by its true empirical consequences. Some authors([48]) have given the following simple definition of this idea:

([48]) Cf. HESSE (1970), p. 50; LENZEN (1974), pp. 25-30.

Def 13: A true (or accepted, respectively)(49) statement S confirms a theory T iff
(1) S is not tautologous,
(2) T is consistent and
(3) $T \vdash S$.

Def 13 leads to the following paradox:

9.2 *Hesse's Confirmation Paradox*(50)

Assume Def 13 and assume the following "condition of strengthening the confirmans": if A confirms T and A' is consistent, true (accepted) and logically implies A, then also A' confirms T. Then it follows: every synthetic T is confirmed by every synthetic and true (accepted) S. [Proof: From $T \vdash T \vee S$ (and the assumptions on T and S) follows that $T \vee S$ confirms T by Def 13. From this and $S \vdash T \vee S$ (and the assumptions on T and S) it follows that S confirms T by the condition of strengthening the confirmans.]

But obviously, confirmation paradox 9.2 relies on an irrelevant deduction, namely,

$T \vdash T \vee S$ — K_2- (K_0-) irrelevant; A_3'-irrelevant; A_2-irrelevant, if predicates occur in S not occurring in T.

So, Def 13 can be repaired by adding the condition: (4) $T \vdash S$ is K_3-relevant and A_3'-relevant.

9.3 *Popper's Definition of Deductive Theory Confirmation (Corroboration)*

The following more complicated definition goes back to Popper and

(49) We do not discuss the question here whether the semantic version of Def 13 (in which the truth of S is required) or the pragmatic version of Def 13 (in which only acceptance of S is required) is the better one. The paradoxes discussed in the following occur in both versions (the same holds for Def 14).

(50) HESSE (1970) has derived this paradox from stronger assumptions, namely (a) Def 15, (b) A confirms B if A logically implies B and (c) transitivity of confirmation. Therefore, Hesse called this paradox the "transitivity paradox of confirmation". Of course it is doubtful whether confirmation really is transitive. Therefore we have derived Hesse's paradox here from a much weaker condition, namely the condition of strengthening the confirmans.

has been interpreted by Käsbauer in the following way: ([51])

Def 14: A class C of true (accepted) observation statements confirms a theory T iff
(1) $\{T\} \cup C$ is consistent, and
(2) there exist two disjoint subclasses C_1 and C_2 for which the following holds: (2a) $C_1 \cup C_2 = C$, (2b) $\{T\} \cup C_1 \vdash C_2$, and (2c) C_2 is not empty. ([52])

As Stegmüller([53]) and others have shown, Def 14 leads to a couple of confirmation paradoxes which are quite analogous to the explanation paradoxes discussed above, namely the following (9.4 – 9.6):

9.4 *Paradox of Deductively Superfluous Theory*

Let O_1 and O_2 be two true (accepted and consistent) observation statements. Let T be any consistent theory which has no predicates in common with O_1 and O_2. Then T is confirmed by $C = \{O_1 \wedge O_2, O_2\}$ (according to Def 15). [Proof: Let $C_1 = \{O_1 \wedge O_2\}$, $C_2 = \{O_2\}$. Then it holds: (1) $\{T\} \cup C$ is consistent (since T and C are consistent and have no predicates in common. Observe that if the language contains identity, T and C must have no nonlogical symbol in common), (2a) $C_1 \cup C_2 = C$, (2b) $\{T\} \cup C_1 \vdash C_2$ and (2c) $C_2 \neq \emptyset$].

Obviously, the deduction relation underlying this paradox is irrelevant according to our criteria:

$T, O_1 \wedge O_2 \vdash O_2$	K_2- (K_0-) irrelevant; A_3'-irrelevant, A_2-irrelevant (since T and $O_1 \wedge O_2$ have on assumption no predicates in common).

([51]) POPPER (1976), p. 212, Käsbauer is reported in Stegmüller (1971), p. 32. Similar definitions have been given by Hempel (1965), p. 26, and others (cf. Lenzen 1974, p. 47).

([52]) "$\{T\} \cup C_1 \vdash C_2$" in Def 14 is to read as "$\{T\} \cup C_1 \vdash \wedge C_2$", with $\wedge C_2$ for the conjunction of all elements of C_2. Notice further that Popper's additional requirement that the confirmation test must be a serious one has been omitted here since it is of no importance for the logical problems discussed here.

([53]) Cf. STEGMULLER (1971), p. 32.

9.5 *Paradox of Irrelevant Theory-Strengthening*

The "general consequence condition" ([54]) is a very plausible condition on confirmation relations which says: if a sentence (or class of sentences) A confirms a sentence B, then A also confirms every logical consequence of B.

Assume this general consequence condition and Def 14. Then it holds: If a given class C of true (accepted) observation statements confirms at least one theory T, then C confirms every theory T', which is consistent and which has no predicates in common with C and T. [Proof: On assumption there exists a C_1 and a C_2 with $C_1 \cup C_2 = C$, $\{T\} \cup C_1 \vdash C_2$ and $C_2 \neq \emptyset$. Now $\{T \wedge T'\} \cup C$ is consistent (since T' and $\{T\} \cup C$ are consistent and have no predicates in common) and $T \wedge T', C_1 \vdash C_2$ holds, too. So C confirms $T \wedge T'$ (according to Def 14). Because of $T \wedge T' \vdash T'$ and the general consequence condition it follows that C confirms T'.]

Again, this paradox is due to the irrelevancy of the underlying deduction relation, namely

$T \wedge T', C_1 \vdash C_2$ (under assumption of $T, C_1 \vdash C_2$)	K_2-irrelevant (A_3- (A_3'-) relevant iff $T, C_1 \vdash C_2$ is A_3- (A_3'-) relevant).

9.6 *Paradox of Irrelevant Confirmans-Weakening*

Assume $\{T\} \cup \{O_1, O_2\}$ is consistent and $T, O_1 \vdash O_2$. Assume that O_1 is a true (accepted and consistent) observation statement and O_2 is some unknown observation statement, i.e. one, which is not accepted and may be false. Let O_3 be any other true (accepted and consistent) observation statement, which has no predicate in common with T and O_1. Then it holds (according to Def 14): (a) T is confirmed by $\{O_1, O_2 \vee O_3\}$. (b) Under assumption of the condition of strengthening the confirmans (cf. 9.2, here applied to the confirmans as a set of sentences) T is also confirmed by $\{O_1, O_3\}$. [Proof: (a) Since O_3 is true (accepted) and $O_3 \vdash O_2 \vee O_3$, also $O_2 \vee O_3$ must be true (accepted).([55]) Furthermore, $T, O_1 \vdash O_2 \vee O_3$ (since $T, O_1 \vdash O_2$) and

([54]) Cf. HEMPEL (1965), p. 31; LENZEN (1974), p. 31.

([55]) If you take acceptance instead of truth you must assume here that acceptance is closed under disjunctive weakening. Furthermore it is presupposed in the proof of the paradox that disjunctions of observation statements ($O_2 \vee O_3$) also count as observation statements.

$\{T\} \cup \{O_1, O_2 \vee O_3\}$ is consistent (since $\{T\} \cup \{O_1, O_2\}$ is consistent). Hence, $\{O_1, O_2 \vee O_3\}$ confirms T. (b) $\{T\} \cup \{O_1, O_3\}$ is consistent (since $\{T, O_1\}$ and O_3 are consistent and have no predicates in common). Because of $\{O_1, O_3\} \vdash \{O_1, O_2 \vee O_3\}$ and the condition of strengthening the confirmans it follows that also $\{O_1, O_3\}$ confirms T.

Once more, this paradox relies on the following irrelevant deduction relation:

$T, O_1 \vdash O_2 \vee O_3$ (under assumption of $T, O_1 \vdash O_2$)	K_2- (K_0-) irrelevant; A_3'-irrelevant, A_2-irrelevant (since on assumption O_3 has no predicates in common with T and O_1)

9.7 *Special Modifications*

Confirmation paradox 9.4 corresponds to explanation paradox 8.2, confirmation paradox 9.5 corresponds to explanation paradox 8.7.2, and confirmation paradox 9.6 corresponds to explanation paradox 8.6.1. In general, every confirmation paradox corresponds to an explanation paradox but not vice versa. For example, arguments of the form "$\forall x(Fx \rightarrow Gx), Fa \wedge Ha \,/\, Ga \wedge Ha$" are inacceptable as explanations because they are partially self-explanatory. But they are probably acceptable as confirmations, i.e., $\{Fa \wedge Ha, Ga \wedge Ha\}$ is a confirmans for $\forall x(Fx \rightarrow Gx)$. Thus the question is whether the criteria of A_3- and K_3-relevance are too strong here. We do not think so. The reason is this: To solve this problem one has to distinguish only between direct confirmation relations and indirect confirmation relations. C is a direct confirmans for T iff it fulfils Def 14 strengthened by the following additional condition: (3) $\{T\} \cup C_1 \vdash C_2$ is A_3'- and K_3-relevant. And C is an indirect confirmans for T iff there exists a direct confirmans C' for T from which the confirmation relation between C and T follows by the mentioned confirmation conditions (like the general consequence condition and the condition of strengthening the confirmans (see 9.2)).([56]) E.g., $\{Fa, Ga\}$ is a direct confirmans for $\forall x(Fx \rightarrow Gx)$, and $\{Fa \wedge Ha, Ga \wedge Ha\}$ is an indirect

([56]) The acceptability of confirmation conditions is a very controversial point (cf. Lenzen 1974, pp. 30-41). Not all conditions are as harmless as the two which we have mentioned. This is one reason more for the separation of the problem of confirmation conditions (indirect confirmation relations) from the notion of direct confirmans.

one because of the condition of strengthening the confirmans ($\{Fa \wedge Ha, Ga \wedge Ha\} \vdash \{Fa, Ga\}$).

There is one paradox of confirmation which cannot be excluded by our relevance criteria, namely Hempel's famous raven paradox. ([57]) This results from the fact that $\forall x(Fx \rightarrow Gx), \neg Ga \vdash \neg Fa$ is relevant and hence $\{\neg Ga, \neg Fa\}$ is an acceptable confirmans for $\forall x(Fx \rightarrow Gx)$ according to Def 14 strengthened by our relevance criteria. However, in the literature on confirmation it is controversial whether $\{\neg Ga, \neg Fa\}$ is really rid of any confirmation power for $\forall x(Fx \rightarrow Gx)$. Hempel ([58]) pointed out that $\{\neg Ga, \neg Fa\}$ has confirmation power for $\forall x(Fx \rightarrow Gx)$ at least in some situations (dependent on the background context). If this is true then the confirmation relation between $\{\neg Ga, \neg Fa\}$ and $\forall x(Fx \rightarrow Gx)$ cannot be regarded in general as paradoxical. ([59])

10. *Paradoxes of Laws of Nature*

10.1 *A Simple Definition of Law of Nature*

According to an old idea which goes back to Aristotle the notion of law of nature is frequently defined in the following way: ([60])

([57]) Cf. HEMPEL (1965), pp. 10ff.

([58]) Cf. HEMPEL (1965), pp. 18f.

([59]) BUNGE (1974) has developed an interesting semantical criterion of relevance which excludes the raven paradox under the following conditions: Let "F" mean "bird", "G" mean "black" and let a be some object which is no bird. Then $T = \forall x(Fx \rightarrow Gx)$ is semantically irrelevant to $C = \{\neg Ga, \neg Fa\}$ (cf. pp. 76, 79f) since $R(C) \cap R(T) = \emptyset$, whereby $R(C)$ and $R(T)$ are the reference classes of C and of T (according to pp. 50-56, $R(C) = \{a\}$ and $R(T)$ = the class of all birds). But Bunge's criterion has problems of its own: for example, if b is some bird, then T would be also semantically relevant to the tautologies $b = b$ and $Hb \rightarrow Hb$, where "H" is any predicate. Further T is semantically relevant to $\forall x[Fx \rightarrow (Gx \vee Hx)]$ if $R(F) \cap R(G) \cap R(H) \neq \emptyset$. On the other hand the respective implications $T \rightarrow (b = b)$, $T \rightarrow (Hb \rightarrow Hb)$ and $T \rightarrow (\forall x)[Fx \rightarrow (Gx \vee Hx)]$ are K_2- and A_3-irrelevant according to our criteria.

([60]) According to Aristotle we would have to add that the law connection is essential and the law is not only true but necessarily true. However, these Aristotelean conditions are still controversial in respect to their interpretation. Therefore we concentrate on the simple version of law of nature which was already proposed by J. St. Mill, Russell, Schlick, Reichenbach, and many other philosophers.

Def 15: L is a law of nature iff

(1) L is a genuine universal sentence (i.e., the universal quantifier(s) is (are) not logically eliminable),

(2) the quantifier-free part of L is of implicational form, and

(3) L is true.

From the numerous paradoxes following from Def 15 we consider only two, namely:

10.2 *The Paradox of Irrelevant Law Specification*([61])

Consider the sentence "all males which periodically take anti-baby-pills will not get pregnant", in formula $\forall x((Mx \wedge Ax) \rightarrow \neg Px)$. Although this sentence fulfils the conditions of Def 15 it cannot be regarded as a natural law, since Ax is causal irrelevant for $\neg Px$ (if Mx is given). This results from the fact that $\forall x((Mx \wedge Ax) \rightarrow \neg Px)$ is not a fundamental but a derivative law, which is a logical consequence of the fundamental law $\forall x(Mx \rightarrow \neg Px)$. However, this inference is obviously irrelevant according to our criteria:

$\forall x(Mx \rightarrow \neg Px) \vdash \forall x((Mx \wedge Ax) \rightarrow \neg Px)$ A_2-irrelevant, K_2'-irrelevant

Notice that A_2- and K_2'-relevance are the appropriate relevance criteria for the deduction of derivative laws, whereas K_2-relevance would be too strong. ([62]) This results from the following fact: a derivative law L inferred from a fundamental law L' is inacceptable only if the inference $L' \vdash L$ contains irrelevant components in L, whereas irrelevant components in L' must be admitted (because they occur in almost all law derivations). For example, if $\forall x(Fx \rightarrow (Gx \wedge Hx))$ is a fundamental law, then $\forall x(Fx \rightarrow Gx)$ will be an acceptable law. Indeed, the inference $\forall x(Fx \rightarrow (Gx \wedge Hx)) \vdash \forall x(Fx \rightarrow Gx)$ is A_2- and K_2'-relevant, but K_2-irrelevant. ([63]) Summarized, we can repair Def 15 by adding the following condition: (4) there exists no law L' fulfilling conditions (1) – (3) for which $L' \vdash L$ is valid but A_2- or K_2'-irrelevant.

([61]) The name of this paradox has been introduced by Stegmüller (1973), p. 285.

([62]) We do not mention here A_3- and K_3'-relevance since for the problem discussed here they bring nothing new against A_2- and K_2'-relevance.

([63]) Notice that K_2'-relevance covers only subformula-occurrences of the premises which are closed. Since all (proper) subformula-occurrences of quantified sentences are not closed, K_2'-relevance covers only irrelevancies in the derivative law but not in the fundamental law.

10.3 *Goodman's Paradox*

Goodman's famous paradox is a paradox of confirmation as well as of law of nature. What we will show is that this paradox is partially due to an irrelevant inference. A simple version of Goodman's paradox given by Hempel(64) runs as follows: Let $B = \{Fa_1t_1 \wedge Ga_1t_1, ..., Fa_nt_n \wedge Ga_nt_n\}$ be a set of observation data (i.e., of accepted observation statements which actually have been observed). Obviously B confirms the law hypothesis $H = \forall x \forall t(Fxt \rightarrow Gxt)$. Let t_0 be some fixed time point which lies in the future. Hence, $t_1 < t_0, ..., t_n < t_0$. Now a "pathological" predicate G^* can be defined in the following way: $G^*xt: \leftrightarrow ((t \leq t_0 \wedge Gxt) \vee (\neg t \leq t_0 \wedge \neg Gxt))$. Assuming this definition and because of $t_i < t_0$ $(1 \leq i \leq n)$, B is equivalent with $B^* = \{Fa_1t_1 \wedge G^*a_1t_1, ..., Fa_nt_n \wedge G^*a_nt_n\}$. B^* confirms the law hypothesis $H^* = \forall x \forall t(Fxt \rightarrow G^*xt)$. But H^* is inconsistent with H since it predicts the F's for all $t > t_0$ to be non-G's.

Many authors have been argued that universal sentences which contain "pathological" predicates must be excluded from the class of potential laws of nature. The crucial problem was to give a non-circular characterization of "pathologicity" of predicates. But focus your attention on the following simple fact: one direction of the equivalence between B and B^* is K-irrelevant. This can be seen by substituting the definiens of G^* for G^*:

$(t_i \leq t_0 \wedge Fa_it_i \wedge Ga_it_i) \vdash$ K_2- (K_0-) irrelevant
$(t_i \leq t_0 \wedge Fa_it_i \wedge Ga_it_i) \vee (\neg t_i \leq t_0 \wedge Fa_it_i \wedge \neg Ga_it_i)$

So, Goodman's paradox can be excluded by a requirement such as the following: A class B of observation data is acceptable as confirmans of a law hypothesis only if there exists no class B' of observation data for which the following holds:

(a) B' contains the same predicates and individual constants like B (where predicates which can be reduced to one another by definition don't count as different)(65),

(b) $B' \vdash B$ and (c) $B' \vdash B$ is K_2- (K_0-) irrelevant. But of course, this is only a partial solution because it works only if we deal with a given

(64) Cf. Hempel (1965), p. 70.

(65) Requirement (a) is necessary since there always exists a B' with new predicates and individual constants for which $B' \vdash B$ is valid and K_2- (K_0-) irrelevant.

language L with fixed predicates as primitives. The deeper aspect of Goodman's problem is that L is translatable into a language L' in which pathological counterparts of L-predicates figure as primitives. This problem cannot be handled by our relevance criteria.

11. *The Paradox of Disposition Predicates* (66)

According to Carnap (67) a disposition predicate Dx can be defined in the following way:

(Definition schema) $Dx: \leftrightarrow \forall t(Cxt \rightarrow Bxt)$

In words: that x has a certain disposition D means that for all times x will behave in a certain way B if certain conditions C are given. E.g., if "Dx" means "x is soluble in water", then "Cxt" means "*x is put into water at time t*" and "Bxt" means "x dissolves within a short time interval beginning with t". The paradox of disposition predicates now results from the fact that in our example every thing x which has never been put into water during its time of existence must be regarded as soluble (according to the above definition scheme). More formally, for every x for which $\forall t \neg Cxt$ holds, Dx follows from the above definition scheme. This is of course counterintuitive. But if we substitute the definiens of Dx for Dx, we see that the underlying inference-relation is irrelevant:

$\forall t \neg Cxt \vdash \forall t(Cxt \rightarrow Bxt)$ $\quad$ A_2-irrelevant, K_2'-irrelevant

A closer consideration shows that this paradox is similar to that of irrelevant law specification and that A_2-, resp. K_2'-relevance are the appropriate criteria here. An elimination of this paradox is possible by restricting the above definition scheme in the following way: $Dx: \leftrightarrow \forall t(Cxt \rightarrow Bxt)$ is an acceptable definition only if every derivation of $\forall t(Cxt \rightarrow Bxt)$ from a true and temporally universal formula $\forall tF(x, t)$ is A_2- and K_2'-relevant.

(66) This paradox goes back to Carnap (1936), p. 440. Cf. also Hegselmann-Raub (1982), pp. 349f.

(67) Cf. CARNAP (1936), p. 440.

12. *Final Remarks*

Since we have proposed different versions of one criterion which differ in strength it might seem that some of the versions are ad hoc. On a closer look, however, it should be appearent that none of the versions depart from the straightforward basic idea of the respective criterion, such that they can be regarded just as modifications of that basic idea for a certain purpose. In this respect it is important to remember once more the basic ideas of the concepts "relevant" or "irrelevant" provided by the A and K criteria:

The Aristotelean Criterion interprets an inference (implication) as irrelevant if the conclusion (consequent) contains something "new" which is not contained in (and therefore in a sense not related to) the premises (antecedent). Thus the premises can be "richer" but must be at least as "rich" as the conclusion. On the other hand the Körner Criterion interprets a statement as irrelevant if it has an inessential component in the sense that it does not matter (for the logical content of the statement) if this component is exchanged by its own negation. Both concepts of relevance made precise by our definitions are simple and straightforward ideas. And though they are different they are both plausible.

Although most of the paradoxes which we have discussed are solvable by both relevance criteria A and K, there exist also paradoxes which are solvable only by one of the two. From a more logical point of view, however, A- and K-criteria differ in important respects as is clear from chapters 2.1.2 and 2.2.2. Therefore the two criteria complement each other.

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