NOTES ON SENTENTIAL LOGIC WITH IDENTITY

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The paper is mainly concerned with the Sentential Calculus with Identity (SCI), i.e. the most simplified version of the non-Fregean Logic (NFL). In a wider perspective, it brings together semantical means of doing sentential logic as elaborated by the late Roman Suszko(†), the inventor of NFL. Partly, it may also be conceived as encouragement for a deeper interest to be taken in the non-Fregean approach to logic which, as I feel it, is still underestimated.

1. *Sentential logic*

The sentential logic is primarily a formal study of relations between sentences depending on their form only. Therefore, the main stress is put on sentential connectives. For the sake of simplicity we shall consider an algebra

(1) $\mathscr{L} = (\mathrm{FOR}, F_1, F_2, \ldots, F_n)$

where FOR represents the totality of sentences and $F_1, F_2, \ldots, F_n$ the stock of connectives under which FOR is closed. The formal representation of propositional language provides room for simple or *atomic* sentences usually referred to as sentential variables $p_1, p_2, \ldots, q_1, q_2, \ldots$. It is also assumed that the set $\mathrm{Var}(\mathscr{L})$ of all variables, $\mathrm{Var}(\mathscr{L}) \subseteq \mathrm{FOR}$, freely generates $\mathscr{L}$. Thus, sentential languages are absolutely free algebras in their similarity class.

While interpreting a language a meaning to each sentence φ $r(\varphi)$ is provided which is its *semantic correlate* or its *referent*, cf. Section 2. At this stage two conditions are required from a mapping r of FOR into the range A of all semantic correlates:

(*) With each $\varphi \in \mathrm{FOR}$ exactly one semantic correlate is associated, i.e. r is a function;

(†) 1919 - 1979.

(**) Two sentences $\alpha, \beta \in$ FOR are interchangeable in any sentential context FOR whenever $r(\alpha) = r(\beta)$, i.e. for any $\varphi \in$ FOR, $p \in Var(\mathscr{L})$
$r(\varphi[\alpha/p]) = r(\varphi[\beta/p])$ if and only if $r(\alpha) = r(\beta)$ (1).

Conditions (*), (**) were assumed by Frege [2]. The second will be referred to as the property of *referential extensionality*. When taken together they imply, cf. [8], the following basic semantical property:

(2) When A is the set of all semantic correlates of sentences of the language $\mathscr{L} = (FOR, F_1, F_2, \ldots, F_n)$, then for each $i = 1, \ldots, n$ the formula $\overline{F}_i(r(\alpha_1), \ldots, r(\alpha_{k_i})) = r(F_i(\alpha_1, \ldots, \alpha_{k_i}))$
defines uniquely a function $\overline{F}_i$ on A of the same arity as F_i.

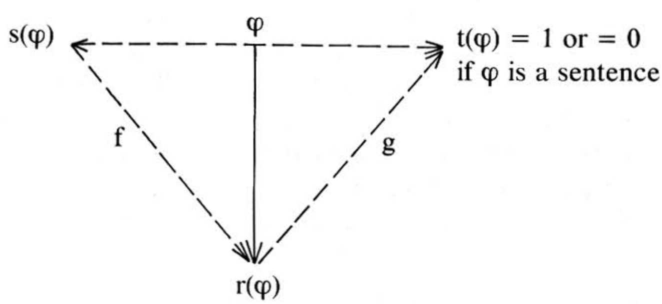
From (2) it immediately follows that each interpretation structure

(3) $\mathscr{A} = (A, \overline{F}_1, \overline{F}_2, \ldots, \overline{F}_n)$

is an algebra similar to $\mathscr{L}$. Algebras of this kind are, somewhat unusually, called *matrices* in [8]. Clearly, homomorphisms from $\mathscr{L}$ into $\mathscr{A}$ are playing the role of *reference assignments* – later on they will be opposed to *logical valuations*.

2. *Fregean Axiom*

Following [2] the Fregean semantic schema can be depicted as follows, cf. [6]:



(1) $\varphi[\alpha/p]$ is the formula which results from φ after substituting α for each occurrence of p.

In the diagram 1 and 0 represent the truth (*das Wahre*) and falsity (*das Falsche*) of sentences, respectively. φ is a name or a sentence and $r(\varphi)$ its semantic correlate – *referent* (i.e. what is given by φ), $s(\varphi)$ – the *sense* of φ (the way in which $r(\varphi)$ is given by φ). And, if φ is a sentence, then $t(\varphi)$ is the logical (*truth*) *value* of φ. These assignments are interrelated as follows:

(4) $s(\varphi) \neq s(\psi)$ whenever $r(\varphi) \neq r(\psi)$

and, for sentences,

(5) $r(\varphi) \neq r(\psi)$ whenever $t(\varphi) \neq t(\psi)$.

In addition, the diagram commutes, since (4) and (5) imply existence of functions f and g such that

(6) $f(s(\varphi)) = r(\varphi)$ and $g(r(\varphi)) = t(\varphi)$.

Being essentially interested in the sentential case we would like, however, to direct the reader's attention to the fact that within the Fregean framework names and sentences are treated in (almost) the same way – all sentences are supposed to be names denoting one of the objects: either 1 or 0. This has found its expression in the so-called *Fregean Axiom*:

(FA) $r(\varphi) = r(\psi)$ whenever $t(\varphi) = t(\psi)$,

(FA) being the converse of (5) together with the latter leads to

(FA′) $r(\varphi) = r(\psi)$ if and only if $t(\varphi) = t(\psi)$,

the condition which says simply that there are exactly two referents for sentences: they may roughly be identified with the *Fregean* 1 and 0.

Suszko baldly rejected (FA) because he did not want to have any restructions concerning either the quantity or the quality of semantic correlates of sentences. And, that was just the turning-point to the construction of the non-Fregean logic (NFL) being a (referentially) extensional paradigm of the classical logic, cf. [6], [5].

What can also be importantly said about Frege is that he did not agree with the converse of (4) thus consenting that for some sentences φ, ψ, it is possible that

(7) $s(\varphi) \neq s(\psi)$ while $r(\varphi) = r(\psi)$,

A nice formal theory of extension and intension extending this idea can be found in [7].

3. *Extensionality versus truth-functionality*

Assuming (FA) we instantly get the following version of (**): For any $\alpha, \beta \in \mathrm{FOR}$

(..) $t(\varphi[\alpha/p]) = t(\varphi[\beta/p])$ for each $\varphi \in \mathrm{FOR}$ and $p \in \mathrm{Var}(\mathscr{L})$ if and only if $t(\alpha) = t(\beta)$.

This is what we call the principle of *truth-functionality*.

If for a sentential language $\mathscr{L} = (\mathrm{FOR}, F_1, F_2, \ldots, F_n)$ (..) is secured, the Fregean assumption of bivalence together with (2) yields that each matrix $\mathscr{A} = (A, \overline{F}_1, \overline{F}_2, \ldots, \overline{F}_n)$ in the sense of Section 1 is isomorphic to a structure $(\{0,1\}, \underline{F}_1, \underline{F}_2, \ldots \underline{F}_n)$such that

(8) $t(F_i(\alpha_1, \ldots, \alpha_{k_i})) = \underline{F}_i(t(\alpha_1), \ldots, t(\alpha_{k_i}))$.

In this case F_i's are *truth-functions* operating on the set of two *logical* values $\{0,1\}$ and each t is an exceptional homomorphism coinciding with a *logical valuation*.

The question on the class of all possible truth functions was answered as early as in 1921 in the classical paper by Emil Post. Here, I am recalling only the conditions for the timely honoured connectives of negation ($\neg$), implication ($\Rightarrow$), disjunction ($\vee$), conjunction ($\wedge$) and material equivalence ($\Leftrightarrow$). A function $t\colon \mathrm{FOR} \to \{0,1\}$ is a classical *truth-valuation* of the language $\mathscr{L} = (\mathrm{FOR}, \neg, \Rightarrow, \vee, \wedge, \Leftrightarrow)$ if and only if for any $\alpha, \beta \in \mathrm{FOR}$:

$$
\begin{array}{llll}
t(\neg\alpha) & = 1 & \text{iff} & t(\alpha) = 0 \\
t(\alpha \Rightarrow \beta) & = 1 & \text{iff} & t(\alpha) = 0 \text{ or } t(\beta) = 1 \\
t(\alpha \vee \beta) & = 1 & \text{iff} & t(\alpha) = 1 \text{ or } t(\beta) = 1 \\
t(\alpha \wedge \beta) & = 1 & \text{iff} & t(\alpha) = 1 \text{ and } t(\beta) = 1 \\
t(\alpha \Leftrightarrow \beta) & = 1 & \text{iff} & t(\alpha) = t(\beta).
\end{array}
$$

If for a sentential language the truth-functionality principle (..) fails, then at least one of its connectives is not truth-functional. Still, however, one may simulate description of the kind described. To this aim, in the language a subset $\mathrm{Ver}(\mathscr{L})$ of true sentences is chosen cf.

[8]. Then, the conditions for $\urcorner$, $\Rightarrow$, $\vee$, $\wedge$ and $\Leftrightarrow$ may be rewritten as below:

$$\begin{array}{rlll}
\urcorner\alpha \in \mathrm{Ver}(\mathscr{L}) & \text{iff} & \alpha \notin \mathrm{Ver}(\mathscr{L}) & \\
\alpha \Rightarrow \beta \in \mathrm{Ver}(\mathscr{L}) & \text{iff} & \alpha \notin \mathrm{Ver}(\mathscr{L}) & \text{or } \beta \in \mathrm{Ver}(\mathscr{L}) \\
\alpha \vee \beta \in \mathrm{Ver}(\mathscr{L}) & \text{iff} & \alpha \in \mathrm{Ver}(\mathscr{L}) & \text{or } \beta \in \mathrm{Ver}(\mathscr{L}) \\
\alpha \wedge \beta \in \mathrm{Ver}(\mathscr{L}) & \text{iff} & \alpha \in \mathrm{Ver}(\mathscr{L}) & \text{and } \beta \in \mathrm{Ver}(\mathscr{L}) \\
\alpha \Leftrightarrow \beta \in \mathrm{Ver}(\mathscr{L}) & \text{iff} & \text{either } \alpha, \beta \in \mathrm{Ver}(\mathscr{L}) & \text{or } \alpha, \beta \notin \mathrm{Ver}(\mathscr{L}).
\end{array}$$

As a consequence of assumption (*) in Section 1 we get that in any semantic interpretation of the language there corresponds to $\mathrm{Ver}(\mathscr{L})$ a subset $D \subseteq A$ of the set A of all semantic correlates. Consequently, to an algebra $\mathscr{A} = (A, \overline{F}_1, \ldots, \overline{F}_n)$ there corresponds a *proper* logical matrix

(9) $\mathfrak{A} = (\mathscr{A}, D)$

(the terminology is, again, borrowed from [8]). Obviously, algebraic functions of possible truth-functional sentential connectives are then satisfying counterparts of the above conditions for $\mathrm{Ver}(\mathscr{L})$. In the matrix $\mathfrak{A}$, the elements of D play the role of *designated elements*.

Without (FA) there is no isomorphism between referents and logical values of sentences. Consequently, none of the truth-functional connectives can express (referential) extensionality. An ingenious construction by Suszko [6] consists in introducing a binary connective of sentential *identity* $\equiv$ characterized by the equality:

(10) $t(\alpha \equiv \beta) = 1$ if and only if $r(\alpha) = r(\beta)$.

Obviously, $\equiv$ is not truth-functional. On the other hand, this connective is (referentially) extensional and due to (**) for any α, β, $\varphi \in \mathrm{FOR}$ and $p \in \mathrm{VAR}(\mathscr{L})$ we have that

(11) $t(\alpha \equiv \beta) = 1$ implies $t(\varphi[\alpha/p] \equiv \varphi[\beta/p]) = 1$

This is a kind of *extensionality principle* expressed in terms of non-truth-functional sentential connective and logical valuations.

4. *SCI and its models*

Suszko rejected (FA) in order to get an unquestionable logic of identity correlated with Leibnitz's ideas. The classical sentential logic extended by the formator of identity characterized by (11) forms the so-called *Sentential Calculus with Identity* (SCI), cf. [6]. So, SCI is formalized in the language

(12) $\mathscr{L} = (\mathrm{FM}, \neg, \Rightarrow, \vee, \wedge, \Leftrightarrow, \equiv)$

containing all previously listed truth-functional connectives and the identity connective.

Let LA = IDA $\cup$ TFA, where TFA is a set of axioms (in the language of SCI) which forms, together with *Modus Ponens*, a complete basis for the classical truth-functional logic, and IDA consists of the following formulas.

(i_1) $\alpha \equiv \alpha$
(i_2) $\alpha \equiv \beta \Rightarrow (\neg\alpha \equiv \neg\beta)$
(i_3) $(\alpha \equiv \beta) \wedge (\gamma \equiv \delta) \Rightarrow [(\alpha \& \gamma) \equiv (\beta \& \delta)$ where & stands for one of the binary connectives $\Rightarrow, \vee, \wedge, \Leftrightarrow$ or $\equiv$
(i_4) $(\alpha \equiv \beta) \Rightarrow (\alpha \Leftrightarrow \beta)$.

The syntactic SCI entailment $\vdash_{\overline{\mathrm{SCI}}}$ is defined as follows:

> $X \vdash_{\overline{\mathrm{SCI}}} \alpha$ if and only if α is derivable by *Modus Ponens* in a finite number of steps from $X \cup \mathrm{LA}$.

Truth-valuational characterization of the SCI-entailment is easy to get. A mapping t: $\mathrm{FM} \rightarrow \{0,1\}$ is an *SCI-valuation* provided that t satisfies familiar conditions for truth-functional connectives (cf. Section 3), $t(\alpha \equiv \alpha) = 1$, t satisfies (11) in the language of SCI and is such that $t(\alpha \equiv \beta)$ implies $t(\alpha) = t(\beta)$. A completeness argument with respect to the class of all SCI-valuations is straightforward.

The characterization of SCI by the use of models given by Bloom [1] not only suits semantical prescriptions of Section 1 as well as their several consequences discussed later but it is also the most informative of all possible approaches as we shall see in the next Section. Actually, an *SCI-model* is a (proper) matrix $\mathfrak{A} = (\mathscr{A}, \mathrm{D})$ consisting of an algebra

(13) $\mathscr{A} = (\mathrm{A}, -, \dot{-}, \cup, \cap, \div, \circ)$

similar to the SCI-language ($\mathrm{A} \neq \emptyset$) and a non-empty set $\mathrm{D} \subseteq \mathrm{A}$ such that D relates to operations in $\mathscr{A}$ as follows: For any a, $\mathrm{b} \in \mathrm{A}$

$$
\begin{array}{llll}
-\, a \in D & \text{iff} & a \notin D & \\
a \div b \notin D & \text{iff} & a \in D & \text{and } -\, b \in D \\
a \cup b \notin D & \text{iff} & a \notin D & \text{and } \; b \notin D \\
a \cap b \in D & \text{iff} & a \in D & \text{and } \; b \in D \\
a \div b \in D & \text{iff} & a \dot{-} b \in D & \text{and } \; b \dot{-} a \in D \\
a \circ b \in D & \text{iff} & a = b. &
\end{array}
$$

The referentially defined entailment $\vdash^{(r)}$ is introduced as follows:

$X \vdash^{(r)} \alpha$ if and only if $X \vdash_{\mathfrak{A}} \alpha$ for every SCI-model $\mathfrak{A}$,

where $X \vdash_{\mathfrak{A}} \alpha$ means that α is satisfied by any valuation over $\mathscr{A}$ which satisfies all formulas in X(2). Obviously, $\vdash_{\overline{SCI}} = \vdash^{(r)}$, cf. [1], [6].

SCI admits a great divergence of models. And, as stated previously, each interpretation (valuation) of the SCI language is a homomorphism of $\mathscr{L}$ into $\mathscr{A}$. Notice that with each such h we may associate a bivalent *logical* valuation $t_h : FM \to \{0,1\}$ so that

$t_h(\alpha) = 1$ if and only af $h\alpha \in D$.

Then, obviously, the truth-functional connectives behave the same with respect to t_h as they did with respect to t in Section 3. This shows how referential assignements are related to logical valuations and how logical *two-valuedness* is opposed to referential *many-valuedness*.

5. *Ontology, modality and SCI*

The non-Fregean logic, of which SCI is the most simplified version, was built as a tool for formalizing an open part of the ontology of *Tractatus Logico-Philosophicus* of Wittgenstein [14], cf. [11] and [12]. Contrary to Fregean view, a distinction was made between names and sentences. Suszko argues in [6] that Fregean scheme applies not only to names and sentences but also to formators of diverse categories and, accordingly, that the famous *Fregean ontological dictum* should sound:

(FOD) everything is either a situation or object or function.

(2) α is satisfied by a valuation h in A provided that h sends α into D (i.e. $h\alpha \in D$).

(FOD) is worded in a terminology elaborated in [11] where referents of sentences and names are called *situations* and *objects*, respectively. Thus one may say that the "non-Fregean logic contains the exact theory of facts i.e. situations described in true sentences or, in other words, situations which obtain. If one accepts the Fregean axiom one is compelled to be an absolute monist in the sense that there exists only one and necessary fact" (p. 218 of [6]). To support this view, one may recall the main ontological thesis of the celebrated *Tractatus* saying that "The real world is a totality of facts and not objects". The crucial argument against (FA) is that the non-Fregean logic suits the ontology of the Tractatus and is the weakest and the most general two-valued logic, cf. also Section 4.

Relations of SCI to other propositional logics are pretty good and they are explaining some important features of the whole logical industry. Firstly, to get the classical sentential calculus it suffices to enrich the set LA of SCI axioms with the formula

(14) $\alpha \Leftrightarrow \beta \ \Rightarrow \alpha \equiv \beta$

or, equivalently, by adding the following *ontological* counterpart of Fregean axiom:

(FA″) $(p \equiv q) \vee (q \equiv r) \vee (p \equiv r)$.

(Then $\Leftrightarrow$ "coincides" with $\equiv$!)(3). On the other hand, SCI proves to be related to some recognized modal and non-classical logics, cf. [9], [10] and [4]. Depending on a case, the comparisons are made syntactically or semantically – each time, however, they show how important role is played by particular invariant *SCI-theories*, i.e. sets of formulas of the SCI-language closed under the SCI-entailment, creating the base for the so-called *axiomatic strengthenings*.

To get a comparison of SCI with modal systems, the formal translation $\alpha \equiv \beta \mapsto \Box(\alpha \Leftrightarrow \beta)$ is employed, cf. [9]. It appears that the two main modal systems S4 and S5 coincide with some invariant theories of SCI: WT and WH, respectively. An axiomatic base for WT is formed by a combination of a group of "logical" axioms for Boolean algebra with $\equiv$ serving as the *equality* (this group contains

(3) More precisely, both the connectives have exactly the same properties, e.g. $\equiv$ is truth-functional, and thus one of them may be excluded from the language.

formulas such as e.g. $((p \wedge q) \vee r) \equiv ((q \vee r) \wedge (p \vee r))$ and $(p \Rightarrow q) \equiv (\neg p \vee q))$ with four formulas for $\Box \alpha =_{df} \alpha \equiv \mathbb{1}$ (with $\mathbb{1}$ representing any classical tautology, say $p \vee \neg p$)(4):

$$\Box(p \Leftrightarrow q) \equiv (p \equiv q)$$
$$\Box(p \wedge q) = (\Box p \wedge \Box q)$$
$$\Box(\Box p \Rightarrow q)$$
$$\Box \Box p \equiv \Box p.$$

WH is received from WF by adding one more axiom:

$$((p \equiv q) \equiv \mathbb{1}) \vee ((p \equiv q) \equiv \mathbb{0})$$

with $\mathbb{0} =_{df} \neg \mathbb{1}$.

In the Introduction to [6], Suszko calls the reader's attention to "somewhat disquieting fact that the strong modal systems ... *are* theories based on an extensional and logically two-valued logic, labelled NFL, exactly in the same sense as axiomatic arithmetic is said to be based on (pure!) logic, created essentially by Frege, hence labelled FL ...". Later on, in Section 11 of the same article it is shown that the square connective $\Box$ *cannot* imitate the metatheoretic predicate of analycity according to the following postulate:

$\Box \alpha$ is true if and only if α is analytic.

6. *Many-valuedness*

SCI-like characterization of many-valued sentential logic was primarily executed on the case of the three-valued logic of Łukasiewicz in [13] and [10], First, the Łukasiewicz matrix $\mathfrak{M}_3 = (\mathscr{A}_3, \{1\})$ with $\mathscr{A}_3$ being an algebra with two operations corresponding to Łukasiewicz negation $\sim$ and implication $\rightarrow$ over the set $\{0,1/2,1\}$ was reproduced as some SCI-model and the set $TAUT_3$ of Łukasiewicz tautologies was shown to be an invariant SCI-theory. As a result, the entailment relation $\vdash_3$:

$$X \vdash_3 \alpha \text{ iff } \forall_{h \in Hom(\mathscr{L}, \mathscr{A}_3)} (h\alpha = 1 \text{ whenever } hX \subseteq \{1\})$$

(4) Obviously, in order to do that we should secure that $p \vee \neg p \equiv q \vee \neg q$ for any p, $q \in Var(\mathscr{L})$. This is actually the case for Boolean SCI-models.

with $TAUT_3 = \{\alpha \in FOR : h\alpha = 1$ for every $h \in Hom(\mathscr{L}, \mathscr{A}_3)\}$ appropriately redefined on the extended propositional language proves to be an axiomatic strengthening of $\vdash_{\overline{SCI}}$, compare p. 348.

The core of the enterprise is the use of the original Łukasiewicz operations $\sim$ and $\rightarrow$ for defining classical connectives (operations): $\urcorner$, $\Rightarrow$, $\vee$, $\wedge$ and $\Leftrightarrow$ over the set $\{0,1/2,1\}$ under the assumption that 1 is the only designated element (the only denotation of true sentences). Recall that (5)

$$\sim a = 1 - a$$
$$a \rightarrow b = \min(1, 1-a+b),$$

cf. [3]. Let us put

$$\urcorner a = a \rightarrow \sim a$$
$$a \Rightarrow b = (a \rightarrow (a \rightarrow b)$$
$$a \vee b = ((a \rightarrow b) \rightarrow b)$$
$$a \wedge b = \sim(\sim a \vee \sim b)$$
$$a \Leftrightarrow b = (a \Rightarrow b) \wedge (b \Rightarrow a)$$
$$a \equiv b = (a \rightarrow b) \wedge (b \rightarrow a).$$

Then, the algebra $\mathscr{A}_3^* = (\{0,1/2,1\}, \urcorner, \Rightarrow, \vee, \wedge, \Leftrightarrow, \equiv)$ is a definitional variant of $\mathscr{A}_3$ and the matrix

$$\mathfrak{M}_3^* = (\mathscr{A}_3^*, \{1\})$$

becomes an SCI-model.

It is worth noticing that in the propositional language to which $\mathfrak{M}_3^*$ corresponds the role of the identity connective is played by the *Łukasiewicz equivalence*, cf. [3]. In turn, when applying the technique of logical valuations mentioned in Section 4, with each homomorphism h: $FOR^* \rightarrow \{0,1/2,1\}$ the valuation t_h: $FOR^* \rightarrow \{\emptyset, \mathbb{1}\}$ (6) is associated so that

$$t_h(\alpha) = \mathbb{1} \text{ iff } h(\alpha) = 1.$$

Then, all considered connectives axcept for $\equiv$ are truth-functional with respect to the set of two logical values $\{\emptyset, \mathbb{1}\}$ (compare Section 4),

To the end, the condition for identity:

$$t_h(\alpha \equiv \beta) = \mathbb{1} \text{ iff } h\alpha = h\beta$$

exemplifies the relation of the realm of Łukasiewicz referents to the universe of two logical values.

Due to what has already been said all the formulas:

$$
\begin{aligned}
\urcorner\alpha &\equiv \alpha \to \sim\alpha \\
\alpha \Rightarrow \beta &\equiv (\alpha \to (\alpha \to \beta)) \\
\alpha \vee \alpha &\equiv ((\alpha \to \beta) \to \beta) \\
\alpha \wedge \beta &\equiv \sim(\sim\alpha \vee \sim\beta) \\
\alpha \Leftrightarrow \beta &\equiv (\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha) \\
\alpha \equiv \beta &\equiv (\alpha \to \beta) \wedge (\beta \to \alpha)
\end{aligned}
$$

belong to $TAUT_3 = \{\alpha : \emptyset \vdash_3 \alpha\}$. On the other hand

$$
\begin{aligned}
\sim\alpha &\equiv (\alpha \equiv \urcorner(\alpha \equiv \alpha)) \\
\alpha \to \beta &\equiv ((\alpha \wedge \beta) \equiv \alpha) \\
\alpha \to \beta &\equiv ((\alpha \vee \beta) \equiv \beta)
\end{aligned}
$$

also are in $TAUT_3$. The latter means that all the properties of Łukasiewicz negation and implication can exhaustively be expressed in terms of the connectives corresponding to the operations of the SCI-model $\mathfrak{M}_3^*$. From that there is only one simple step to be made to come to the conclusion that actually $Ł_3$ is an axiomatic strengthening of SCI, cf. [13].

The reference of the properties of $Ł_3$ just discussed to the general framework presented in the article may show what was distinctly expressed by Suszko in [10] where the first half of the main thesis says that:

> "The construction of so-called many-valued logics by Jan Łukasiewicz was the effective abolition of the Fregean Axiom".

To complete the view it should perhaps be mentioned that the results concerning $Ł_3$ were subsequently generalized onto the case of all finite-valued Łukasiewicz sentential logics in [4]. So, we may maintain that Łukasiewicz was not only the first to abolish (FA) but to construct an infinite family of *non-Fregean* logics as well.

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