NEW SEMANTICS FOR THE LOWER PREDICATE CALCULUS

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Two non-standard semantics, S^p and S^f, are presented in terms of which the lower predicate calculus is proven strongly sound and complete. In S^p monadic predicates are interpreted as set-theoretic elements, while individual terms are interpreted as sets of these elements. In S^f individual terms and monadic predicates are interpreted as sets. In the course of discussing the philosophical motivation for these systems of semantics, formal semantics is distinguished from the theory of reference, the extension and reference of individual terms are distinguished, and analogical relationships between metaphysical systems and systems of formal semantics are introduced. A non-standard treatment of relations is also presented which permits sentences in which polyadic predicates occur to be interpreted without positing semantic values for such predicates.

The lower predicate calculus (LPC) is usually shown complete relative to a set-theoretic semantics in which variables range over members of a domain of entities while monadic predicate constants are assigned subsets of this domain. An atomic sentence is then interpreted as true if and only if the assignment of the individual term is a member of the assignment of the predicate letter. In what follows two semantic systems which depart from this procedure will be presented. In S^p individual terms will be interpreted as sets of entities assigned to monadic predicates. S^f will provide interpretations of the same set-theoretic type for both individual terms and monadic predicates. A non-standard treatment of polyadic predicates will also be introduced which makes possible the interpretation of relational sentences on the same pattern as that of monadic predication.

It will be helpful to begin with Church's distinction between a logistic system and a formalized language. The former is an abstract calculus for which no interpretation is fixed. LPC is a logistic system. A formalized language, according to Church, is a logistic system augmented by *semantic rules* which assign meanings "in some sense" to the well-formed expressions of the system. [1] In what follows it will be important to distinguish the senses in which a logistic system may be provided with semantic rules. In particular, care must be taken to differentiate between a system of formal semantics and a theory of reference. A system of formal semantics for a logistic system provides an interpretation function from the expressions of the logistic system to the components of some other abstract structure, e.g. set theory, algebraic theory. The value of the interpretation function for an expression is the *extension* of the expression. A theory of reference relates linguistic expressions to the objects to which those expressions refer. In what follows, the term "semantics" will be used in the sense of a formal semantic system. Theories of reference will be explicitly designated as such.

The exposition of the formal details of the semantic systems, S^p and S^f, will be preceded by a discussion of three interrelated theses. All three claims concern the relationship between semantics and metaphysics. The significance of the proposed semantic systems, and the motivation for their construction, will be explicated with reference to these three theses.

1) Neither the traditional Tarskian semantics nor either of the proposed systems requires a commitment to an ontology of any given kind of entity. Also, the adoption of a semantics with a certain structure does not commit one to a metaphysics which shares this structure. One may accept Hintikka's game theoretic semantics while denying that the world has the structure of a game. First-order logic, including both syntax and formal semantics, is in this sense metaphysically neutral in content and form.

2) The metaphysical neutrality of the proposed semantics may not be apparent if one assumes that the extension of an individual term is identical with its reference. The relationship between this assumption and Quine's dictum, "To be is to be the value of a bound variable," will be explained below. It will be shown that the assumption of the identity of the reference and extension of individual terms threatens the ontological neutrality of the proposed semantic systems. The adequacy of S^p and S^f with respect to LPC indicates that there is no need to identify extension and reference, and that the assumption that

(1) CHURCH (1951), p. 100.

the identification holds unneccessarily limits the variety of acceptable semantic interpretations of LPC.
3) Although various systems of semantics may be metaphysically neutral, the different structures of these systems may be found to *model* opposing metaphysical claims. Thus, a semantic system may provide an analogue for a certain metaphysical position without committing one to an acceptance of that position. The notion that semantic systems play a role as models for metaphysical claims will be elucidated with respect to the proposed systems.

There are at least two senses in which first-order logic is metaphysically neutral. First, our use of a system of semantics with a certain structure does not commit us to the attribution of that same structure to the world. In Tarski's semantics for first-order logic, individual terms are interpreted as the elements of sets which serve as the values of predicates. This feature of the interpretation function by no means implies the metaphysical claim that the objects over which our variables range are ontologically prior to, or simpler than, those things with which we may associate predicates (e.g. universals). The elements and relations of a given formal semantics need not be held to correspond to existing entities and their relations in the world. The success of a semantic theory depends not upon isomorphism with real world structures, but upon applicability in the world and upon the coherent systematization of restrictions on truth-value claims for various sentences, dependent upon their syntactic structure.

A similar concern to avoid confusion of logical issues with metaphysical issues has been voiced by such philosophers as Carnap (1947)([2]), Cornman (1963), Nagel (1944), and Tarski (1944)([3]). Cornman, for example, urges that logic be distinguished from the theory of reference. He, Carnap and Nagel deny that logic brings with it ontological commitments. Tarski suggests that not only the use of syntactic systems, but the employment of systems of formal semantics as well, do not commit one to any particular metaphysical position. This paper may be read as a further attempt to distinguish formal semantics from metaphysics.

Second, even those who would go so far in linking ontology to

([2]) "Empiricism, Semantics and Ontology," in Carnap (1947), pp. 205-221.
([3]) Cf. especially section 19, "Alleged Metaphysical Element in Semantics."

semantics as to agree with Quine that "to be is to be the value of a bound variable," may do so without deciding whether or not to quantify over spacetime points, material objects, sense data, or some other kind of entity. In this sense also, first-order logic is neutral.

Strictly speaking, then, the standard semantics for LPC is metaphysically neutral. However, this semantics has a certain noteworthy feature in relation to the theory of reference which is customarily assumed in connection with it, viz. there is a fundamental assymetry in its treatment of individual terms and monadic predicates. Individual terms are typically understood in such a way that they form the major link between LPC and its applications to the world. Individual terms are taken to refer to individuals in the domain of discourse. Formal semantics and the theory of reference coincide in their assignments to individual terms. Carnap, for example, offers the definition:

> The *extension of an individual expression* is the individual to which it refers (hence the descriptum, if it is a description). (4)

Predicates, on the other hand, are not understood in such a way that they must be taken to refer at all. The *extension* of a monadic predicate will be a set of individuals, but typically this is not taken to mean that, just as individual terms refer to individuals, predicates refer to sets. The *extension* of a given monadic predicate may be a set of individuals, but reference is another matter. Of course, there is no general agreement among philosophers of language and metaphysicians on the issue of reference, but if one is prepared to accept the claim that individual terms refer, one is faced with a fundamental assymetry in the standard treatment of individual terms and monadic predicates. Individual terms are taken to refer to the objects which serve as their extensions, while monadic predicates either do not refer at all, or, if they do, they do not refer to their extensions.

The identification of the object of reference and the extension of an individual is not in itself objectionable. However, it is not necessary and it should not be made a criterion for the acceptability of a semantic system for two reasons:

1) The requirement that to be acceptable, a semantic system must identify the object of reference of a given term with the extension of

(4) CARNAP (1947), p. 40.

that term needlessly limits the range of acceptable semantic systems. The requirement is unnecessary because alternative semantics may be constructed which fail to identify the object of reference with the extension of an individual term though they provide sound and complete interpretations of first-order logic, with clear conditions for determining the truth values of sentences. To require that the extensions and objects of reference of individual terms be indentified is to violate the metaphysical neutrality of semantic theory in the first of the senses explicated above. By linking semantics and reference with respect to individual terms, the requirement imposes a specific correspondence between the world and semantic theory.

The distinction between reference and extension has already been generally accepted with regard to predicates. At one time predicates were thought to refer to properties. Today predicates are often taken not to refer at all, but in any case, there is no confusion between the reference of a predicate and its extension. Just as progress in semantic theory was made by divorcing reference from extension with respect to predicates, the distinction between reference and extension for individual terms may also aid in the development of new semantic theories. An example may help to clarify the point. One way to look at the formal semantics of LPC is as follows. Consider two syntactic structures, that of LPC and that of Zermelo-Frankel set theory. The semantics for LPC defines a function from terms and formulae of LPC to terms and formulae of ZF. In the standard semantics the individual terms of LPC are mapped onto individual terms of ZF. It is natural to assume that the individual terms of ZF refer to the same objects as those to which the corresponding individual terms of LPC refer. But this is not the only means by which ZF may be utilized to interpret LPC. We may distinguish the set theoretic individuals which serve as the extensions of individual terms from the individual objects to which the individal terms of LPC refer. It is well known, for example, that the natural numbers may be used to construct models for LPC, even where the intended interpretation is not arithmetic. So there is no temptation here to confuse extension and reference. But one might also correlate the individual terms of LPC with sets of numbers, and the predicates with individual numbers. This is like what is done in S^p. The alternative semantic systems proposed here, however, are not merely technical routines for finding unintended interpretations.

These systems provide adequate means for determining the truth values of sentences without interfering with the intended reference of the terms used in these sentences.

2) Questioning the assumption that the extensions of individual terms are identical with their objects of reference throws new light on Quine's dictum of ontological commitment, "To be is to be the value of a bound variable." The dictum is ambiguous. The "value" of a bound variable might be taken to be either its extension, or its reference, and, as we have seen, the two need not be identified. I would suggest that if Quine's dictum is true at all, it is with regard to reference, and not extension, that ontological commitment occurs. The sentence, "Someone is human," comes out true, according to the semantics, because certain conditions are fulfilled by the extension of the predicate, "is human," and the domain from which extensions for individual terms are drawn. Suppose we use sets or numbers as the extensions of individual terms which refer to human beings. Does that mean that we are committed to the existence of sets or numbers when we hold it true that someone is human? It does not if ontological commitments are made with respect to reference but not necessarily with respect to extension. How then are we to understand the formulae of the metalanguage? Sentences within the formal semantics need not be considered to be literally true; rather they are the vehicles by means of which the sentences of the object language are assigned truth value. We might agree with Quine's dictum as a measure of ontological commitment on the object-language level, while denying it on the metalogical level. If I assert, "There is an x such that x is identical to Socrates." I commit myself to the existence of the referent of "Socrates". What the referent of "Socrates" is, is to be determined through an investigation of the use of the term. The fact that the number 470,399, or the set of properties which Socrates exemplifies, serves as the extension of "Socrates" in my favorite semantics does not commit me to the existence of numbers, properties, or sets, and still less does it commit me to the identification of Socrates with the number or set which I use as the extension of his name. The fact that in the standard semantics there is an "intended interpretation" in which the extension of each individual term is identified with its object of reference should not blind us to the fact that for some purposes, systems of semantics might be preferred for

which the intended interpretation does not require the identification of extension and reference for individual terms. We may pursue a formalist interpretation of the study of formal semantics, free from questions of metaphysics.

Although LPC, augmented by its customary formal semantics, is metaphysically neutral, analogies may be drawn between the semantics and more traditional metaphysical theories. For example, if individual terms are taken to represent substances and monadic predicate constants represent properties, the standard semantics for LPC may be said to *illustrate* the view that substances are ontologically prior to their properties. Properties, in this illustration, are like set theoretic constructions out of substances. A system of formal semantics may be said to illustrate or to be analogous to a metaphysical system when elements and relations of the semantics model elements and relations of the metaphysical doctrine. The fact that a system of formal semantics illustrates a metaphysical doctrine suggests that alternative metaphysical doctrines may provide clues for the construction of non-standard systems of formal semantics. Conversely, the consistency of certain metaphysical claims may be demonstrated through their use in models for LPC, and other consistent syntactical structures.

In short, systems of formal semantics are metaphysically neutral, provided that it is not required that extension and reference be identified. Metaphysical bias is introduced into the study of logic when a system of formal semantics and a theory of reference are conflated. Systems of formal semantics may, however, illustrate metaphysical theses without being committed to them. These claims will be reiterated below with respect to the two semantic systems, S^p and S^f.

In S^p each monadic predicate takes as its extension a set theoretic element. The extension of an individual term in S^p is a set of these elements. The extension of an atomic sentence, 'Pa', is truth if and only if the extension of the predicate, 'P', is a member of the extension of the individual term, 'a'. Thus we find in S^p a simple reversal of the standard semantics with regard to the interpretations of individual terms and monadic predicates. Although the extension of an individual term in S^p is a set, this does not mean that those who use S^p must hold that individuals are really sets. There need be no identification of extension with the object of reference. Although we may use individual terms to refer to individual objects, we need not identify these individual objects with the extensions of the individual terms which refer to them.

While S^p is metaphysically neutral, it may be used to *illustrate* the traditional empiricist view of the relation between substances and qualities.

> The idea of a substance... is nothing but a collection of simple ideas, that are united by the imagination, and have a particular name assigned them, by which we are able to recall, either to ourselves or others, that collection. (5)

So writes Hume in Section VI, Part I, Book I of his *Treatise*. In order to serve as a rough analogue to the empiricists' "bundle theory", a semantics may interpret a particular name (individual term) as a collection, instead of as a simple element. The simples of such a semantics will be the properties or qualities which serve as the values of monadic predicates. The domain over which the quantifiers range will be, not the set of all collections of properties, since not every set of properties corresponds to a substance, but instead, those sets of properties which would be "united by the imagination." The domain of discourse will thus be a subset of the power set of the set of properties. Notice that S^p, the property-based semantics, may in this way *illustrate* the "bundle theory" without committing one to this or any other metaphysical doctrine. This point is likely to be obscured by the widespread tendency to assume that the extension of a term is identical with its object of reference.

As will be seen in the proofs below, S^p is not significantly more complicated than the standard semantics. For some purposes S^p may even be preferred. Although this line of investigation will not be pursued here, I suggest that S^p will serve nicely for the interpretation of free logics. Non-denoting singular terms may be interpreted as sets which are not included in the domain of discourse. One of the major problems with fictional entities is their incompleteness. Santa Claus wears a red suit, but the suit is of no particular size. This may be

(5) HUME (1740), p. 16.

reflected in S^p by taking the extension of Santa Claus to be a set which includes the extension of "wears a red suit," but which does not include the extensions of predicates of the form, "wears a red suit of size x."

In S^f both individual terms and monadic predicates take as their extensions sets of set-theoretic elements. The extension of an atomic sentence, 'Pa', is truth if and only if the intersection of the extension of the individual term, 'a', with the extension of the predicate, 'P', is a singleton. In S^f, unlike in the standard semantics, both individual terms and monadic predicates have extensions of the same set-theoretic type. Once again a word of caution is in order concerning reference and extension. If the reference and extension of individual terms are identified, adoption of S^f will bring with it commitment to the unsavory metaphysical position that all objects of reference of individual terms are sets. No such view need be accepted by the proponent of S^f, provided care is taken to distinguish reference from extension. The elements and relations of a system of formal semantics, such as S^f, need not be held to correspond to elements and relations of the actual world. Formal semantics may in this way be treated instrumentally rather than realistically.

The adoption of an instrumentalist attitude toward S^f should not prevent one from appreciating that this formal semantics may be used to illustrate various metaphysical positions. For example, Frank Ramsey(6) has suggested that the difference between universals and particulars is subjective, and that either may be treated as a propositional function. Whether we say "Socrates is wise," or "Wisdom is a characteristic of Socrates," is a matter of style. Both sentences express the same proposition, of which Socrates and Wisdom are components. Ramsey noted that while Russell thought of the universal expressed by "... is wise" as a propositional function which would take particulars as arguments, one might also take "Socrates ..." as a propositional function which would take universals as arguments. This suggests that we might interpret individual terms and monadic predicates as sets of propositions.

Ramsey cites with approval the Wittgensteinian view that the objects of a proposition fit together like the links of a chain. If

(6) Ramsey (1925), pp. 17-39.

individual terms and monadic predicates are interpreted as sets of propositions this linkage could be modelled by the intersection of two sets at a unique proposition, or state of affairs. In the *Tractatus* Wittgenstein writes that "the possibility of the state of affairs must be written into the thing itself." (7) It makes no difference in the structure of the model whether the elements of S^f are taken as propositions or as states of affairs, or facts. The "writing into the thing itself" mentioned by Wittgenstein could be modelled by set membership. Monadic predicates may then be interpreted as sets of facts. For example, "is wise" may be interpreted as a set which contains the facts that Socrates is wise, that Plato is wise, etc. Individual terms may be interpreted in the same way. That is, "Socrates" may be interpreted as a set which includes the facts that Socrates is wise, that Socrates is snubnosed, etc. The link between Socrates and wisdom, which makes the sentence "Socrates is wise" true, is the fact that Socrates is wise.

Interpreting individual terms and monadic predicates as sets of facts is analogous to certain features of Logical Atomism, but it conflicts with some elements of that philosophy as developed by each of its proponents. The point here is merely to indicate certain analogous features which the formal semantics has with some views of the philosophers alluded to above.

S^f may also be used to illustrate a version of the Quinian view that sentences are the primary units of meaning.

> First we learn short sentences, next we get a line on various words through their use in those sentences, and then on that basis we manage to grasp longer sentences in which those same words recur. (8)

Although Quine would frown on a semantic theory which posited facts at its foundation, the fact-based semantics illustrates certain aspects of the Quinian position. Certain patterns of stimulation are associated with sentences containing a certain word, "Fido", for example. The association of the sentences containing "Fido" together with their corresponding stimulations leads to the ability to recognize circums-

(7) WITTGENSTEIN (1921), 2.012, p. 5.
(8) QUINE (1982), p. 3.

tances as being of the Fido type, and hence to the hypostatization of Fido. The same sort of thing might go on in the learning of certain adjectives, and other simple predicates. This may be expressed in terms of the fact-based semantics by letting certain facts be initially included in the interpretation of "Fido", on the basis of which other facts or circumstances may be judged as also belonging to their type.

Acceptance of S^f does not commit one to a form of Logical Atomism, to a Quinian view of language acquisition, or to an ontology of facts. While the semantics is analogous in certain respects to the theories mentioned above, it remains metaphysically neutral. What it requires is no more than a willingness to allow that the extensions of individual terms and the extensions of monadic predicates are sets. The members of these sets may be most naturally understood as propositions, states of affairs, or facts, but there is no more need to accept an ontology of such entities in order to employ S^f, than there is a need to accept an ontology of substances in order to utilize the standard semantics.

Some final comments regarding the treatment of relations in S^p and in S^f are in order. The distinctive features of S^p and S^f concern the interpretation of individual terms and of monadic predicates. There is no natural extension of the non-standard treatment of monadic predicates to relations. Some philosophers may feel that the difference between the interpretations of monadic predicates and relations is awkward. These philosophers see polyadic and monadic predicates as two species of the same genus, all of whose members deserve uniform treatment. This view is questionable. However, in order to accommodate those who wish a uniform treatment of all atomic sentences, regardless of whether they contain monadic or polyadic predicates, a non-standard treatment of relations will be introduced.

The basic idea is to provide no direct interpretation for polyadic predicates at all. Sentences containing polyadic predicates are to be interpreted by assigning values to strings consisting of an n-place predicate followed by $n-1$ individual terms and a place holder. These strings may then be interpreted in the same manner as are monadic predicates. Both this non-standard treatment of relations, as well as the customary practice of assigning to each n-place relation a set of n-tuples could be given for S^p, for S^f, or for the standard semantics. In the formal work to follow the non-standard treatment of relations is incorporated into the presentation of S^f, while relations are given the customary treatment in the presentation of S^p.

In what follows it is shown how, for any model of the standard semantics given for LPC, S^s, a model of S^p (S^f) can be constructed which preserves the truth assignments of the standard model. From this it follows that if there is no S^p (S^f) model for G+{-A}, where G is any set of wffs, there is no standard model for G+{-A}. By the maximal consistency of the set of sentences true in a model, then $G \models^p A \Rightarrow G \models^s A$ ($G \models^f A \Rightarrow G \models^s A$). The strong completeness of LPC relative to the non-standard semantics is a corallary of this result. Since LPC is strongly complete relative to the standard semantics, i.e. $G \models^s A \Rightarrow G \vdash A$, it follows that $G \models^p A \Rightarrow G \vdash A$, ($G \models^f A \Rightarrow G \vdash A$). The converse is also demonstrated. For any non-standard model, there is a standard one which preserves truth assignments. Hence, $G \models^s A \Rightarrow G \models^p A$ ($G \models^s A \Rightarrow G \models^f A$). Since LPC is valid, or sound, relative to the standard semantics, LPC is valid relative to the non-standard semantics. Hence, $G \models^p A \Leftrightarrow G \models^f A \Leftrightarrow G \vdash A$. Note that these proofs occur entirely on the semantic level; no essential use is made of provability results.

The Property-Based Semantics: S^p

An S^p model is a pair <**P**,J> where **P** is a non-empty set whose members will serve as the extensions of monadic predicates. The members of **P** will be called properties in order to emphasize the analogy between the formal structure of S^p and the bundle theory. This is just an analogy. It should be recalled that the use of S^p need not commit one to the existence of universals, nor to anything else. J is an interpretation function defined as follows:

1. J(**P**) is a subset of the power set of **P**, minus the null set, and -(J(**P**) = { }). J(**P**) is the domain of discourse for <**P**,J>. The empty domain is not countenanced. The domain of discourse is a non-empty subset of the set of all the non-empty sets of properties. The extensions of individual terms are sets of properties which are members of this domain, over which the quantifiers range.
2. If a is an individual term, i.e. an individual constant or an individual variable, J(a) ∈ J(**P**).

3. If P is a monadic predicate, J(P) $\in$**P**.
4. If P is an n-place predicate, n > 1, J(P) $\subseteq$ (J(**P**))^n, that is, J(P) will be a set of n-tuples of members of the domain of discourse, just as in canonical models. Relations are given a standard interpretation.
5. If A is a wff of the form Pa, so that P is a monadic predicate, J(A) = T iff J(P) $\in$ J(a). Monadic predication is true iff the property for which the predicate stands is a member of the set of properties which is the interpretation of the individual term.
6. If A is a wff of the form $Pa_1 \ldots a_n$, n > 1, then J(A) = T iff $<J(a_1), \ldots, J(a_n)> \in J(P)$.
7. If A is a wff of the form (x)B, J(A) = T iff for each *d* which is a member of J(**P**), Jx/*d*(B) = T, where Jx/*d* is the interpretation which differs from J only by assigning *d* to x.
8. If A is a wff of the form -B, J(A) = T iff -(J(B) = T).
9. If A is a wff of the form (B$\rightarrow$C), J(A) = T iff -(J(B) = T) or J(C) = T.
10. If A is a wff and -(J(A) = T) by clauses 1 through 9, J(A) = False.

S^p Completeness

In order to prove the completeness of LPC relative to S^p it will be shown how for any S^s model to construct an S^p model in which exactly the same wffs are true. Begin with an S^s model <**D**,I> where **D** is the domain of quantification and I is the standard interpretation function. The first step in the construction of an S^p model <**P**,J> corresponding to <**D**,I> is to find a set of properties corresponding to each member of **D**. It is not sufficient to look merely at monadic predicates in the search for the appropriate properties, since the same set of monadic predicates may apply to different entities. Even the inclusion of properties designated by relational predicates will not provide sufficient distinctions among entities. An example will help to clarify the point.

Example 1: Suppose that <**D**,I> and <**D**′,I′> are two standard models. **D** = $\{d_1, d_2\}$. **D**′ = $\{d_1, d_2, d_3\}$. For all individual terms a, I(a) = I′(a) = d_1. $I(P^2) = \{<d_1,d_1>, <d_2,d_2>\}$, while $I'(P^2) = \{<d_1,d_1>, <d_2,d_2>, <d_3,d_3>\}$. For all n-place predicates P other than P^2, let I(P) = I′(P) = { }. In <**D**′, I′> d_2 and d_3 are indiscernible in the following sense: for every individual term a and wff A which contains an occurence of a, $I'a/d_2(A) = I'a/d_3(A)$. In both <**D**, I> and <**D**′, I′> this wff is true: $(x)(P^2xx)$, but with respect to the sentence, (1), the models differ. In <**D**, I>, (1) is false, while it is true in <**D**′, I′>.

1) $(Ex)(Ey)(Ez)(-P^2xy \,\&\, -P^2xz \,\&\, -P^2yz)$.

The construction of an S^p model for which the same wffs are true as are true for <**D**′, I′> will have to distinguish between d_2 and d_3, even though d_2 and d_3 are indiscernible relative to <**D**′, I′>, in the sense given above. This means that among the properties associated with d_2 and d_3 must be found at least one for which no predicate or sentence abstract stands, by means of which d_2 and d_3 may be differentiated. The analogue of this point in metaphysics is that the bundle of properties associated with Socrates, for instance, should include not only properties which Socrates shares with other things, but a property unique to Socrates, Socrateity, even if there is no word for such a property.

In order to construct an S^p model <**P**, J> for which the same wffs are true as are true for an S^s model <**D**, I>, the notion of a property will first be introduced. If P is a monadic predicate, P will be called a property. If $d \in \mathbf{D}$, d will also be called a property; more specifically, d may be called an haecceity, although such language will not be used in the completeness proof. Since relations receive the standard treatment here, there is no need to include relational properties among the properties.

Let **P** be the set of all properties.

J(**P**) will be the domain of discourse, and will be constructed by finding sets of properties which correspond to each of the members of **D**. If $d \in \mathbf{D}$, d^p will be called the property-set which corresponds to d, and will be defined as follows:

a) If A is of the form Pa, P is a monadic predicate, and $Ia/d(A) = T$, let P be a member of d^p.
b) If $d \in \mathbf{D}$, let d be a member of d^p.
c) Let nothing be a member of d^p except by (a) and (b).

1′. Let J(**P**) be the set of all and only such d^p as specified in (a), (b) and (c), that is, J(**P**) is the set of all and only those property-sets which

correspond to the members of **D**. Note that the correspondence between **D** and J(**P**) is one to one.
2′. If a is an individual term and I(a) = d, let J(a) = d^{p}.
3′. If P is a monadic predicate, J(P) = P.
4′. If P is an n-adic predicate, n > 1, and I(P) is a set of n-tuples such that $<d_1, \ldots, d_n> \in$ I(P), let J(P) be a set of n-tuples such that $<d^{\mathrm{p}}_1, \ldots, d^{\mathrm{p}}_n> \in$ J(P).

This completes the definition of the model <**P**, J>.

In order to prove that all and only those wffs which are true for <**D**, I> are true for <**P**, J>, it is first demonstrated that for any I′ and J′ which are like I and J with the possible exception of the assignments made to the individual terms, such that I′(a) = d iff J′(a) = d^{p}, for all wffs A, I′(A) = J′(A). From this it follows that I(A) = J(A). The proof is by induction on the complexity of wffs. Degree of complexity is to be understood in the usual manner. The inductive hypothesis is that for all such I′ and J′, for every wff B, whose degree of complexity is less than n, I′(B) = J′(B). On the basis of this it will be found that for all such I′ and J′, for any wff A whose complexity is of degree n, I′(A) = J′(A).
5′. If A is a wff of the form Pa, I′(A) = T iff I′(a) ∈ I(P). (Since I(P) = I′(P).) By (1′), (2′), (3′), and the specification of the d^{p} by clauses (a), (b) and (c) above, I′(a) ∈ I(P) iff J(P) ∈ J′(a), iff J′(A) = T.
6′. If A is of the form $\mathrm{Pa}_1 \ldots \mathrm{a}_n$, I′(A) = T iff $<d_1, \ldots, d_n> \in$ I(P), where for all i, $1 \leq \mathrm{i} \leq \mathrm{n}$, $\mathrm{I}'(\mathrm{a_i}) = d_{\mathrm{i}}$, iff $<d^{\mathrm{p}}_1, \ldots, d^{\mathrm{p}}_n> \in$ J(P), by (2′), (4′), (a), (b) and (c), iff J′(A) = T.
7′. If A is a wff of the form (x)B, I′(A) = J′(A) iff for each d which is a member of **D**, and for each d^{p} which is a member of J(**P**), I′x/d(B) = J′x/d^{p}(B). This is provided for by the inductive hypothesis since I′x/d and J′x/d^{p} differ from I and J only in virtue of the assignments made to individual terms.
8′. If A is a wff of the form -B, I′(A) = J′(A) iff I′(B) = J′(B) which follows from the inductive hypothesis.
9′. If A is a wff of the form (B → C), I′(A) = T iff -(I′(B) = T) or I′(C) = T, by induction iff -(J′(B) = T) or J′(C) = T, iff J′(A) = T.

It is thus established that for all wffs A, I′(A) = J′(A), and hence that for any standard model, <**D**, I>, there is a property based model, <**P**, J>, such that for any wff A, I(A) = J(A).

The strong completeness of LPC relative to S^p follows from the above proof. If G is a set of wffs and G+ {-A} is consistent, then by the strong completeness of LPC with respect to S^s, G+ {-A} has a standard model; so by the above proof, G+ {-A} has an S^p model. So, by contraposition, if $G \models^p A$, then $G \vdash A$.

Note that for the weak completeness of LPC relative to S^p it need only be found that for any wff A, iff A is true in a standard model, it is true in an S^p model. To establish this, there is no need to include haecceities among the properties, that is, there is no need to include the members of **D** in **P**. If A is a wff and <**D**, I> is a standard model with some indiscernible members of **D** (that is, **D** contains members, d and d', such that for every wff A, $Ia/d(A) = Ia/d'(A)$), then an S^p model, <**P**, J>, may be constructed such that I(A) = J(A), by defining <**P**, J> as above in (a), (b), (c) and (1′) through (4′) except that (b) should be replaced by:

> b′) For each d which is a member of **D**, take a different monadic predicate P, such that P does not occur in A, and let P be a member of d^p.

With suitable adjustments in the proof given it should not be difficult to show by induction on the complexity of A, that for any wff A, if there is a standard model for which A is true, there is a property-based model for which A is also true.

Strong completeness cannot be established by this method. Where G is an infinite set of wffs there is no appropriate adjustment of (b′) which would insure that G has a model with no indiscernible members. The compactness theorem for LPC states that every finite subset of G has a model iff G has a model, but this result does not extend to models which exclude indiscernibles. That is, it does not follow from the fact that every finite subset of G has a model which contains no indiscernible members that G has a model which contains no indiscernible members. This point may be shown with reference to Example 1, given above. Suppose G is the set of wffs which are true on model <**D**′, I′> of the example. Then (1) is a member of G. Any finite subset of G will be true in a model which distinguishes d_2 and d_3 by means of some predicate which does not occur in that particular subset of G. But where G is taken in its infinite entirety, there are no predicates or terms by means of which the indiscernible members of **D**′ may be distinguished.

S^p Validity

The proof to follow will demonstrate that for each S^p model, there is an S^s model for which exactly the same wffs are true. Begin with an S^p model, <**P**, J>. A corresponding standard model <**D**, I> may be defined as follows:

1″. Let $\mathbf{D} = J(\mathbf{P})$.
2″. For each individual term a, let I(a) = J(a).
3″. If P is a monadic predicate, let $I(P) = \{d : d \in \mathbf{D} \ \& \ J(P) \in d\}$.
4″. If P is an n-adic predicate, $n > 1$, let I(P) = J(P).

In order to show that I(A) = J(A) for all wffs A, it is first demonstrated that for all I′ and J′ which are like I and J with the possible exception of the assignments made to the individual terms, I′(A) = J′(A). The proof is by induction on the complexity of A. Assume for induction that I′(B) = J′(B) for all I′ and J′ where B is less complex than A.

5″. If A is a wff of the form Pa, J′(A) = T iff $J(P) \in J'(a)$, by (1″), (2″) and (3″), iff $I'(a) \in I(P)$, iff I′(A) = T.
6″. If A is a wff of the form $Pa_1 \ldots a_n$ by (2″) and (4″), I′(A) = J′(A).
7″. If A is a wff of the form (x)B, by the inductive hypothesis, (1″) and (2″), for all members of **D**, *d*, $I'a/d(B) = J'a/d(B)$, so I′(A) = J′(A).
8″-9″. These cases follow from the inductive hypothesis.

Together with the previous result of the strong completeness of LPC relative to S^p, it is established that for any set of wffs G′, G′ has a property-based model if and only if it has a standard model, and hence LPC is strongly complete and sound relative to S^p,

$$G \models^p A \Leftrightarrow G \models^s A \Leftrightarrow G \vdash A.$$

The Fact-Based Semantics: S^f

An S^f model is a pair <**F**, H> where **F** is a nonempty set whose members are called facts, and H is an interpretation function. The definition of H will employ an undefined metalinguistic symbol, "*", which may be called a plug. The introduction of the plug and certain other peculiarities of H pertain to the treatment of relations which is presented here. This treatment of relations is *not* an essential part of the fact-based semantics. Relations could be treated here as in the standard semantics, as was done for the property-based semantics. Also, the treatment of relations could be incorporated in an otherwise standard semantics, or in a property-based semantics. The idea behind interpreting relations with a plug is that relational predicates are not directly interpreted at all. Instead, n-place relations, followed by a sequence which includes a plug among n−1 individual terms are to be interpeted in the same way that monadic predicates are interpreted. H will be a function which takes as arguments: **F**, individual terms, wffs, but instead of predicates, H will interpret n-place predicates followed by a sequence including a plug among n−1 individual terms; these sequences will be called "plugged predicates".

The device of "plugging up" relations is utilized in Parsons (1980) both syntactically and semantically, although for different ends than those which constitute the aim of his exercise.

Since plugged predicates will be interpreted which contain individual variables, the treatment of quantification will not be as straight forward as is usual. It will not do, for instance, to say that a wff such as (x)Rxx is true in model M iff for every *d* which is a member of H(**F**), Hx/*d*(Rxx) = T. This will not do because as the value of x changes from model to model, the values which are assigned to the plugged predicates R*x and Rx* must also be made to change accordingly. It should not be required that for (x)Rxx to be true, Rxx must be true no matter how R*x and Rx* are interpreted. What is needed is a set of models which are such that no two models in the set will differ in their interpretations of plugged predicates containing certain variables, yet give the same values to these variables themselves. The term "model structure" will be used (somewhat unconventionally) for the appropriate sets of models.

A fact-based model structure, μ, is a set of models such that where <**F**, H> and <**F**′, H′> are both members of μ, the following conditions are satisfied:

i) $F = F'$
ii) $H(\mathbf{F}) = H'(\mathbf{F}')$
iii) if β is a plugged predicate; and if for all individual variables x wich occur in β, $H(x) = H'(x)$, then $H(\beta) = H'(\beta)$.
iv) for each sequence of individual variables, $<x_1, \ldots, x_n>$, and each sequence of members of $H(\mathbf{F})$, $<d_1, \ldots, d_n>$, there is a model $<\mathbf{F}, H''>$ which is a member of μ, such that for all i, $1 \leq i \leq n$, $H''(x_i) = d_i$.

All and only those sets of models of the fact-based semantics which satisfy the above conditions are fact-based model structures.

Models may be defined independently of model structures for nonsentential arguments. The values of wffs will be relative to a model structure. Note that model structures are defined as sets of models which have certain features regarding their interpretations of nonsentential arguments.

Where μ is a fact-based model structure, an S^f model in μ is a pair, $<\mathbf{F}, H>$, where **F** is a nonempty set whose members are called facts, and H is defined as follows:

1. $H(\mathbf{F})$ is a nonempty subset of the set of all nonempty subsets of **F**. $H(\mathbf{F})$ is the domain of discourse for $<\mathbf{F}, H>$. The individuals over which the quantifiers range are represented as sets of facts.
2. If a is an individual term $H(a) \in H(\mathbf{F})$.
3. If P is an n-place predicate and $a_1, \ldots, a_n$ are individual terms, then $H(Pa_1 \ldots a_{i-1} {}^*a_{i+1} \ldots a_n) \subseteq \mathbf{F}$. Plugged predicates are interpreted, like individual terms, as sets of facts. Suppose, for example, that P is a monadic predicate. Then technically P goes uninterpreted, but the interpretation of P* is a set of facts.
4. If A is a wff of the form $Pa_1 \ldots a_n$, $H(A) = T$ iff for each i, $1 \leq i \leq n$, there is an f which is a member of **F** such that the intersection of $H(a_i)$ with $H(Pa_1 \ldots a_{i-1} {}^*a_{i+1} \ldots a_n)$ is $\{f\}$. A couple examples will help to make this clear. It is true that Socrates is human iff the set of facts associated by H with "Socrates" and the set of facts associated with "* is human" have exactly one member in common. It is true that Socrates is the teacher of Plato iff there is exactly one fact which Socrates has in common with the property of being a teacher of Plato, and there is exactly one fact which Plato has in common with the property of being one of whom Socrates is the teacher.

5. If A is a wff of the form (x)B, H(A) = T iff for all members of H(**F**), d, Ha/d(B) = T, where Ha/d is here and in what follows an interpretation like H in that $\langle \mathbf{F}, \mathrm{Ha}/d \rangle \in \mu$, and if a is an individual term other than x, Ha/d(a) = H(a), but Ha/d(a) = d.
6. If A is a wff of the form -B, H(A) = T iff -(H(B) = T).
7. If A is a wff of the form $(B \rightarrow C)$, H(A) = T iff -(H(B) = T) or H(C) = T.
8. If A is a wff and -(H(A) = T) by 1-7, H(A) = False.

S^f Completeness

The strong completeness of LPC relative to S^f will be established through the intermediary of the standard semantics by showing that for any set of wffs G, if there is a standard model for G, there is an S^f model structure, μ, which contains a model for G.

Begin with an S^s model $\langle \mathbf{D}, \mathrm{I} \rangle$. If $\beta \subseteq \mathbf{D}$ and $d \in \beta$, call $\langle d, \beta \rangle$ a fact. If for all i, $1 \leq i \leq n$, $d_i \in \mathbf{D}$, and $\langle d_1, \ldots, d_n \rangle \in \beta$ where $\beta \subseteq \mathbf{D}^n$, call the triple $\langle d_i, \langle d_1, \ldots, d_n \rangle, \beta \rangle$ a fact. An S^f model $\langle \mathbf{F}, \mathrm{H} \rangle$, corresponding to $\langle \mathbf{D}, \mathrm{I} \rangle$ may be defined as follows. Let **F** be the set of all facts. For each d_i which is a member of **D**, let $d^f{}_i$ be the set of all facts whose first member is d_i. There is thus a one-one correspondence between the members of **D** and the set of all d^f.

1′. Let H(**F**) be the set of all and only those d^f constructed as specified above. Since **D** is not empty and for each d which is a member of **D** there is a β such that $d \in \beta$, H(**F**) is nonempty and H(**F**) is a subset of the nonempty subsets of **F**.
2′. If a is an individual term and I(a) = d, let H(a) = d^f, where d^f is as specified above.
3′a. If P is a monadic predicate, let $\mathrm{H}(\mathrm{P}^*) = \{\langle d, \beta \rangle : d \in \mathbf{D} \ \& \ d \in \beta \ \& \ \beta = \mathrm{I}(\mathrm{P})\}$.
3′b. If P is an n-adic predicate and $a_1, \ldots, a_n$ are individual terms, let $\mathrm{H}(\mathrm{P}a_1 \ldots a_{i-1} {*} a_{i+1} \ldots a_n) = \{\langle d_i, \langle d_1, \ldots, d_n \rangle, \beta \rangle :$ for all j, $1 \leq j \leq n$, if $-(i = j)$, $d_j = \mathrm{I}(a_j)$, $d_i \in \mathbf{D} \ \& \ \mathrm{I}(\mathrm{P}) = \beta \ \& \ \langle d_1, \ldots, d_n \rangle \in \beta\}$. So each n-adic predicate followed by a plug among $n-1$ individual terms is correlated with a set of facts.

Let μ be the set of all S^f models $\langle \mathbf{F}, \mathrm{H}' \rangle$ such that H′ differs from H at most with regard to the values it assigns to the individual variables, plugged predicates, and wffs in which there is some occurrence of individual variables, as follows: if $H'(a_i) = d^f{}_i$, for all i, and H′ differs from H with regard to the assignments it makes to some variables which occur in sequences of the form $Pa_1 \ldots a_{i-1} {*} a_{i+1} \ldots a_n$, let $H'(Pa_1 \ldots a_{i-1} {*} a_{i+1} \ldots a_n)$ be the set of all triples, $\langle d_i, \langle d_1, \ldots, d_n \rangle, \beta \rangle$, where $1 \leq i \leq n$, $\langle d_1, \ldots, d_n \rangle \in \beta$, and $\beta = I(P)$.

In order to prove that $I(A) = H(A)$, it is shown that for all I′ which are like I with the possible exception of the assignments made to individual terms, and for all H′ such that H and H′ are members of μ, and I′ and H′ correspond in their assignments in the manner indicated above, for all wffs A, $I'(A) = H'(A)$. The proof is by induction on the complexity of A. Assume that for all wffs B which are not as complex as A, $I'(B) = H'(B)$.

4′a. If A is a wff of the form Pa, and $I'(A) = T$, then there is a d which is a member of **D** such that $I'(a) = d$ and $d \in I(P)$. If $I'(a) = d$, and $I(P) = \beta$, $\langle d, \beta \rangle \in H'(a)$ and $\langle d, \beta \rangle \in H'(P^*)$, by (2′) and (3′a). Since only pairs with first member d are members of $H'(a)$, and only pairs with second member β are members of $H'(P^*)$, $\langle d, \beta \rangle$ is the one and only member which $H'(a)$ has in common with $H'(P^*)$, so $H'(A) = T$. If $I'(A) =$ False, then $I'(a)$ is not a member of $I(P)$, so $\langle I'(a), I(P) \rangle$ is not a fact, so $H'(A) =$ False.

4′b. If A is a wff of the form $Pa_1 \ldots a_n$, and $I'(A) = T$, then $\langle I'(a_1), \ldots, I'(a_n) \rangle \in I(P)$. Then for all i, $1 \leq i \leq n$, if $f = \langle I'(a_i), \langle I'(a_1), \ldots, I'(a_n) \rangle, I(P) \rangle$, f is a fact and by (2′) and (3′b), $f \in H'(a_i)$ and $f \in H'(Pa_1 \ldots a_{i-1} {*} a_{i+1} \ldots a_n)$. For any f' which is a member of $H'(Pa_1 \ldots a_{i-1} {*} a_{i+1} \ldots a_n)$, f' is a triple, $\langle d_i, \langle I'(a_1), \ldots, I'(a_{i-1}), d_i, I'(a_{i+1}), \ldots, I(a_n) \rangle, I(P) \rangle$, and if $f' \in H'(a_i)$, $I'(a_i)$ is the first member of f', so $f' = f$. If $I'(A) =$ False, $\langle I'(a_1), \ldots, I(a_n) \rangle$ is not a member of $I(P)$, so f is not a fact, so $H'(A) =$ False.

5′. If A is a wff of the form (x)B, $I'(A) = T$ iff for all d of **D**, $I'(B) = T$, by the inductive hypothesis, iff $H'a/d^f(B) = T$ for all d^f of H(**F**), by (1′) and the specification of μ, iff $H'(A) = T$.

6′-7′. The cases for negation and the conditional are trivial.

This completes the proof from which it follows that LPC is strongly complete relative to S^f, $G \models^f A \Rightarrow G \vdash A$.

S^f Validity

To show that LPC is sound relative to S^f it will be shown that for each S^f model structure μ, if $<\mathbf{F}, H> \in \mu$, there is an S^s model, $<\mathbf{D}, I>$, such that for any wff A, I(A) = H(A). Begin with an S^f model structure, μ, and a model in this structure $<\mathbf{F}, H>$. A corresponding standard model, $<\mathbf{D}, I>$ may be defined as follows:

1″. Let $\mathbf{D} = H(\mathbf{F})$.
2″. If a is an individual term let I(a) = H(a).
3″a. If P is a monadic predicate let I(P) be the set of all d of $\mathbf{D}$ such that the intersection of H(P*) with d is a singleton.
3″b. If P is an n-adic predicate, $n \geq 2$, let I(P) be the set of n-tuples $<d, \ldots, d_n>$, where $d_i \in \mathbf{D}$, $1 \leq i \leq n$, such that there is a model $<\mathbf{F}, H'>$ in μ, and a sequence of individual terms $<a_1, \ldots, a_n>$, such that for each i, $1 \leq i \leq n$, there is a fact, f of $\mathbf{F}$, and $H'(Pa_1 \ldots a_{i-1} {}^*a_{i+1} \ldots a_n)$ and d_i have f as their sole common member.

In order to show that for all A, I(A) = H(A), it is shown first that for all H′ of μ and I′ like I with the possible exception of the assignments made to individual terms, where for all individual terms I′(a) = H′(a), I′(A) = H′(A). The proof is by induction on the complexity of A. It is assumed that the theorem holds for all wffs of complexity less than that of A.

4″a. If A is a wff of the form Pa and H′(A) = T, then the intersection of H′(P*) with H′(a) is $\{f\}$, where $f \in \mathbf{F}$. By (1″) and (3″a), $I'(a) \in I'(P)$, so I′(A) = T. If H′(A) = False, the intersection of H′(P*) with H′(a) is not a singleton, so I′(a) is not a member of I(P), so I′(A) = False.
4″b. If A is a wff of the form $Pa_1 \ldots a_n$ and H′(A) = T, then for each i, $1 \leq i \leq n$, the intersection of $H'(a_i)$ with $H'(Pa_1 \ldots a_{i-1} {}^*a_{i+1} \ldots a_n)$ is $\{f\}$ for some f of $\mathbf{F}$. Hence $<I'(a_1), \ldots, I'(a_n)> \in I(P)$ by (1″), (2″), and (3″b), so I′(A) = T. If H′(A) = False, then there is no appropriate f of $\mathbf{F}$, so by (3″b), I′(A) = False.
5″. If A is of the form (x)B, H′(A) = T iff for all d of H(**F**), $H'a/d(B) = T$, by (1″) and the inductive hypothesis iff $I'a/d(B) = T$, for all d of **D**, iff I(A) = T.
6″-7″. The clauses for negation and the conditional are trivial.

This completes the proof of the strong validity of LPC relative to S^f. LPC is strongly complete and sound relative to both S^p and S^f,

$$G \models^f A \Leftrightarrow G \models^p A \Leftrightarrow G \models^s A \Leftrightarrow G \vdash A.$$

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