DATA SEMANTICS FOR ATTITUDE REPORTS*

Fred LANDMAN

1. *Direct and indirect perception*

Suppose you are sitting in your room, reading a book. A friend comes in and says: 'I just heard on the radio that a famous operasinger lost her voice.' Compare this situation with the following one: again you are sitting in your room, reading a book. A friend comes in and says: 'I just heard on the radio a famous operasinger lose her voice.' There seems to be quite a difference between the two situations.

A rather straightforward explanation for this difference runs along the following lines: in uttering the second sentence your friend reports that she has a direct relation to what happened: she actually heard the painful silence! With the first sentence she describes an indirect relation to the events in the concert-hall: perhaps she has only heard the newsreader and inferred from his words what happened.

A semantic analysis of perception verbs based on this distinction was presented by Jon Barwise and John Perry in a semantic framework called Situation Semantics(1). I will consider their analysis in some detail (section 2), formulate some criticism (section 3) and propose a different analysis (sections 4, 5, 6).

2. *Situation Semantics*

In Situation Semantics: 'Situations are basic and ubiquitous. We

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(1) BARWISE, J. and J. PERRY, 1981, 'Situations and Attitudes', in *The Journal of Philosophy* Vol. LXXVIII, 11, p. 668-691.

are always in some situation or other. Human cognitive activity categorizes these situations in terms of objects having attributes and standing in relations to one another at locations – connected regions of space-time.'(2) A situation is a partial specification of what the world is: it specifies what is true, false or not yet decided on the basis of some part of the world.(3) A situation is characterized by its location and by its type.

(1) A *situation type* is a partial function from (primitive) objects and (primitive) relations between objects to truth values 0 and 1.(4)

If S is the set of all situation types, and B the domain of S and $\mathfrak{L}$ a propositional language (a language based on propositional symbols, negation $\neg$, conjunction $\wedge$, and disjunction $\vee$) then the notions 'true on the basis of situation type s' and 'false on the basis of situation type s' can be defined in two steps: first an interpretation for $\mathfrak{L}$ is given; this is a function *i* which assigns to every propositional symbol of $\mathfrak{L}$ an element of B, the domain of S.

Secondly 'truth (falsity) on the basis of s(rel *i*)' is defined recursively. Here several alternatives are possible. They all share the following basic step:

(2) If p is a propositional symbol then:
p *is true on the basis of* s(rel *i*) iff s(*i*(p)) = 1
p *is false on the basis of* s(rel *i*) iff s(*i*(p)) = 0

I will not go into clauses for complex formulas here.

As said above, a situation is characterized by a location and its

(2) BARWISE and PERRY, op. cit. p. 668.

(3) Barwise and Perry often use the terminology 'true in a situation'. I prefer 'true on the basis of a situation'. The reason for this is that 'true in' might lead to some confusion. For instance, if it is true in our world that Russia has a partyleader then one could argue that it is also true in the situation you are in now, because that is just part of the world. However, it is not true *on the basis of* that situation (but rather undetermined by it, because it is not related to it at all).

(4) So the domain of these functions is the set $R_1 x D \cup R_2 x D^2 \cup \ldots \cup R_n x D^n, \ldots$, where D is the set of objects, and R_n the set of n-place relations. An element of this domain is an n+1-tuple $<r_n, d_1, \ldots, d_n>$. If one calls such a tuple a fact (in line with Russell's *Philosophy of Logical Atomism*) then a situation type can be regarded as a special set of positive and negative facts. Cf. Marsh, R. (ed.), 1956 *Bertrand Russell, Logic and Knowledge*, Allen and Unwin, London, p. 175-281.

type. If L is the set of space-time locations, and S is the set of situation types, then the set of all situations $\mathbb{S} = L \times S$.

(3) A *situation* is a pair $\langle l,s \rangle$.

The type s indicates what holds objectively of the situation. The location l functions as a kind of perspective: what is true on the basis of s can be obscured by the conditions of the light, etc. according to l, i.e. l can cover up certain things that hold on the basis of s. We can express this by associating with l a function p_l, the perspective according to l, which fulfils this covering function:

(4) p_l is a function that maps every situation type s onto some situation type s′ such that $s' \subseteq s$.

With this, truth/falsity on the basis of *situation* $\not{s}$ can be defined as follows: Let $\not{s} = \langle l,s \rangle$:

(5) ϕ is true on the basis of $\not{s}$ *iff* ϕ is true on the basis of (type) $p_l(s)$.
ϕ is false on the basis of $\not{s}$ *iff* ϕ is false on the basis of (type) $p_l(s)$.

So what is true on the basis of the *type* of $\not{s}$, need not be true on the basis of $\not{s}$ itself, because l (or p_l) can cover up relevant details.

The basic idea of Barwise and Perry's analysis of perception reports is that all perception verbs (see, hear, feel, etc.) are relations between individuals and situations. For this they introduce primitive relations *see*, *hear*, ... between objects and situations. ([5]) Instead of giving a precise extension of the propositional language with sentences expressing attitude reports, this extension will here be assumed and their semantics will be introduced with an example.

We are concerned with the difference between sentences (6) and (7) (which correspond to the two sentences in our first example).

(6) John sees Bill walk.
(7) John sees that Bill walks.

In (6) the embedded sentence is a so-called Naked Infinitive; in (7)

([5]) This is in fact easier to say than to do. There should be situation types mapping these relations, individuals and situations onto truth values. Such situations take situations as arguments. To guarantee that they are *sets* one either has to require that these complex types are essentially partial, are not defined and cannot be defined for certain arguments, or introduce reflexive domains for situation types.

it is a that-complement. We will call (6) itself a Naked Infinitive (NI)-sentence, and (7) a That-complement (Th-c)-sentence.

The truth conditions for Naked Infinitive sentences are roughly:

(8) Let $\mathfrak{s} = \langle l,s \rangle$ be a situation.
'John sees Bill walk' is true on the basis of $\mathfrak{s}$ *iff* for some $\mathfrak{s}_1$ ($= \langle l_1,s_1 \rangle$): $s(\textit{john}, \textit{see}, \mathfrak{s}_1) = 1$ and $s_1(\textit{Bill}, \textit{walk}) = 1$

So John sees Bill walk if he is visually connected with a situation such that it holds on the basis of its type that Bill walks.

In the analysis of the that-complement sentence extensions of situations play a prominent role. 'John sees that Bill walks' will be analyzed as a relation *see* between John and a situation which – according to John – has only extensions on the basis of which Bill walks. So we are not concerned with all possible extensions, but only with those that John would take into consideration as extensions. According to Barwise and Perry there are certain constraints on situations to which people can be attuned, and these determine what according to John are possible extensions of a situation. For our purposes we can introduce this as one constraint c_{john}, the conjunction of everything John is attuned to. With their concept of extension:

(9) $\mathfrak{s}_2 \supseteq \mathfrak{s}_1$ *iff* $l_2 = l_1$ and $s_2 \supseteq s_1$

we can introduce the semantics for that-complement-sentences: (6)

(10) Let $\mathfrak{s} = \langle l,s \rangle$ be a situation.
'John sees that Bill walks' is true on the basis of $\mathfrak{s}$ *iff* for some $\mathfrak{s}_1$ ($= \langle l_1,s_1 \rangle$): $s(\textit{john}, \textit{see}, \mathfrak{s}_1) = 1$ and for every $\mathfrak{s}_2 \supseteq \mathfrak{s}_1$: if $\mathfrak{s}_2$ fulfils c_{john} then 'Bill walks' is true on the basis of $\mathfrak{s}_2$. (7)

So John sees that Bill walks is true if John stands in the see-relation to a situation $\mathfrak{s}_1$, and in every extension of $\mathfrak{s}_1$ to which John is attuned it holds that Bill walks.

Applying this to our first example we can say that your friend hears a singer lose her voice if she stands in the hear-relation to a situation on the basis of which a singer loses her voice; your friend hears that a singer loses her voice if she is confronted with a situation that can only

(6) The definition is somewhat simplified but the differences are not relevant here.

(7) I.e. if $\mathfrak{s}_2$ fulfills c_{john} then $P_{l_2}(s_2)(\textit{Bill}, \textit{walk}) = 1$.

be extended (relative to what she is attuned to) to a situation in which a singer loses her voice. In the first case the situation she is in contact with gives her direct evidence for the truth of 'an operasinger loses her voice'; in the second case the situation she is in contact with gives indirect evidence for the truth of this sentence.

Though they do not defend it, Barwise and Perry present this distinction on several places as an epistemic distinction: the NI-sentence is epistemically neutral, deals with what there is to see, while the Th-c-sentence involves awareness, deals with what the perceiver is aware of.(8) To defend this claim they should explain how the direct-indirect-contrast, as formalized in (8) and (10), yields an epistemic distinction as well. On the basis of their definitions and some of their remarks we could motivate this claim as follows:
Claim I: the clause for see-that involves truth on the basis of $\mathfrak{s}_2$ instead of s_2, so the perspective is involved. Truth relative to a perspective involves some kind of awareness.
Claim II: the information that a singer loses her voice is in some way inferential: the clause of hear-that says that there is no way to complete the situation heard that does not lead to a singer losing her voice. To hear that, one has to be aware of the special nature of the situation; this involves inference, awareness.

It will be apparent that these claims are formulated somewhat metaphorically. What we shoud ask is: is this metaphor captured by the mathematical model offered by Barwise and Perry; that is, can we show *formally* that the direct-indirect distinction is an epistemic contrast as well? I will argue in the next section that the answer must be negative.

3. *Two problems*

I will argue that Barwise and Perry's analysis of attitude reports does not yield an epistemic distinction between NI-sentences and Th-c-sentences, and that this is a most problematic aspect of their theory. I will do this by arguing against claims I and II.

(8) See for this also Barwise, J. 1981, 'Scenes and other Situation', in *The Journal of Philosophy*, Vol. LXXVIII, 7, p. 369-397, in particular p. 375.

Claim I was the claim that there is an epistemic difference between 'truth on the basis of a situation type' and 'truth on the basis of a situation, relative to the perspective'. But in the ontology the perspective is just a space-time location, the place-time where the situation type is located. Situations thus are as objective as are situation types: the perspective function of a space-time region is not epistemic.

One could meet this objection by making the perspective more subjective. If the perspective not only reflects the conditions of the light, and other aspects of the location, but also the perceptual capacities of the one who is perceptually related to the situation, then there is some right to claim an epistemic distinction between what is objectively true on the basis of a situation one is perceptually connected with and what one – given ones perceptual capacities – can be aware of. Perspective then is a function from locations, individuals and situation types to situation types:

(11) $p_{l,john}$ (s) is some s′ such that $s' \subseteq s$.

The perspective obscures certain aspects of a situation relative to the location and an individual.

This would meet the objection, but it won't be of any help. The first problem for situation semantics is that this modified perspective is already needed in the analysis of naked infinitive-sentences!

Consider the following quotation from Frege's 'thoughts'.(9)
'I go for a walk with a companion. I see a green field. I thus have a visual impression of the green. (...) The field and the frogs in it, the sun which shines on them are there no matter whether I look at them or not, but the sense-impression I have from the green exists only because of me, I am its owner. (...) My companion and I are convinced that we both see the same field; but each of us has a particular sense-impression of the green. I glimpse a strawberry among the green strawberry-leaves. My companion cannot find it, he is colour-blind. The colour-impression he gets from the strawberry is not noticeably different from the one he gets from the leaf.'

(9) P. GEACH (ed.), 1977, *Gottlob Frege. Logical Investigations*, Blackwell, Oxford, p. 14.

Take the situation sketched by Frege as assumption. Then consider the following NI-sentence:

(12) Frege sees a strawberry dangle between the leaves.

This sentence is true on the basis of the sketched situation $\mathfrak{s}$. Thus for some $\mathfrak{s}_1$: s(Frege, see, $\mathfrak{s}_1$) = 1 and s_1 (a strawberry, dangle) = 1. Now Frege's assumption is that he and his companion see the same field; the strawberry is there! This means that the objective situation they are visually attached to is the same. Together with the claim of Situation Semantics that NI-sentence are neutral, concern what (objectively) holds of the situation we can draw the following conclusion: there is some situation, namely the *same* situation $\mathfrak{s}_1$ such that s(Frege's companion, see, $\mathfrak{s}_1$) = 1 and s_1(a strawberry, dangle) = 1. But this means that Frege's colour-blind companion sees a strawberry dangle among the leaves, and that is, of course, exactly what he does not see!

If we use the modified perspective of (11) in the definition of NI-sentences then this problem is solved: though the objective situation Frege and his companion are confronted with is the same, what they can see of it differs:

(13) 'Frege sees a strawberry dangle' is true on the basis of $\mathfrak{s}$ *iff* for some $\mathfrak{s}_1$: s(Frege, see, $\mathfrak{s}_1$) = 1 and $p_{l,\mathrm{Frege}}(s_1)$(a strawberry, dangle) = 1.

Now the conclusion does not follow, because $p_{l,\mathrm{Frege}}(s_1) \neq p_{l,\mathrm{Frege's\ companion}}(s_1)$.

An obvious consequence for Situation Semantics itself is that apparently there are no completely neutral attitude reports. But attitude reports are presented as an argument for objective situations in the semantics of natural language instead of (subjectively) interpreted situations. It is questionable whether attitude reports form a firm basis for this kind of realism.

The conclusion should be that the perspective does not evoke a difference in awareness between NI-sentences and Th-c-sentences.

Claim II was the claim that if one sees of situation that relative to what one is attuned to has only extensions on the basis of which the complement is true, one must in some sense be aware of that. But

extensions are just ordinary situations. The second problem takes advantage of that.
Consider the following two sentences: ([10])

(14) Nixon sees miss Woods erase the crucial part of the tape.
(15) Nixon sees that Miss Woods erases the crucial part of the tape.

We imagine a situation $\mathfrak{s}$ = <l,s> that satisfies the following assumptions:
(a) The NI-sentence (14) is true on the basis of $\mathfrak{s}$.
(b) The conditions of the light are perfect.
So Nixon is in the best position to watch it. His reaction cannot possibly be: 'From where I was sitting I could not see what she was doing.' But
(c) Nixon does not realize what she is doing.
Instead his reaction is: 'I know I should have seen it, but I did not know that what she was doing was just *that*.' In this situation the Th-c-sentence (15) is *not* true, because (assumption c) Nixon was not aware.

Now look at (8) and (10) and assumptions a-c. NI-sentence (14) is true on the basis of $\mathfrak{s}$ (assumption a). Thus, for some $\mathfrak{s}_1$: s(Nixon, see, $\mathfrak{s}_1$) = 1 and s_1(Woods, erase, tape) = 1. Assumption b tells us that the conditions of the light are perfect. This means that the perspective does not cover up any aspects of the situation. But this means that 'Miss Woods erases the tape' is not only true on the basis of situation type s_1, but also on the basis of situation $\mathfrak{s}_1$.

But if a sentence is already true on the basis of situation $\mathfrak{s}_1$, then it is also true on the basis of every extension $\mathfrak{s}_2$ of $\mathfrak{s}_1$. However, if it is true on the basis of every extension of $\mathfrak{s}_1$, then certainly it is true on the basis of those extensions of $\mathfrak{s}_1$ to which Nixon is attuned (because that is just a subset). Thus with assumptions a and b we can conclude:

(16) For some $\mathfrak{s}_1$: s(Nixon, see, $\mathfrak{s}_1$) = 1 and for all $\mathfrak{s}_2 \supseteq \mathfrak{s}_1$: if $\mathfrak{s}_2$ satisfies c_{Nixon} then 'Miss Woods erases the tape' is true on the basis of $\mathfrak{s}_2$.

This would, mean that (15) is true.
If assumptions a and b are fulfilled (15) is true: if Nixon sees miss Woods erase the tape and Nixon has good sight, then Nixon sees that

([10]) From Barwise, J., op. cit., p. 373.

miss Woods erases the tape. Nixon's defence 'I was not aware of it' is not acceptable. But then, of course it is!

The direct reason for this failure is that awareness should come in with extensions, while there is no epistemic difference between being true on the basis of $\mathfrak{s}$ and being true on the basis of extensions of $\mathfrak{s}$. The counterexample makes use of the fact that direct evidence is the borderline case of indirect evidence (i.e. the NI-sentence is the borderline case of the Th-c-sentence). The example suggests that a more profound criticism of Situation Semantics can be given.

Indirectness (having indirect evidence) is a gradual notion. Evidence is indirect to a certain degree, and some evidence is more indirect than other. Direct evidence is the borderline case of indirect evidence: the difference between the two also is a matter of degree. If the difference between the NI-sentence and the Th-c-sentence is in the first place a difference in indirectness then their difference is essentially gradual and the counterexample is fatal.

However, consciousness, awareness is not gradual in the same sense as indirectness. If the difference between the NI-sentence and the Th-c-sentence is in the first place a difference in awareness then one can say that the Th-c-sentence involves a different kind or level of awareness, a different type of complexity than does the NI-sentence. The direct-indirect contrast does not yield an epistemic distinction. If one turns matters around, and starts with an epistemic distinction between NI-sentences and Th-c-sentences it seems not a priori improbable that in the end this will yield a distinction in directness as well. This direction will be taken in the remaining part of this paper.

4. *Facts and propositions*

The starting point of Barwise and Perry's analysis of NI-sentences and Th-c-sentences was that both types of sentences express relations between individuals and situations. Consequently the Th-c-sentence expressed a complex, indirect relation between individuals and situations. However, we saw that this analysis was not capable of creating an epistemic difference between NI-sentences and Th-c-sentences.

Starting from the other side our first step is to take the syntactic analysis of that-complement sentences seriously. Sentence (17) is analyzed as (18) and not as (19).

(17) Bill sees that Mary comes.
(18) [Bill [sees [that Mary comes]]].
(19) [Bill [sees-that [Mary comes]]].

If we take this as our guide for the semantics, it suggests that we should not analyze (17) as a complex relation *see-that* between Bill and the interpretation of 'Mary comes' (a situation), but rather as a simple relation *see* between Bill and the interpretation of 'that Mary comes'. The latter cannot just be a situation. Consider the following examples (broadening the discussion to th-c-sentences in general).

(20) Mary regrets that both Carnap is invited and Heidegger is invited.
(21) Mary regrets that Carnap is invited.

There are many situations in which Carnap and Heidegger are invited, but (20) cannot express a relation between Mary and any one of them, because that also is a situation in which Carnap is invited. Then (20) would entail (21), which for Mary, who dreams of a conference of positivists, is false.

Situation semantics claims that Mary has *a more complex attitude* of regret. Our point of view here is that what is more complex is *what* it is that Mary regrets: the interpretation of 'that Carnap is invited and Heidegger is invited'. (20) does not express a relation to a situation in which the complement is true, but rather a relation to what is common to such situations, a relevant aspect of ordinary situations that can have properties like being regrettable for someone (sentence (20)), being visible for those that have the capacity of detecting it, etc. The importance of such entities should not be underestimated: we are not just confronted with situations and talk about situations, but we have the capacity to recognize (sometimes very subtle) aspects of the situations we are confronted with, and we refer to those aspects as well.

So there are semantic entities that function as the interpretation of the that-complements, and these entities are more complex than (say) situations.

Data semantics as a theory of semantic objects is based on two principles:
1) Semantic entities play a role in representations of information about the world.

2) If we want to understand how a complex level of semantic entities can develop out of a simple level we should regard semantic entities as constructions out of that simple level. Semantics, in this view, does not deal with the relation between language and the world, but rather with the way our information about the world is structured; the question what is true in a possible world is replaced by the question what is true on the basis of our information about the world. ([11]) Possible worlds come in as borderline cases of information representations: states of complete information. Normally an information representation is partial. Semantic entities, from now on *propositions*, are units of information; this determines a larger part of their properties.

A further assumption is that our direct information about the world is represented in the form of facts. ([12]) The possession of a property by an object is a fact; the holding of a relation between two objects is a fact; in general, facts are composed of objects and relations between them.

In the light of our information perspective, it will be clear that (unlike Russell's facts) facts are not the ingredients the world is made of, but rather the ingredients of our direct information about the world (and consequently, so are objects and relations); the level of facts is a level of representation. This should not imply idealism about the world: the level of facts is in no way an arbitrary level of representing the world. It is a level that is at least biologically real: it has developed through our interacting with the world.

An important observation is that if facts are representations of the world, there is nothing against *possible* facts! This is important for negation.

([11]) Data semantics, and in particular the idea to define truth/falsity on the basis of a set of facts, interpreted as an information set is developed in Frank Veltman's paper 'Data Semantics'. In fact, for many ideas expressed here that paper was the main source of inspiration. (Differences are mainly due to the fact that Veltman does not treat complements, and hence needs no propositions.)

Cf. VELTMAN, F., 1981, 'Data Semantics', in Groenendijk, J., JANSSEN, T., and M. STOKHOF (eds.), *Formal Methods in the Study of Language*, Mathematical Centre Tracts, Amsterdam, p. 541-565.

([12]) Related in a sense to what Russel calls atomic facts in *The philosophy of Logical Atomism*. Cf. Marsh (ed), op. cit. p. 199.

In 'The Philosophy of Logical Atomism' mr. Demos suggests to define: '$\neg$p is true iff fact p is incompatible with some fact in the world'. For Russell this was unacceptable, because there are only *actual* facts in the world: if p is incompatible with some fact in the world, then p itself is not a fact in the world, and hence is nothing. Russell's solution was to accept negative facts in the world as the interpretation of $\neg$p. [13]

If we have possible facts, we can also accept incompatible facts: these facts are facts that cannot both be contained in an information representation (like the facts that 'this is green' and 'this is red'). You do not need to accept a negative fact if p is false; there can be a positive fact with which p is incompatible.

Now the question can be asked: why this parsimony if we know from the start that we will need things like negative facts as semantic entities anyhow?

The reason is that propositions are incomprehensable mysteries if they are not related to the simple level of direct information, as stated above. Propositions can be regarded as properties (subtle logical aspects) of sets of facts. The level of propositions is a level of higher complexity than that of facts. Propositions are not elements of our direct representations of the world; facts, relations between objects are more basic; the world as our senses represent is consists of facts: parsimony about facts is no vice. But if propositions are not in some sense constructions out of facts, then our theory of propositions gives us no idea at all about how such a complex level could have developed out of the simple level. If however, the proposition that this berry is red or this berry is poisonous is a construction out of facts, a complex property of ordinary situations (direct representations), then we get some idea why there is such a proposition: it can be very profitable to recognize that an ordinary situation has this property, justifies this proposition.

This might also give some idea why, if the referent of a that-complement is a proposition, it induces awareness: I can only stand in relation to a construction out of facts if I have the capacity to construe it and be aware of some very subtle aspects of ordinary situations. This is an intellectual capacity that we have (and that we make

[13] Marsh (ed), op. cit. p. 211-215.

extensive use of) that can be regarded as a complex specialization of the perceptual system we share with other animals. My cat shares with me the way the direct information about the world is ordered; she also has in some degree the simple constructive capacity (that perhaps lower animals like amoebas do not have) to extract the fact that I am fixing her dinner out of the total direct information she is presented with: but it seems doubtful that she has the general capacity to construct more complex aspects of situations and stand in relations to them. For instance, it seems implausible that my cat can find it embarrassing that there is not enough fish to feed her both tonight and tomorrow night.

5. *Data semantics*

A data semantics for complements should present: a set of possible facts; operations with which propositions are constructed out of facts; an information representation in which propositions are the basic units. Propositions should not in general be reducible to facts, but semantic relations exist between certain sets of facts (direct representations) and propositions: 'truth/falsity on the basis of a set of facts'.

We will start with some general requirements of constructions, and discuss some specific requirement of constructions out of facts later on.

We seek operations with which we can construct the set of all propositions out of the set of all facts. Turning matters around we can say that we seek within the set of all propositions a special subset, the set of facts, out of which we can construct the whole set of propositions. But we won't be satisfied with any such set: propositions should really be constructions and facts constructors: if you leave some facts away then there are propositions which you cannot construct. Moreover, facts themselves cannot be constructed.

What we describe here is in fact a well-known algebraic notion, namely that of an independent set of generators. We define it after introducing some basic concepts from universal algebra. (14)

(14) All these notions can be found in Grätzer, G. 1978, *Universal Algebra*, 2nd edition, Springer, New York.

(22) An *n-place operation on* set A is a function from A^n to A.
(23) An *algebra* is a pair $\mathfrak{A} = \langle A,F\rangle$, where A is a non-empty set and F is a set of operations on A.
(24) Let $\mathfrak{A} = \langle A,F\rangle$ and $\mathfrak{B} = \langle B,F\rangle$ be two algebras.
$\mathfrak{B}$ is a subalgebra of $\mathfrak{A}$, $\mathfrak{B} \subseteq \mathfrak{A}$, iff B is a subset of A and $F_{\mathfrak{B}}$ is the set of operations of $F_{\mathfrak{A}}$, restricted to B.
(25) Let $\mathfrak{A} = \langle A,F\rangle$ be an algebra and $B \subseteq A$.
The subalgebra of $\mathfrak{A}$ *generated by* B, [B], is the intersection of all subalgebras of $\mathfrak{A}$, containing B (i.e. $[B] = \bigcap_{j \in J} (\mathfrak{A}_j : \mathfrak{A}_j \subseteq \mathfrak{A}$ and $B \subseteq A_j)$

If you select in the set of propositions a set of facts, then you can regard the algebra generated by this set of facts as the set of constructed propositions. We thus seek a set of facts out of which we can construct every proposition:

(26) B *is a set of generators for* $\mathfrak{A}$ iff $[B] = \mathfrak{A}$.

As said above, this is not enough, we want this set of facts to be somehow independent in $\mathfrak{A}$: if we leave some facts away we cannot construct every proposition:

(27) B *is an independent set of generators for* $\mathfrak{A}$ iff $[B] = \mathfrak{A}$ and for all $b \in B$: $b \notin [B\text{-}\{b\}]$.

An independent set of generators is also called a minimal set of generators (because no proper subset of B can generate $\mathfrak{A}$). So a general requirement on the relation between facts and propositions is that the set of facts is an independent set of generators for the algebra of propositions.

We can also give a general definition of the notion of construction. Let $\mathfrak{A} = \langle A,F\rangle$ be an algebra, B an independent set of generators for $\mathfrak{A}$, $a \in A$, $b_1 \ldots b_n \in B$:

(28) $\{b_1 \ldots b_n\}$ is a set of generators for a iff $a \in [\{b_1 \ldots b_n\}]$.
(29) $\{b_1 \ldots b_n\}$ is an independent set of generators for a iff $a \in [\{b_1 \ldots b_n\}]$ and for all $b_i \in \{b_1 \ldots b_n\}$: $a \notin [\{b_1 \ldots b_n\}\text{-}\{b_i\}]$.
(30) a *is a construction out of* $b_1 \ldots b_n$ iff $\{b_1 \ldots b_n\}$ is an independent set of generators for a.

A proposition is a construction out of facts $f_1 \ldots f_n$, if applying operations to $f_1 \ldots f_n$ yields a and moreover all of $f_1 \ldots f_n$ are needed. We will consider some applications of this general notion of construction later on.

Propositions, we have said, are units of information. Apart from their relations to facts we are quite generally interested in the question how the information expressed in a proposition is related to the information expressed in other propositions. Let us consider the following relation between propositions:

(31) $p_1 \leq p_2$ *iff* the information expressed in p_1 *contains* the information expressed in p_2.

It is quite simple to see that this relation is reflexive, transitive and anti-symmetric: $\leq$ is a partial order on the set of propositions.

Now consider two facts *it rains* and *it snows*, and in particular the information they express. We can ask the following question: is there a *fact* that contains exactly the information shared by these two facts?

The answer must be negative. Facts are relations between objects, the fact described above should consist of a relation between objects as well. Even if one could imagine it to have this form, it would look like a complicated construction out of the old facts: parsimany about facts then tells us that such facts do not exist.

On second thought, however, we can say that it is precisely the function of disjunction to postulate for two facts a quasi-fact (proposition) which is the smallest unit of information that contains exactly the information that is part of the information expressed in both those two facts. From the facts *it rains* and *it snows* we can construe the proposition *it rains or it snows* as the smallest proposition of which the information is contained in the information expressed in *it rains* and in the information expressed in *it snows*. This seems to present a plausible view on the function of disjunction.

A similar story can be told for conjunction: there is no fact which expresses exactly the information that contains the information expressed in *it rains* and *its snows*. Conjunction postulates the existence of a proposition *it rains and it snows* as the largest unit of information, that both contains the information expressed in *it rains* and the information expressed in *it snows*.

The set of propositions, partially ordered by 'contains' now forms a lattice:

(32) A *lattice* is a partially ordered set <A,≤> such that for every a, b ∈ A there is a largest element $a \cdot b$ such that $a \cdot b \leq a$ and $a \cdot b \leq b$ and there is a smallest element $a + b$ such that $a \leq a + b$ and $b \leq a + b$.

We have talked about facts and propositions without distinguishing them. For that we have to regard the lattice of propositions as an algebra, a set with operations. This is no problem, because every lattice can be given the form of an algebra: (32) and (33) are equivalent.

(33) A *lattice* is an algebra <A,+,·>, with two-place operations + (join) and · (meet), satisfying the following postulates:

1. idempotency : $(a \cdot a) = a$; $(a + a) = a$
2. commutativity : $(a \cdot b) = (b \cdot a)$; $(a + b) = (b + a)$
3. associativity : $(a \cdot b) \cdot c = a \cdot (b \cdot c)$; $(a + b) + c = a + (b + c)$
4. absorption : $a \cdot (b + a) = a$; $a + (b \cdot a) = a$

The lattice-order can be reintroduced as:

(34) $a \leq b$ iff $a + b = b$.

If we want propositions to be constructions out of facts, then it will be clear from our earlier discussion that we require that the lattice of propositions is independently generated by the set of facts.

This, however, can be regarded as a rather formal requirement: it does not constrain the notion of fact. Until now, we let some set of propositions play the role of facts. In what follows we will discuss some properties, distinguishing facts from propositions.

The first property concerns disjunctive information. Remember that we did not accept negative, conjunctive or disjunctive facts as real facts: facts are relations between individuals, and as such they are positive simple representations of the world. Think of facts here as inventories of the world, small lists of relations that hold between objects. We can generalize this to other representations: if facts $f_1,\ldots,f_n$ are thought of as lists of what relations hold in the world, then we can call the conjunction $f_1 \cdot \ldots \cdot f_n$ also a list, an inventory of the relations that hold in the world. In the lattice so far we have not incorporated conditions on facts dealing with such properties at all. It would go far beyond the scope of this paper to try to define such notions. Whatever notion will be developed, however, it will count pure disjunctive information as non-inventory: if a proposition contains disjunctive information a+b, but not trivially, because it contains, say, a then this proposition cannot in any serious sense be regarded as a list of relations between objects in the world. If we take the idea that facts are inventories seriously, then we have to require that facts or more general lists of facts do not contain purely disjunctive information: if such a representation contains disjunctive information, then it contains one of the disjuncts. We formulate this as:

(35) Let $\mathfrak{A} = <A,+,\cdot>$ be a lattice, independently generated by set B. B is a set of facts iff $\mathfrak{A}$ satisfies fact condition 1.
Fact-condition 1: for all $b_1,\ldots,b_n \in B$, p, $q \in A$: if $b_1 \cdot \ldots \cdot b_n \leq p+q$ then $b_1 \cdot \ldots \cdot b_n \leq p$ or $b_1 \cdot \ldots \cdot b_n \leq q$.

So the set of propositions is constructed with operations + and · from the (independent) set of facts which are inventories of the world.

Turning to negation. We wanted to explain negation with help of the notion of incompatibility. Consider again some fact p = *the grass is green*.
Question: is there a fact incompatible with p?
Answer: lots of them! For instance, q = *the grass is blue*.
What does incompatibility mean? Our first try, in section 4 was: two facts are incompatible if they cannot both be part of a representation of the world. The problem with this explanation is that it is circular, because we want to built representations of the world with facts. A second, more promising try is the following: two facts are incompatible if their combination is impossible: p and q are incompatible if p·q is impossible. So we have reduced the question to: what does it mean that p·q is impossible?

We could keep the notion of impossibility of the combination of two facts a primitive notion, but that has the disadvantage that it obscures the notion of negation: the observation that the combination of two facts is impossible is a complex observation. How can we understand that people can learn to see a thing of such a complexity if there is nothing simpler on which it is based?

The grass is green (p) expresses information about the world, it expresses that the world is organized in a certain way. The same holds for *the grass is blue* (q). A crucial observation is that *the grass is blue* expresses that the world is organized *in a way different from* what *the grass is green* tells us. The capacity to see that the world differs from what p says is an elementary capacity, that, for one thing, is basic for every notion of correction. This basic capacity is the foundation for the notion of incompatibility:

(36) Two facts are incompatible if the one expresses that the world differs from what the other expresses.

There are many facts that express more or less that the world differs from what p says.
Question: is there a fact that expresses *exactly* that the world differs from what p says? A first straightforward answer is: no, parsimony about facts forbids the existence of such a weird fact. On second thought, however, we can say that it is precisely the function of negation to postulate with a fact a unique quasi-fact (proposition) that expresses exactly that the world differs from what p says. In this view negation is essentially a complement-notion. Introducing negation with these properties lays some more restrictions on the set of propositions. The lattice is distributive, because the negation of p is unique. If the negation of p is the proposition that exactly expresses that the world differs from what p says, then the proposition that exactly expresses that the world differs from what the proposition that exactly expresses that the world differs from what p says expresses is p itself! So we have the law of double negation as well. A similar argument can justify the laws of De Morgan:

(37) A *De Morgan lattice* is an algebra $\mathfrak{A} = \langle A,+,\cdot,^{-}\rangle$ where $\langle A,+,\cdot\rangle$ is a lattice and $^{-}$ a 1-place operation (complementation), such that:
1. Distributivity: $a\cdot(b+c) = (a\cdot b)+(a\cdot c)$;
   $a + (b\cdot c) = (a+b)\cdot(a+c)$
2. De Morgan: $\overline{a\cdot b} = \overline{a}+\overline{b}$; $\overline{a+b} = \overline{a}\cdot\overline{b}$
3. Double Negation: $\overline{\overline{a}} = a$

We have said that a and b are incompatible if the one expresses that the world is organized differently from what the other expresses. We have also said that a and b are incompatible if their combination is impossible. Informally, we can say that $a\cdot b$ is impossible if the information expressed in $a\cdot b$ contains the information that the world is

organized in a certain way *and* at the same time is organized differently.
In a definition:

(38) a is *possible* iff for no $x \in A$: $a \leq x \cdot \overline{x}$, *impossible* otherwise.
a and b are *incompatible* iff $a \cdot b$ is impossible

With these notions we can lay some further restrictions on the set of facts and the information representation in general. Suppose we have a set of compatible information D and a proposition p. We find out that p is compatible with every list of propositions in D. This of course does not mean that D already *contains* the information expressed in p, but it should be possible to *extend* D to a set of information that does contain this information. A De Morgan Lattice does not guarantee that this is possible. If we are not only interested in partial information about the world, but also in extensions of information, in particular states of complete information, we have to impose a condition that guarantees that partial information can be extended to total information: ([15])

(39) A *completion lattice* is a De Morgan lattice $\langle A, +, \cdot, ^{-} \rangle$ such that for all a, b, $c \in A$ if a and b are possible and $a \cdot c$ is impossible and $b \cdot \overline{c}$ is impossible then $a \cdot b$ is impossible.

This tells us *that* extension of information is always possible, it does not tell us *how* it takes place. It seems plausible that if D is a set of facts, a direct representation of the world, information will be added to it either by adding *new facts*, or by adding propositions that express information that is contained in facts which are already added to D. On this view, however, there is a problem with negative information (complements of facts). If no fact contains the information that it doesn't rain, then we will never reach a situation in which we actually add the proposition *it doesn't rain*, even if we know that it is

([15]) At first sight one should expect that this means that we can always add more information. That is not meant here, however. We have made the idealization that all possible information comes from our set of propositions; adding more information to information complete with respect to this set of propositions simply means changing the set of propositions. In a De Morgan lattice, however, information can be essentially partial: there can be propositions p of which neither p nor $\overline{p}$ can be added to the information.

compatible with our information. This shows what is missing up to now: the addition of negative information should be related to the addition of *facts*. In building up a complete representation of the world we do not add negative information, but we add *facts* containing this information. The information expressed in *it doesn't rain* does not appear out of thin air: *it doesn't rain* is added to our information through the addition of some *fact* containing *it doesn't rain*:

(35′) Let $\mathfrak{A} = <A,+,\cdot,^{-}>$ be a De Morgan lattice, independently generated by set B. B is a set of facts iff $\mathfrak{A}$ satisfies fact-condition 1 (see 35) and fact-condition2:
fact-condition 2: Let $D \subseteq B$ and $b \in B$: if for all $f_1,\ldots,f_n \in D$: $f_1 \cdot \ldots \cdot f_n \cdot \bar{b}$ is possible, then there is a fact $f \leq \bar{b}$ such that for all $f_1,\ldots,f_n \in D$: $f_1 \cdot \ldots \cdot f_n \cdot f$ is possible.

(40) We will call a completion lattice independently generated by a set of possible facts, satisfying the fact conditions 1 and 2 a *constructive completion lattice*.

We will define a direct representation of the world as a set of mutually compatible facts. ([16]) We first define what it means that the information expressed in proposition p is contained in information representation X (set of propositions) even if p itself is not an element of X:
Let $<\mathfrak{A},B>$ be a constructive completion lattice, $X \subseteq A$, $p \in A$:

(41) *p is founded on* X *iff* for some $x_1,\ldots,x_n \in X$: $x_1 \cdot \ldots \cdot x_n \leq p$.

So p is founded on X if there is a representation of elements of X that contains the information expressed in p. In general a set of compatible information has the finite consistency property:

(42) X *has the finite concistency property* (fcp) iff for no $a \in A$: $a \cdot \bar{a}$ is founded on X.
(43) A *dataset in* $<\mathfrak{A},B>$ is a set $D \subseteq B$ (a set of facts) which has the fcp

A dataset is a set of compatible partial information about the world. This information can be made total in the following sense. A dataset D

([16]) Cf. VELTMAN, F., op. cit. p. 544.

has the fcp, so no contradiction is founded on D. Consider the set of all propositions founded on D. Trivially D is a subset of this set and this set does not contain any contradictions. It can be proved that this set is a pure filter, in fact the smallest pure filter of which D is a subset(17):

(44) A *filter in* $<\mathfrak{A},B>$ is a nonempty set $F \subseteq A$ such that
 1. if $x, y \in F$ then $x \cdot y \in F$
 2. if $x \in F$ and $x \leq y$ then $y \in F$

(45) A filter F is *pure* iff for no $a \in A$ $a \cdot \overline{a} \in F$

(46) Let $D \subseteq A$, *the filter generated by* D, G_D is the set $\{a \in A$: a is founded on D$\}$

It can also be proved that every pure filter can be extended to a maximal pure filter.

(47) A pure filter F is *maximally pure* if there is no pure filter G such that $F \subseteq G$ and $F \neq G$.

Finally, in a completion lattice a filter is maximally pure iff it is an ultrafilter:

(48) A pure filter U is *an ultrafilter* iff for all $a \in A$: either $a \in U$ or $\overline{a} \in U$ (not both)

An ultrafilter is a set of complete information, it takes a decision on every proposition. In this sense every dataset can be extended to complete information (an ultrafilter). We can finally define a possible world as a direct complete representation of the world:

(49) A dataset D is a *possible world* if the generated filter of D is an ultrafilter.

Call a dataset D maximal if no dataset D′ properly contains D. Then every maximal dataset is a possible world. So, also in this sense a dataset can be extended to complete (direct) information.

Let us turn to semantics. We will define what it means that sentence ϕ is true/false on the basis of dataset D. For that, we interpret ϕ as a proposition and ask whether that proposition is founded on D.

(17) All proofs here are analogous to similar proofs for Boolean Algebras, see for instance Bell, J. and A. Slomson, 1969, *Models and Ultraproducts*, North Holland, Amsterdam.

(50) Let $\mathfrak{L}_0$ be a propositional language. A *model for* $\mathfrak{L}_0$ is a triple $M = \langle \mathfrak{A}, B, [\,] \rangle$, where $\langle \mathfrak{A}, B \rangle$ is a constructive completion lattice and $[\,]$ is an interpretation function from $\mathfrak{L}_0$ to A such that:

1. $[p] \in B$, for all propositional symbols p
2. $[\neg\phi] = \overline{[\phi]}$
3. $[\phi \wedge \psi] = [\phi] \cdot [\psi]$
4. $[\phi \vee \psi] = [\phi] + [\psi]$

a is founded on D iff $a \in G_D$, the generated filter of D.
Writing $D \models_M \phi$ for 'ϕ is true on the basis of D' and $D \; {}_M\!\!=\!\!| \; \phi$ for 'ϕ is false on the basis of D' we define:

(51) Truth definition:
$D \models_M \phi$ iff $[\phi] \in G_D$
$D \; {}_M\!\!=\!\!| \; \phi$ iff $\overline{[\phi]} \in G_D$
Validity: let Δ be a set of sentences, ϕ a sentence of $\mathfrak{L}_0$: $\Delta \models \phi$ iff for no M, D it holds that for all $\delta \in \Delta$: $D \models_M \delta$ and $D \not\models_M \phi$.

(51) has the following consequences:

(52) a) $D \models_M \neg\phi$ iff $D \; {}_M\!\!=\!\!| \; \phi$
$D \; {}_M\!\!=\!\!| \; \neg\phi$ iff $D \models_M \phi$
b) $D \models_M \phi \wedge \psi$ iff $D \models_M \phi$ and $D \models_M \psi$
$D \; {}_M\!\!=\!\!| \; \phi \wedge \psi$ iff $D \; {}_M\!\!=\!\!| \; \phi$ or $D \; {}_M\!\!=\!\!| \; \psi$
c) $D \models_M \phi \vee \psi$ iff $D \models_M \phi$ or $D \models_M \psi$
$D \; {}_M\!\!=\!\!| \; \phi \vee \psi$ iff $D \; {}_M\!\!=\!\!| \; \phi$ and $D \; {}_M\!\!=\!\!| \; \psi$

One can formalize the distinction between direct evidence and indirect evidence as follows.

(53) D presents direct evidence for ϕ iff $D \models_M \phi$
D presents indirect evidence for ϕ iff there is no extension D′ of D, $D' \supseteq D$ such that $D' \; {}_M\!\!=\!\!| \; \phi$

This distinction is directly relevant for the characterization of informational aspects of modal expressions like *must* and *may*.(18) Such an extension of $\mathfrak{L}_0$ falls outside the scope of this paper.

Let us come back to constructions. We have argued that proposi-

(18) Cf. VELTMAN, F., op. cit. p. 552 f.f.

tions are constructions out of facts. We have given a general definition of construction in (30). If propositions are constructions out of facts, then it obviously matters what facts they are constructions out of. The facts a proposition is a construction out of can be regarded as the content matter of the proposition([19]), what the proposition is about([20]).

Applying our general notion of construction to <$\mathfrak{A}$,B>, we can observe that all of $b_1 \cdot b_2$, $b_2 \cdot b_1$, $(b_1+b_2) \cdot (b_1 \cdot b_2)$ are construction out of b_1 and b_2. Interesting is that $(b_1 \cdot b_2)+b_1$ and $(b_1+b_2) \cdot b_1$ are constructions out of b_1, and not out of b_1 and b_2: b_2 is inessential. This means that the absorption laws do not have the consequence that proposition p is a construction out of everything whatsoever. Something like this is the main difference between a De Morgan lattice and a Boolean Algebra. In a Boolean Algebra all contradictions are identified: $p \cdot \overline{p} = q \cdot \overline{q}$, similar for tautologies. In a De Morgan lattice contradictions are not identified (neither are tautologies). All propositions are constructions out of facts, for all propositions it matters what they are constructions out of. In a constructive completion lattice $p \cdot \overline{p}$ is a construction out of p, and $q \cdot \overline{q}$ is a construction out of q. There is no reason whatsoever to suppose that $p \cdot \overline{p}$ and $q \cdot \overline{q}$ are the same construction. In the information system presented here tautologies and contradictions have some content.

However, though contradictions express different propositions ($[p \wedge \neg p] = [q \wedge \neg q]$ is not generally valid), they still are equivalent.([21])

(54) For all D: $D \models_M p \wedge \neg p$ iff $D \models_M q \wedge \neg q$.

Similarly, though tautologies are not identified, though on partial information they are not even equivalent, in the borderline case of total information they are equivalent:

(55) For all possible worlds w: $w \models_M p \vee \neg p$ iff $w \models_M q \vee \neg q$.

([19]) Cf. PERRY, J., forth, 'Contradictory Situations', in the proceedings of the fourth Amsterdam Colloquium held in September 1982.

([20]) Cf. ZEEVAT, H., forth, 'Propositions', in the proceedings of the Fourth Amsterdam Colloquium held in September 1982.

([21]) Though not strongly equivalent in the sense of Veltman, op. cit. and Barwise, op. cit.

In fact, if one considers as datasets only possible worlds, then the truth definition is just the classical truth definition (and the logic is classical logic).

These observations on contradictions and tautologies are also relevant for the analysis of attitude reports.

6. *Attitude reports*

This section deals with some general properties of that-complement-sentences. Fine distinctions between different kinds of complement-taking verbs are ignored here. Thus it only presents half of the analysis of attitude reports (an analysis of the particular properties that attitude reports do not share with other Th-c-sentences has to be postponed to another paper).

We will introduce complements with one predicate symbol P, extension to all kinds of relations with complements is straightforward.

$\mathfrak{L}_0$ was a language with propositional symbols $p_0, p_1, p_2, \ldots$, closed under $\neg$, $\wedge$ and $\vee$. $\mathfrak{L}_1$ is a language based on $\mathfrak{L}_0$ in the following way:

(56) $\mathfrak{L}_1$, a language with $\mathfrak{L}_0$-complements, is the smallest set such that:
1. $\mathfrak{L}_0 \subseteq \mathfrak{L}_1$
2. if $\phi \in \mathfrak{L}_0$ then P(that-ϕ) $\in \mathfrak{L}_1$
   (if $\phi \in \mathfrak{L}_0$ then P(ϕ) $\in \mathfrak{L}_1$)
3. $\mathfrak{L}_1$ is closed under $\neg$, $\wedge$, $\vee$

P(that-ϕ) also abbreviates see (mary, that-it rains).
P(ϕ) abbreviates see(mary, it rain).

A model for our new language $\mathfrak{L}_1$ will contain the following components: a (primitive) property $\mathfrak{P}$ as the interpretation of predicate P; an $\mathfrak{L}_0$-model to interpret the $\mathfrak{L}_0$-part of $\mathfrak{L}_1$; a constructive completion-lattice and an interpretation which extend the $\mathfrak{L}_0$-one, based on a set of $\mathfrak{L}_1$-facts, which is the set of $\mathfrak{L}_0$-facts, together with relations towards sets of $\mathfrak{L}_0$-propositions (the interpretations of complements P(that-ϕ); P(ϕ)).

(57) Let $M_0 = \langle \mathfrak{A}_0, B_0, [\,]_0 \rangle$ be an $\mathfrak{L}_0$-model.

An $\mathfrak{L}_1$-*model based on* M_0 is a quadruple $M_1 = <\mathfrak{A}_1, B_1, [\,]_1, \mathfrak{P}>$ where

1. $\mathfrak{P}$ is a (primitive) property
2. B_1, *the set of* $\mathfrak{L}_1$-*facts*, is the smallest set such that:
   a. $B_0 \subseteq B_1$
   b. if X_0 is a set of $\mathfrak{L}_0$-propositions $(X_0 \subseteq A_0)$ then $<\mathfrak{P}, X_0> \in B_1$
3. $<\mathfrak{A}_1, B_1>$ is a constructive completion lattice
4. $\mathfrak{A}_0 \subseteq \mathfrak{A}_1$
   $[\,]_1 \restriction \mathfrak{L}_0 = [\,]_0$
5. a. $[P]_1 = \mathfrak{P}$
   $[\ ]_1$-clauses for $\neg$, $\wedge$, $\vee$ as for $[\,]_0$
   b. Th-c-sentences: $[P(\text{that-}\phi)]_1 = <\mathfrak{P}, \{[\phi]_1\}>$

(57) only defines a model for a language with $\mathfrak{L}_0$-complements. To get all kinds of complements, especially iterations like:

(58) John sees that (Barry knows that (Henry is ill) and Mary leaves)

one only needs the following observation. Definitions (56) and (57) can be regarded as the first step of a whole hierarchy of languages and models, with successively more complements. One only needs to substitute $\mathfrak{L}_{n+1}$ and $\mathfrak{L}_n$ for $\mathfrak{L}_1$ and $\mathfrak{L}_0$ and M_{n+1} and M_n for M_1 and M_0 to get a chain of languages $\mathfrak{L}_0 \subseteq \mathfrak{L}_1 \subseteq \ldots \subseteq \mathfrak{L}_n \subseteq \mathfrak{L}_{n+1} \subseteq \ldots$ of which each language $\mathfrak{L}_{n+1}$ is a language with $\mathfrak{L}_n$-complements. Similarly one gets a chain of models $M_0 \subseteq M_1 \subseteq \ldots \subseteq M_n \subseteq M_{n+1} \subseteq \ldots$ of which each model M_{n+1} is based on a set of facts B_{n+1} which is the set of B_n-facts, extended with relations to sets of B_n-propositions.

The full language of iterations is the union of this chain of languages $\bigcup_{n \in \mathbb{N}} \mathfrak{L}_n$ and the full model is the union of this chain of models: $\bigcup_{n \in \mathbb{N}} M_n = <\bigcup_{n \in \mathbb{N}} \mathfrak{A}_n, \bigcup_{n \in \mathbb{N}} B_n, [\,], \mathfrak{P}>$. This model is a constructive completion lattice. In this model (58) would be interpreted as the fact (59).

(59) $<\text{see}, \text{john}, \{<\text{know}, \text{barry}, \{<\text{ill}, \text{Henry}>\}> \cdot <\text{leave}, \text{mary}>\}>$

Let us compare NI-sentences and Th-c-sentences. We could introduce NI-sentences (for comparisons only) with the semantics of Barwise and Perry as in (60):

(60) $D_1 \models_M$ see(john, it rain) *iff* for some dataset D_0: <see, john, D_0> $\in G_{D_1}$ and $[\text{it rains}]_1 \in G_{D_0}$

The Th-c-sentences are analyzed as follows:

(61) $D \models_M$ see(john, that-it rains) iff <see, john, {[it rains]}> $\in G_D$

If John sees it rain he has a visual relation to a set of facts (an interpreted situation) one of which happens to be the fact that it rains (a situation in which it rains). If John sees that it rains he has a relation to the set containing just the fact that it rains, the constructed situation that characterizes 'it rains'; he has been able to *isolate* this particular fact out of the total perceived situation.

To isolate a fact from a situation is an activity of intelligent, conscious beings: I can isolate the fact that Mary is cooking fish from a perceived situation, my cat sometimes can do that, but it is doubtful whether the goldfish in the aquarium at the kitchen table, who also watches Mary, can do that. Isolation of a fact is a rather simple intellectual activity, but still it requires *awareness* of special characteristics of the situation you see. This explains the difference in *type* between naked-infinitive sentences and that-complement sentences: the first involves visual capacities, that we share with other animals; the second essentially involves also more complex intellectual capacities, some of which, like construction of a disjunctive proposition, seem highly sophisticated. Having a visual relation to a situation is having your eyes open; having a visual relation to a construction is seeing complexity in the world. That is why to see it rain does not entail to see that it rains. The distinction in awareness is fundamental.

Returning to the direct-indirect distinction, we see that this is not the distinction on which the semantic analysis of attitude reports is based. However, there obviously is a relation between a construction out of facts and sets of facts on the basis of which this construction is true. If your friend hears that an operasinger loses her voice she has a relation to that constructed proposition. Given the way her perception and intelligence works, she won't have such a relation, if she has not got some other relation to a real situation (direct representation) that presents as least indirect evidence for the proposition. One can formulate it pragmatically: someone uttering sentence (62) *states* that Mary has a relation to a proposition,

(62) Mary hears that an operasinger loses her voice

and *assumes* that this relation is based on a relation of Mary to ordinary situations which at least presents some evidence for the truth of the proposition. This evidence can vary from direct perception to hearing a newsreader saying it.

As said above, this analysis should be extended, for each type of complement taking verb, with an analysis of the particularities of that type of verb. Up to now *see* is a relation between an individual and a proposition. A more fine-grained analysis of seeing will induce constraints om *what* kind of relation *see* is, therewith fixing the particular inference pattern of *see*.

We will finally make some remarks about substitution within Th-c-sentences in general, and the inferences that can and cannot be drawn with complements.

Complements and commutativity: in data semantics the following identity holds:

(63) [it rains] + [it snows] = [it snows] + [it rains].

Hence, the following also holds:

(64) $\langle\mathfrak{P}, \{[\text{it rains}] + [\text{it snows}]\}\rangle = \langle\mathfrak{P}, \{[\text{it snows}] + [\text{it rains}]\}\rangle$

This means that commutativity holds for that-complements. The following inference is valid in data semantics:

(65) Mary regrets that it rains or it snows

Mary regrets that it snows or it rains

Consequences of complements. Though $p \cdot q \leq p$, it does not generally hold that if $\{p \cdot q\}$ has a property $\mathfrak{P}$, then also $\{p\}$ has the property $\mathfrak{P}$. It is not generally the case that $\langle\mathfrak{P}, \{p \cdot q\}\rangle \leq \langle\mathfrak{P}, \{p\}\rangle$ (though certain relations may impose such a restriction). To give an example, in data semantics the following inference is not valid:

(66) It is embarrassing that Heidegger is invited and Carnap is invited

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x It is embarrassing that Carnap is invited

Complements and tautologies: here lies a difference with possible world semantics, where the set of propositions forms a Boolean algebra. In data semantics it is not generally the case that $p = p\cdot(q+\overline{q})$ (which is true in a Boolean algebra). The following inference is invalid in data semantics: ([22])

(67) Mary sees that Bill enters

---

x Mary sees that Bill enters and John leaves or John does not leave

Complements and contradictions: as was explained before, contradictions are not identified. So it is not generally the case that $p\cdot\overline{p}$ is $q\cdot\overline{q}$, and thus it is not generally true that $\langle \mathfrak{P}, \{p\cdot\overline{p}\}\rangle = \langle \mathfrak{P}, \{q\cdot\overline{q}\}\rangle$. The following inference is invalid in data semantics:

(68) Mary says that it rains and it does not rain

Mary says that Einstein is a genius and Einstein is not a genius

Indeed, Mary might not have heard of Einstein at all!

If attitude reports are analyzed as (perhaps complex) relations between individuals and direct, simple representations of the world, be it datasets or situations, instead of relations between individuals and (constructed) propositions, inference (68) will unavoidably turn out valid (even if Mary firmly denies to have ever said anything about Einstein). Datasets, or situations, as direct representations of the world, cannot distinguish between contradictions. It is not the world that distinguishes between contradictions, we do.

University of Amsterdam
Centrale Interfaculteit
Grimburgwal 10, gebouw 13
1012 GA Amsterdam

Fred LANDMAN

([22]) It is also invalid in Situation Semantics.