FORM AND IMPLICATION

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I. *Introduction*

The concept of logical form has long been among the most important factors in the science of logic. Not only is it employed in the classification of statements and arguments, it is a basic ingredient in systematic accounts of argument validity, logical relations, and logical modalities. Some writers virtually characterize logic in terms of form,(1) but apart from such a sweeping perspective it is safe to say that an almost universally-held hypothesis is that the logical properties of a statement are fully determined by its logical form or structure. Quine, for one, holds that a sentence is a logical truth or falsehood in virtue of its logical structure and, extending this idea to include relations like implication, he writes; "one closed sentence logically implies another when, on the assumption that the one is true, the structures of the two sentences assure that the other is true." (2) More generally, the hypothesis can be formulated as follows:

(LF) For each logical property or relation R holding of statements $A_1, \ldots, A_n$, $n \geq 1$, there are logical forms $F_1, \ldots, F_n$

(1) See, for example, A. CHURCH (1956), p. 1, who writes that "logic is concerned with the analyses of sentences or of propositions and of proof with attention to the *form* in abstraction from the *matter*." Again, A. N. WHITEHEAD once commented that "Aristotle founded the science of logic by conceiving the idea of the form of a proposition, and by conceiving deduction as taking place in virtue of forms," (1916-17), p. 73. Compare, also, REICHENBACH (1968), p. 216; HENKIN (1967), p. 61; and CARGILE (1979), p. 10.

(2) QUINE (1970), p. 48. See, in addition, BOLZANO (1973), pp. 204-216; LEWIS and LANGFORD (1959), p. 340; and CASTAÑEDA (1975), p. 71. Even TARSKI (1956), despite his model-theoretic definition of logical consequence, insisted that logical consequence is "... uniquely determined by the form of the sentences between which it holds," pp. 414 and 417.

such that the fact that A_1 is of form F_1 and ... and A_n is of form F_n is sufficient for R holding of $A_1, \ldots, A_n$.

Though this principle may seem quite ordinary and intuitive at first glance a problem arises in its application. We agree that the statement "this is white and this is rectangular" logically implies "this is white" but not "this is scarlet," assuming uniformity of demonstrative reference. According to (LF), there should be some explanation for this difference in terms of a difference in logical forms. And here is the problem; "this is white" and "this is scarlet" appear to be totally alike in their logical form or structure, so how do *logical structures* assure the truth of "this is white" given the truth of "this is white and this is rectangular" while not that of "this is scarlet"? Where *is* the relevant difference in logical form?

An initial response might be that the statement "this is white" corresponds to "this is white and this is rectangular" in a way that "this is scarlet" does not, that since the conjunction has the form "p & q" then "this is white" is the *corresponding* statement of form "p" while "this is scarlet" is not. To this it can be added that the two forms reflect a sharing of content between the first two statements upon which the correspondence is based. Such a response is fine, as far as it goes, but given that there is a sense in which "this is scarlet" is also a statement of form "p," as is every statement, and that it too shares content with the said conjunction, as indicated by the demonstrative, then more must be said about correspondence and shared content if (LF) is to be so defended. The present essay attempts to do just that.

The discussion will, however, be complicated by a further issue whose resolution may very well limit theoretical options in dealing with the problem at hand. There is an old debate whether the ultimate terms of logical relationships are *sentences* or what sentences express, viz., *propositions*. Though this is not a debate to be settled herein, I do wish to offer proposals broad enough to accomodate the nod towards propositions, while, at the same time, not ruling out a more linguistic approach. Employment of the terms 'statement' and 'statement form' will be indicative of this hedge. The decision might render the account more philosophical than the project warrants, but *if* propositions are the bearers of logical properties and relations then, given (LF), *propositional forms* are philosophical entities that logical theory cannot afford to ignore.([3]) As such, something must be said about the conditions under which a proposition "has" a given form, about how a proposition can have many forms, how forms are to be differentiated from one another, and how (LF) can be anchored in a theory of *propositional* form. Here, then, is our concern: to resolve the problem raised by the application of (LF) with an account flexible enough to adjust to a variety of philosophical perspectives on logical form.

II. *The plurality of forms*

It is helpful to have some device for representing both statements and forms as objects for thought, i.e., as subjects of predication, by means of appropriate nominalizations. Let us allow the result of enclosing an indicative sentence within asterisks to constitute a name of the statement expressed by that sentence in a given context of interpretation. Noting that logical forms are commonly represented by matrices containing logical constants and freely occurring variables (or schematic letters), we extend the convention and stipulate that a matrix enclosed in asterisks is a name of a form – provided that the matrix specifies a logical form in the first place.

Though philosophers often talk about *the* logical form of a statement or argument, the notion that a statement has only one form must be resisted if logical form is to have the systematic value imputed to it in the study of logical properties and relations. The statement $*\sim$(I am hungry) $\supset$ ((I am hungry) $\supset$ (I am unhappy))*, for instance, not only has the general conditional form $*p \supset q*$ but also the more complex form $*\sim p \supset (p \supset q)*$, and it is in virtue of the latter that the statement is a tautology. Still deeper forms turn on the predicative structure of

([3]) Several philosophers have written of the logical forms of non-linguistic items, for instance, BOLZANO, *op. cit.*, who spoke of the forms of *Satz* (propositions); STRAWSON (1952), chp. 2; and CASTAÑEDA (1975), chp. 3 and (1977), pp. 298-299 and 327-329. One who takes seriously the idea that forms belong to propositions could hold that sentences have their forms only derivately, and similarly, that logical relationships among sentences are parasitic upon logical relationships among propositions expressed by those sentences.

its components, allowing that even the terms 'hungry' and 'unhappy' may conceal logical complexity. In the direction of greater generality, it is not uncommon for authors of logic texts to construe a single propositional variable as a logical form inasmuch as they employ it in displaying argument forms and in stating inference rules.(4) A uniform treatment of validity, coupled with the use of matrices in representing form, seems to require that each single propositional variable portrays a logical form that is shared by all statements. Let us call this 'the minimal logical form' and, by the same reasoning, we may acknowledge a minimal negative form $*{\sim}p*$, a minimal conjunctive form $*p \,\&\, q*$, and so forth, noting that different variables can be used to represent these forms. In sum, a statement may possess a plurality of logical forms.(5)

The term 'refinement' can be used to express a relation among forms such that form F_1 is a refinement of form F_2 just in case any statement that has F_1 also has F_2.(6) For example, the form $*{\sim}p \supset q*$ is a refinement of $*p \supset q*$, though not conversely, while $*{\sim}p \supset (p \supset q)*$ is a refinement of both. Refinement is a reflexive, antisymmetric and transitive relation in its field and, hence, a partial order. Its converse can be called 'approximation' and its strict (irreflexive) counterpart 'deeper refinement'. We note that the minimal form is an approximation of all other forms, and if a statement has a form F, but none of the refinements of F, then F is a *specific* form of that statement.

Define a form as *valid* just in case every statement having it is true; *satisfiable* just in case some statement having it is true; and *unsatisfiable* just in case no statement having it is true. The following principle relates these notions to the concept of refinement:

(1) For any logical forms F_1 and F_2, if F_1 is a refinement of F_2

(4) COPI (1978), pp. 300-301, provides a clear instance of one who uses a single variable, e.g., 'p', to portray a statement form. Compare, as well, CASTAÑEDA (1975), p. 68, for a more philosophically explicit recognition of the minimal form of propositions.

(5) CASTAÑEDA, in particular, has argued for this, *op. cit.*, pp. 68-69; but see also STRAWSON (1952), pp. 53-56 and LEWIS and LANGFORD (1959), p. 315.

(6) This relation is discussed in detail by CASTAÑEDA, *op. cit.*, p. 69.

then (a) if F_2 is valid so is F_1; (b) if F_2 is unsatisfiable so is F_1; and (c) if F_1 is satisfiable so is F_2.

It is precisely because of this principle that the search for properties like logical truth, implication, consistency, etc., can occasionally be terminated by examining just *some* of the logical forms of the relevant statements. Satisfiability and unsatisfiability can also be understood in terms of validity:

(2) For any logical form F, F is satisfiable just in case it is not the case that $\sim$F is valid, and F is unsatisfiable just in case $\sim$F is valid.

To understand this principle, however, requires a clear explication of how one can determine what the negative form $\sim$F is once one has identified F. Predictably, this issue is one with the problem of correspondence and shared content mentioned above, and is settled in section VI below.

III. *Implication*

Our initial attempt to apply (LF) to logical implication has run aground upon a relatively simple example, and it was suggested that a way out of the difficulty requires clarification of the sense in which statements can "correspond" to each other in virtue of their forms. It is important to realize how deeply the notion of correspondence affects the study of logic from the standpoint of (LF). Consider, for instance, the following argument:

A1 I am honest and it is not the case that i am happy ∴ It is not the case that if I am honest, then I am happy,

which is valid on any plausible interpretation of the conditional. If it exhibits the pattern $A \; \& \sim B \; \therefore \sim(A \supset B)$, say, it can be mated to the following rule of inference: from $A \; \& \sim B$ infer $\sim(A \supset B)$. The presence of schematic letters suggests an implicit appeal to underlying logical form, but, mindful of the demands of (LF), how can this be brought out more directly in the statement of the rule? It will not do, clearly, to phrase it as follows: from a proposition of the form $*p \& \sim q*$ infer a proposition of the form $*\sim(p \supset q)*$. Such a rule would sanction not only *A1* but also,

A2 I am honest and it is not the case that I am happy ∴ It is not the case that if $2 + 4 = 6$, then I am angry,

since its conclusion is of the form $*\sim(p \supset q)*$ as well. Nor does it help to include reference to *the* statement of form $*\sim(p \supset q)*$ for there is no such statement as long as forms are shared by distinct statements – a claim that is not ruled out here. Instead, what is needed is something like this: from a statement of the form $*p \& \sim q*$ infer the corresponding statement of the form $*\sim(p \supset q)*$. Thus, given the common premise, the conclusion of *A1* is, while that of *A2* is not, the *corresponding* statement of form $*\sim(p \supset q)*$.

How is this concept of correspondence to be explicated? Linguistically, a corresponding statement of a form expressed by matrix *M* is one where, in a given context, each free variable in *M* replaces one and the same constant throughout its free occurrences in that context. However, within our present designs, a non-linguistic account is desired, and here the problem emerges in yet another light. Contrast the rules, (a) from a statement of form $*p \& q*$ infer the corresponding statement of form $*p*$, and (b) from a statement of form $*p \& q*$ infer the corresponding statement of form $*q*$. The distinctness of variables points to a clear difference between (a) and (b), but if we acknowledge that the *forms* $*p*$ and $*q*$ are identical, i.e., that there is only *one* minimal form, then it is not easy to differentiate (a) and (b). To point out that the descriptions 'the corresponding statement of form $*p*$' and 'the corresponding statement of form $*q*$' have different meanings in their respective contexts serves merely to identify the problem, not to resolve it.

The issue of correspondence resurfaces in the very analysis of implication in terms of form. According to one standard version, a statement *A logically implies B* if and only if the statement "if *A* then *B*" has a valid logical form. Though, admittedly, this is a generic account until the proper sense of 'if-then' is clarified – and for convenience we will use the material conditional – there is something lacking in the analysis. According to (LF), the implicational relationships of a statement are a function of *its* logical forms, *not* of the forms of some other statement of which it is a proper part. To accomodate this, one might propose that *A* logically implies *B* just in case *A* has a form F_1, *B* has a form F_2 and the form "if F_1 then F_2" is valid. But what is this conditional form? *Can* forms combine to constitute compound forms in the way that statements combine to yield compound statements? This seems dubious. The premise of *A2* does not logically imply its conclusion, yet the premise has the form *p & ~q*, the conclusion *~(p ⊃ q)*, and the form *(p & ~q) ⊃ ~(p ⊃ q)* is valid. Though the conditional formed from the two statements *does* have the form *(p & ~q) ⊃ ~(r ⊃ s)* it does *not* have the form *(p & ~q) ⊃ ~(p ⊃ q)*, but if compound forms were composites of simpler forms then these two forms would be identical since *p & ~q* and *r & ~s* are one and the same form. By contrast, the conditional formed from the premise and conclusion of *A1* does have the form *(p & ~q) ⊃ ~(p ⊃ q)*. Again, correspondence is the deciding factor, and we now see that this notion underlies an important ontological difference between compound forms and compound statements while proving fatal for the proposal at hand.

A similar development affects any account that appeals to a relation between logical forms, that is, to views which hold that *A* implies *B* because *A* has a form which stands in some "implication-making" relation *N* to a form of *B*, a relation which we can call 'form-necessitation'.(7) Thus, it will not do to characterize this relation as follows: $N(F_1, F_2)$ just in case if any statement having F_1 is true then some statement having F_2 is true. Such a construal, like its immediate predecessor, fails to provide sufficient conditions for implication as it would render valid both *A1* and *A2*. An alternative is more accurate: $N(F_1, F_2)$ just in case if any statement having F_1 is true then there is a corresponding statement of form F_2 which is true. With this, *one* specification of (LF) in regards to implication is straightforward:

(3) *A* logically implies *B* if and only if there are forms F_1, F_2 such that *A* has F_1, *B* has F_2, and $N(F_1, F_2)$.

(7) WITTGENSTEIN seemed to opt for some such view when he wrote; "If the truth of one proposition follows from the truth of others, this finds expression in relations in which the forms of the propositions stand to one another" (1961), 5.131.

This principle might well be true, but it is significant that the notion of correspondence has again made an appearance, and it is difficult to escape the conclusion that it must do so on any reliable account of form-necessitation.

Let us define the logical powers of a statement A as the class of statements logically implied by A, representing this by 'P(A)', and let 'F(P(A))' symbolize the union of the classes of logical forms of the members of P(A). Taking (3) seriously, one interesting consequence of it is,

(4) A and B share all their logical forms if and only if F(P(A)) = F(P(B)).

From this principle, in turn, it follows that,

(5) If A logically implies B and B has a form which no member of P(C) has, for some C, then A has a form which C does not have.

Form-necessitation is connected to the relation of refinement mentioned above. Letting 'R' abbreviate 'is a refinement of', we have the following principles:

(6) If $N(F_1, F_2)$ and $R(F_3, F_1)$, then there is a form F_4 such that $R(F_4, F_2)$ and $N(F_3, F_4)$.
(7) If A logically implies B, A has F_1, B has F_2, and $N(F_1, F_2)$ then if A has F_3 where $R(F_3, F_1)$, for any form F_3, then there is a form F_4 such that B has F_4, $R(F_4, F_2)$ and $N(F_3, F_4)$.

To illustrate, if A, B and C are distinct statements then the statement $(A \& B) \supset C$ logically implies $\sim C \supset \sim(A \& B)$, and, the first has the form *p ⊃ q*, the second has *~q ⊃ ~p*, and N(*p ⊃ q*,*~q ⊃ ~p*). Now $(A \& B) \supset C$ has the further form *(p & q) ⊃ r* which is a refinement of *p ⊃ q*. By (6) there is a form which is a refinement of *~q ⊃ ~p*, necessitated by *(p & q) ⊃ r* and which, by (7), is possessed by $\sim C \supset \sim(A \& B)$. The obvious candidate for this fourth form is *~r ⊃ ~(p & q)*.

The centrality of the notion of correspondence should, by this point, be clear; not only is it a vital ingredient in the characterisation of form-necessitation, but, by (3), in implication itself. Its linkage to refinement, revealed in (6) and (7), indicates that we must probe more deeply into the *structures* of statements and forms in order to place (LF) on a satisfactory basis.

IV. *Form and structure*

We now digress from direct consideration of (LF) and approach logical form from another angle. It has already been suggested that a form is an abstract aspect or characteristic that distinct statements can have in common. But this must be contrasted with the view that a form is, in some sense, the *structure* of a statement. However, since 'structure' is nearly synonymous with 'form',([8]) more must be said about structure if this idea is to deepen our understanding of form. In one respect, any complex entity – other than a mere set or heap – is a structure, i.e., a totality constituted by parts arranged in a certain way. Statements themselves, whether sentences or propositions, are structures, and so, it does little to classify forms as structures and leave it at that.

Each statement is a real togetherness of entities, independent of its truth-value. Moreover, there are different levels at which a statement can be viewed as a totality. The statement *I am hungry & I am angry*, for example, is a togetherness of the component statements *I am hungry* and *I am angry*, but it is also a togetherness of the components expressed by 'I', 'hungry' and 'angry'. Analogously, a sand castle is a complex whose parts are, at one level, grains of sand while, at another, molecules making up those grains. In general, a given complex can be constituted by a certain group of elements at one level of composition and by a distinct, though related, group at yet another level.

([8]) BERGMANN (1964), p. 51, has noted this, and it should be added that Bergmann has advanced further into the ontological dimension of logical form than almost anyone who has ventured to speak of the forms of non-linguistic entities. The view that form is structure has been championed by many, for example, RUSSELL (1961), p. 52; STEBBING (1961), pp. 35 and 126; and LANGER (1953), p. 34.

There seems to be at least two common uses of 'structure', one in which we mean a complex consisting of parts and the other in which we single out some feature of the complex, that is, *its* structure or the structure it *has*. Call the former a 'concrete structure' or 'c-structure' and the latter a 'structural attribute' or 'a-structure', and represent their connection by saying that the c-structure 'has_i' the a-structure. A house, for instance, is a c-structure; a blueprint depicts an a-structure that it has_i. A c-structure may $have_i$ many a-structures of varying degrees of abstracteness; a house might have several blueprints, some with more information about its composition, e.g., about the walls, that they are so thick, made of wood, or joined in a certain manner.

Where there are exactly *n* elements making up a c-structure at a certain level of composition we can also distinguish the way or mode in which the parts are arranged to constitute that c-structure. The mode – which we will call a 'structuring' – is an abstract network of connectedness whereby the parts enter into and comprise a complex. Unlike an a-structure, a structuring is not an attribute of a c-structure, but each a-structure is associated with a single structuring and distinct a-structures may be associated with one and the same structuring. A blueprint, for example, also conveys *how* parts are arranged so as to constitute a house, but distinct blueprints might be isomorphous in this respect while differing only in their specification of varying types or thicknesses of materials to be used in the construction.

If we recall that a complex entity can be viewed as constituted by different levels of composition, then it is a mistake to *identify* an array of parts *in* a given arrangement or structuring with a c-structure, even though the c-structure is constituted by this array. The point is that each structuring determines *a* level of composition, and so we must distinguish between the massive complex (the c-structure), on the one hand, and the complex-at-a-given-level on the other. Labeling the latter a 'c-level' let us say that each c-level *constitutes* a c-structure and allow that a single c-structure may be constituted by several c-levels (thus, the sand castle by an array of sand grains or by an array of molecules, etc.). A c-level is a complex *at* a certain level of composition; but this is an 'is' of constitution, not identity, and, in a sense, a c-level is an abstraction from the concrete complex.(9) Where

(9) Here we broach a philosophical problem that relates not only to the metaphysics

X is a c-level and Y a c-structure, we abbreviate 'X constitutes Y' by 'X is $_c$ Y'.

These distinctions can readily be applied to our subject matter. Statements are c-structures; logical forms are among their structural attributes, that is, forms are a-structures. Each form is associated with exactly one *propositional structuring* and each statement is constituted by selected components in given structurings, i.e., by *propositional c-levels*, of which there is always a plurality. Such structurings will be represented by expressions resulting from uniform replacements of free variables in form-specifying matrices by numerals, expressions which can be called 'frames'. To illustrate;'1 ⊃ 2' is a frame designating the structuring associated with *p ⊃ q* while '1 ⊃ 1' designates that associated with *p ⊃ p*. A frame reveals two magnitudes of the structuring; the number of distinct occurrences of numerals determines its *adicity* while the number of distinct numerals its *degree*. '*1* ⊃ 2', for instance, portrays a dyadic structuring of the second degree, but '1 ⊃ 1' a dyadic structuring of the first degree. Roughly, the degree of a structuring is the maximal number of distinct entities that can constitute a statement through that structuring, and the adicty reflects the number of *occurrences* of components in a statement at the level of composition determined by that structuring.

A form-specifying matrix is a blueprint of a statement and, hence, a model for a structural attribute of the statement, i.e., for a logical form. The structuring associated with a form is depicted by the syntax of a matrix, that is, by the juxtaposition of variables with logical constants. But can it be claimed that the freely occurring variables themselves represent additional aspects of form, or are they mere place-holders akin to blanks or empty gaps? More to the point, does a matrix like 'p ⊃ q', say, represent anything more than does the frame '1 ⊃ 2', or, for that matter, '– ⊃ . . .'? Is there anything to a logical form beyond a structuring?

of complex entities but also to the issue of reference to many-faceted entities. The point is that it is occasionally necessary to speak of a complex thing with an eye to one, or a few, of its aspects only; and so it is that philosophers speak of an entity "under a description" or, to use the terminology of Castañeda (1977), of a "guise" of an entity, or, as we do, of a "c-level" of a complex. Concerning propositions, our own proposals come closest to Castañeda's theory (1977) inasmuch as our c-levels are analogous to his propositional guises.

These questions call for some remarks about variables. The use of variables to portray form is familiar; it permeates scientific writings suggesting that the notion of *form* extends beyond the boundaries of logic. In this usage, moreover, it is the free occurrences of variables that are crucial in expressing the abstractions under scrutiny. Allied to this usage, at least as concerns statements, is the employment of variables in setting forth generalizations, regardless if variable-binding operators are made explicit, suggesting a connection between form and generality.([10])

It is obvious that some statements are *generalizations upon* others, e.g., *everyone was crafty* is a generalization upon *Churchill was crafty*, and if *A* is a generalization upon *B* then there is some component of *B* that is *generalized with respect to* so as to yield *A* – in the case at hand this is the referent of 'Churchill'. Perhaps not every statement component can be generalized with respect to, but label those that can 'g-components'. Also, where *A* is a generalization upon *B* with respect to g-component X it seems clear that *A* embodies a concept, category, or kind, typically expressed by a common noun, under which X is to be classified. Thus, *everyone was crafty* embodies the kind (concept) being a person, expressed by the term 'one', and the referent of 'Churchill' is an entity of this kind.

Variables are the formal analogues of common nouns, at least when locked into quantifier phrases with a quantifier expression, as in 'for any x'.([11]) This is more readily seen in formalizing restricted generalizations when it becomes necessary to utilize restricted variables in order to give exact expression to these generalizations.([12]) It is

([10]) Russell's writings are rich with suggestions that the connection between form and generality is intimate, for example, in (1962), pp. 248-251; (1919), pp. 198-199; and, especially, (1964), p. 7, where Russell writes that "the process of transforming constants in a proposition into variables leads to what is called generalization, and gives us, as it were, the formal essence of a proposition."

([11]) RUSSELL (1962), p. 228, writes that "the function of variables is exactly that of general words."

([12]) GUPTA (1980) includes a nice discussion of common nouns and restricted variables. Gupta argues against the familiar reduction of restricted generalizations to unrestricted counterparts and goes so far as to suggest that the term 'thing' is not a common noun to be coupled with quantifiers. See, however, my review of Gupta, in *Noûs* (1984), vol. 18, nr. 1.

difficult to resist the suggestion that variables represent precisely the content that common nouns of quantification signify, namely, sortal or kind attributes (concepts). If this assumption is correct then we can speak of the sortals so designated as *intensions* of the variables. Moreover, we can form names of such intensions by prefixing '!' to the variables representing them, e.g., where 'p' ranges over all and only propositions then '!p' signifies the category of "being a proposition." Two variables may be said to be of the *same type* if they have identical intensions, and an item falls within the *range* of a variable only if it falls under the intension of the variable.

The free variables in matrices specifying logical forms do not lose their representative roles. This can be seen by the fact that only *certain sorts* of variables can occur in such matrices, namely, those whose intensions embody only *logical* content, i.e., content characterizables in purely logical terms. Within the present framework, a language suitable for the expression of logical form provides for three general sorts of logical variables; those ranging over all and only *individuals* (subjects of predication), those over *predicables* (properties and relations), and those over *propositions* (or, more generally, *affirmables* ([13])). There is no barrier to the introduction of variables harboring yet more restriction, for instance, those ranging over predicables of a specified adicity, or over sets, or over all and only conjunctive propositions. It is misleading, therefore, to regard logical variables as mere place-holders or blanks; the latter do not discriminate among different logical categories nor among possible substituends. The intensions of logical variables seem as important in identifying logical form as constants and linear juxtapositions, and, if so, then forms cannot be identified with their associated structurings, otherwise we could not distinguish the form $*x \in y*$ from $*x \in A*$, or $*x = y*$ from $*A = B*$, where 'x' and 'y' range over all individuals and 'A' and 'B' over sets, or, again, $*p \supset q*$ from $*p_c \supset q*$ where 'p_c' ranges over conjunctions only.

This sprinkling of ontology supplies a basis for accommodating the

([13]) By an *affirmable* is meant whatever can be affirmed, asserted, or endorsed. Propositions are paradigmatic affirmables, but one might also include questions and practical thought-contents (imperatives, intentions) if these are not propositional (cf., CASTAÑEDA (1975) *passim*).

observations of section II, for resolving the problem of correspondence distinguished in section III and, ultimately, for setting (LF) upon a more secure philosophical foundation.

V. *A set-theoretic representation of form*

Let M be a matrix which specifies a logical form F and let $(u_1, \ldots, u_n)$ be a sequence comprised of the n distinct variables occurring freely in M in the order in which they occur in M from the left, $n \geq 1$. Correlated to this n-adic sequence is another, $(!u_1, \ldots, !u_n)$, where $!u_i$ is the intension of u_i for each i, $1 \leq i \leq n$, and label this the 'intensional array of F'. Where s(F) is the propositional structuring associated to F, specified by the frame obtained from M, we will call the pair $((!u_1, \ldots, u_n), s(F))$ the 'formal coupling of F'. Given that variable intensions are sortals then *formal couplings* constitute a subclass of pairings of n-tuples of sortals with propositional structurings of the nth degree, and a principle is available for demarcating this subclass. First, let 'f', with or without subscripts, range over all sortals and, for natural numbers m and n, '$f^{m,n}$' over m-adic propositional structurings of the nth degree. Next, where n elements $a_1, \ldots, a_n$ are arrayed in a structuring $f^{m,n}$ to constitute a statement A, let us extend an earlier convention to allow the result of enclosing in asterisks the concatenation of an expression for $f^{m,n}$ with an expression of the n-tuple $(a_1, \ldots, a_n)$ to be a name of the relevant propositional c-level, and permit such an expression to be a context open to quantification into. We have the following:

(P1) For any sortals $f_1, \ldots, f_n$ and any structuring $f^{m,n}$, the ordered pair $((f_1, \ldots, f_n), f^{m,n})$ is a formal coupling *if and only if* for each n-tuple of entities $(a_1, \ldots, a_n)$, if a_i has (falls under) f_i $1 \leq i \leq n$, then there is a statement A such that $*f^{m,n}(a_1, \ldots, a_n)*$ is_c A.

Where C is the class of all and only formal couplings, as determined by (P1), let C^* be that subclass each of whose members contains only logical sortals, i.e., the intensions of logical variables, in its first member and a structuring specifiable by a frame containing only logical constants (and numerals) as its second member. The next principle is fundamental:

(P2) There is a one-to-one correspondence mapping C^* onto the class of all and only the logical forms of statements.

With this, an expression designating a member of C^* can double as a representation of the corresponding form, an allowance underscored by the following differentiating principle:

(P3) For any logical forms F_1 and F_2, $F_1 \neq F_2$ *if and only if* F_1 and F_2 differ in their associated structurings or in their intensional arrays.[14]

Some examples serve to illustrate (P1)-(P3). Corresponding to the form $*(p \& q) \supset r*$ is the pair $((!p, !q, !r), (1 \& 2) \supset 3)$. This form differs from the valid $*(p \& q) \supset p*$, whose formal coupling is $((!p, !q), (1 \& 2) \supset 1)$, and also from the form $*p_c \supset q*$ whose formal coupling is $((!p_c, !q), 1 \supset 2)$. In both contrasts the intensional arrays differ as well as the structurings, though this is not always so; $*p_c \supset q*$ differs from $*p \supset q*$ only in arrays while $*(p \& q) \supset p*$ differs from $*p \& q*$ only in associated structurings.

How is the minimal form *p* to be interpreted in these terms? In particular, how is *p* a "structure" of a statement at all? A passage from Wittgenstein's *Tractatus* is suggestive:

> The existence of a general propositional form is proved by the fact that there cannot be a proposition whose form could not have been forseen (i.e., constructed). The general form of a proposition is: This is how things stand.[15]

[14] Logical forms cannot be differentiated by what were earlier called the 'logical powers' (section III) that such forms confer upon statements having them. Thus, distinct forms might confer upon their statements exactly the same logical powers, for instance, $*\sim(p \& q)*$ and $*\sim p \vee \sim q*$. Cf., CARGILE (1979), p. 14, who makes a similar point.

[15] WITTGENSTEIN, *op. cit., 4.5.*

This is highly abstract, but perhaps the phrase 'this is how things stand' reveals an important aspect of a statement. Note, first of all, that the intensional array of *p* is, simply, (!p), where !p is the categorical kind of being a statement. What is the associated structuring? Perhaps it is aspect of a statement which renders it *more* than a mere "object" of thought, that is, a subject of predication; a statement is something that *can be affirmed*, and it is this that is conveyed by the phrase 'this is how things stand'. In the present context, such an aspect is a way in which every statement is "structured" and, so, can be labeled 'the structuring of affirmability'. Tentatively, then, the minimal form corresponds to the pair ((!p), 1), where '1' signifies this general structuring.

A statement has a given form just because it contains g-components which, at a certain level of composition, fall under the logical sortals that they do and are arrayed in a structuring so as to constitute that statement. The principle governing the having$_i$ of forms by statements is as follows:

(P4) For any statement A and logical form F whose formal coupling is $((f_1, \ldots, f_n), f^{n,n})$, A has$_i$ F *if and only if* there is a sequence $(a_1, \ldots, a_n)$ of k distinct g-components of A such that (1) a_i has (falls under) f_i for each i, $1 \leqslant i \leqslant k \leqslant n$, and (2) there is a c-level $*f^{n,n}(a_1, \ldots, a_n)*$ which is$_c$ A.

The statement *~(I am hungry ⊃ I am happy)*, for example, has the forms *~p* and *~(p ⊃ q)*. It has the first since it is constituted by the c-level *~1(*I am hungry ⊃ I am happy*)*, where *I am hungry ⊃ I am happy* falls under the category !p, and it has the second since it is also constituted by *~(1 ⊃ 2)(*I am hungry*, *I am happy*)*, where each of the simple statements falls under the category of being a statement. Why does this statement fail to have the form *~(p ⊃ p)* which *~(I am hungry ⊃ I am hungry)* has? In brief, the latter has both forms *~(p ⊃ q)* and *~(p ⊃ p)* since it is constituted by the c-level *~(1 ⊃ 2)(*I am hungry*, *I am hungry*)* as well as by *~(1 ⊃ 1)(*I am hungry*)*. The former, on the other hand, fails to have the form *~(p ⊃ p)* since '~(1 ⊃ 1)' does not specify a structuring of any sequence of its g-components in such a way to constitute it.

VI. *Implication and correspondence, again*

Having explicated the conditions under which a statement possesses a given form and having accounted for the sense in which one statement can have many forms, we can now consider the issues raised in section III. Of central importance is the notion of a corresponding statement of a given form, and in the present approach to it we must first develop means for determining when compound forms are values of certain functions applied to simpler forms, the functions in question being the statement connectives.

If statement *A* has_i a form F whose formal coupling is $((f_1, \ldots, f_n), f^{n,n})$ then let us use '*A*/F' to stand for that sequence $(a_1, \ldots, a_n)$ such that the c-level $*f^{n,n}(a_1, \ldots, a_n)*$ is_c *A*. Where *O* is any *monadic* connective then *O*(*A*) is the statement determined by applying *O* to *A*, *O*(F) is the form determined by applying *O* to F, and, where *S* is any structuring, then *O*(*S*) is the structuring specified by applying *O* to *S*. The functional character of *O* when applied to forms and structurings is revealed in the following:

(P5) For any statement *A* and form F whose formal coupling is $((f_1, \ldots, f_n), f^{n,n})$, if A has_i F and $(a_1, \ldots, a_n)$ is *A*/F then there is exactly one *m*-adic *n*th degree structuring $O(f^{n,n})$ such that (1) there is just one c-level $*O(f^{n,n})(a_1, \ldots, a_n)*$ which is_c *O*(*A*) and (2) there is just one form *O*(F) whose formal coupling is $((f_1, \ldots, f_n), O(f^{n,n}))$ such that *O*(*A*) has_i *O*(F).

To illustrate; if *p & q* is ((!p, !q), 1 & 2) then the value of the negation operator applied to this form is ((!p, !q), ~(1 & 2)), i.e., *~(p & q)*, and if *I am glad and he is sad* has_1 *p & q* then *~(I am glad and he is sad)* has_i the form *~(p & q)*. With (P5) we have the means for accomodating the principle (2) put forth in section II since the existence and uniqueness of ~F is assured once we have identified F.

The situation is more complicated for non-monadic connectives of which only the dyadic case will be treated. Three further notions must be introduced, labeled respectively 'merger', 'proxy' and 'synthesis'. Where x_i is the *i*th term in an *n*-termed sequence $(x_1, \ldots, x_n)$ let '$e(x_i)$' represent the entity that occupies the *i*th position – noting that fewer than *n* distinct entities might occur in the sequence – and let '$\bar{x}_n$'

abbreviate '$(x_1, \ldots, x_n)$'. Given sequences $\bar{x}_n$ and $\bar{y}_m$ their *merger* – represented by '$M(\bar{x}_n, \bar{y}_m)$' – is a $(n+j)$-adic sequence, $0 \leqslant j \leqslant m$, constituted by $x_1, \ldots, x_n$ in that order and followed by exactly j of the terms in $y_1, \ldots, y_m$ whose associated elements are distinct from any of $e(x_1), \ldots, e(x_n)$ taken in order of their occurrence in $(y_1, \ldots, y_m)$. For example, the merger of (1, 2, 1) with (1, 2, 3, 4) is (1, 2, 1, 3, 4). Where each $e(y_i)$ is identical to some $e(x_j)$, $1 \leqslant i \leqslant m$ and $1 \leqslant j \leqslant n$, then $M(\bar{x}_n, \bar{y}_m)$ is simply $\bar{x}_n$.

Let F_1 be a form with formal coupling $((f_1, \ldots, f_n), f^{m,n})$ that is had_i by a statement A where sequence $\bar{a}_n$ is A/F_1. Where f_i is the ith term in $(f_1, \ldots, f_n)$ and a_i the ith term in $\bar{a}_n$, $1 \leqslant i \leqslant n$, let us call f_i the ',proxy' of a_i with respect to F_1 and A, and let '$P(x)$' represent the *proxy function* so that $P(a_i) = f_i$. An exact characterization of this function is beyond reach at present because more must be said about structurings than has so far been offered. Suppose that F_2 is a form with a formal coupling $((g_1, \ldots, g_k), f^{h,k})$ that is had_i by a statement B where sequence $\bar{b}_k$ is B/F_2. A *synthesis* of F_2 to F_1 is any sequence $(f_1, \ldots, f_n, g_i, \ldots, g_j)$ where $0 \leqslant i \leqslant j \leqslant k$. The *minimal synthesis* of F_2 to F_1 relative to the pair (A, B) is the $(n+k)$-termed sequence $(\bar{f}_n, \bar{g}_k)$, and the *maximal synthesis* of F_2 to F_1 relative to (A, B) is that synthesis which is similar (isomorphic) under the converse of the proxy function to $M(\bar{a}_n, \bar{b}_k)$.([16]) That is, this sequence – represented by '$S^{+}_{A.B}(\bar{f}_n, \bar{g}_k)$' – is a sequence $(P(a_1), \ldots, P(a_n), P(b_i), \ldots, P(b_j))$ where $0 \leqslant i \leqslant j \leqslant k$. Obviously, if the merger $M(\bar{a}_n, \bar{b}_k)$ is $(n+j)$-adic, $0 \leqslant j \leqslant k$, then so is $S^{+}_{A.B}(\bar{f}_n, \bar{g}_k)$.

Where O is any dyadic connective then $O(A, B)$ is the statement determined by applying O to the pair (A, B). Now there may be several different structurings and forms determined by the application of O to the pairs $(f^{m,n}, f^{h,k})$ and (F_1, F_2) respectively, in fact, as many as there are distinct syntheses of F_2 to F_1. We are connected, primarily, with the structuring and form posited in the following:

(P6) For any forms F_1 and F_2 whose formal couplings are, respectively, $((f_1, \ldots, f_n), f^{m,n})$ and $((g_1, \ldots, g_k), f^{h,k})$, and for any statements A and B where A has_i F_1, B has_i F_2, A/F_1 is $\bar{a}_n$, and B/F_2 is $\bar{b}_k$, there is exactly one $(m+h)$-adic $(n+j)$th

([16]) See SUPPES (1972), p. 128, for a definition of similarity under a function.

degree structuring $O(\mathfrak{f}^{n,n}, \mathfrak{f}^{h,k})$, $0 \leq j \leq k$, such that (1) there is just one c-level $*O(\mathfrak{f}^{n,n}, \mathfrak{f}^{h,k})(M(\bar{a}_n, \bar{b}_k))*$ which is$_c$ $O(A,B)$ and (2) there is just one form $O(F_1, F_2)$ whose formal coupling is $\mathfrak{f}^{n,n}, \mathfrak{f}^{h,l}$)) such that $O(A,B)$ has$_i$ $O(F_1, F_2)$.

Given statements A and B, forms F_1 and F_2 and structurings $\mathfrak{f}^{n,n}$ and $\mathfrak{f}^{h,k}$ as determined in this principle, we can represent the posited structuring and form by '$O(\mathfrak{f}^{n,n}, \mathfrak{f}^{h,k})/O(A,B)$' and '$O(F_1,F_2)/O(A,B)$' respectively so as to emphasize that compound structuring and form are fixed only in reference to the compound statement $O(A,B)$. From (P6) together with (P4) it follows that $O(A,B)$ has$_i$ $O(F_1,F_2)/O(A,B)$.

Given (P6) the central notion of correspondence is readily characterized. At its core, correspondence presupposes a sharing of g-components so that if A is a statement of form F_1 then B is a corresponding statement of form F_2 only if some element in B/F_2 is also an element in A/F_1. We have;

(P7) For any statements A and B, if A has$_i$ a form F_1 then B is a corresponding statement of form F_2 *if and only if* where $\mathfrak{f}^{n,n}$ and $\mathfrak{f}^{h,k}$ are, respectively, the associated structurings of F_1 and F_2 then, for any dyadic connective O, $O(\mathfrak{f}^{n,n}, \mathfrak{f}^{h,k})/O(A,B)$ is a structuring of degree i, where $i < (n+k)$.

The use of the indefinite article 'a' instead of 'the' is due to the fact that for given A and F_1 there may be more than one corresponding statement of form F_2, e.g., if A has *p* then not only is AvB a corresponding statement of form *p v q* but so is A v C where B and C are distinct. The definite article is appropriate only if B/F_2 contains no elements other than those that occur in A/F_1, for instance, if A & B has$_i$ *p & q* then there is only one corresponding statement of form *p*, namely, A. To return, now, to the example considered at the outset of this paper; if *this is white and this is rectangular* has$_i$ *p & q* why is not *this is scarlet* a corresponding statement of form *p* since it has$_i$, after all, *p*? By (P6) there is a structuring S such that the posited c-level *S(M((this is white and this is rectangular), (this is scarlet))* is$_c$ the statement O((this is white and this is rectangular), this is scarlet), where O is any dyadic connective, and this structuring is O((1 & 2), 3)/O((this is white and this is rectangular), this is scarlet). Since this is a structuring of the *third* degree, however, then, by (P7),*this is scarlet* is *not* a corresponding statement of form *p*.

The notion of correspondence embodied on (P7) hinges upon some degree of content sharing between corresponding statements, specifically, a sharing of g-components. With it, a more accurate definition of form-necessitation is,

Def. 1 $N(F_1,F_2) =_{df}$ if any statement which has_i F_1 is true then any corresponding statement of form F_2 is true.

The advantage of this over the previous construal of section III is the allowance that a given statement of form F_1 corresponds to more than one statement of form F_2. Principle (3) of section III can be retained with this adjustment, but it is to be observed that the type of implication fostered by this principle, given (P7) and *Def. 1*, is one that requires content sharing between implicans and implicandum, that is, some type of *relevant* implication.([17]) If, however, one alters (P7) by making the index *i* equal to *n*, thereby assuring the uniqueness of a corresponding statement, then (3) supports a brand of *analytic* implication where the implicandum merely "unpacks" the implicans.([18])

It would be undesirable if an attempt to resolve one problem facing the principle (LF), namely, that of correspondence, should preclude the more familiar construal of implication in terms of the impossibility of the implicans being true while the implicandum is false. Fortunately, an account of implication is available which, though conforming to (LF), does not require content sharing and is, to this extent, independent of correspondence. The construal is based directly upon (P6):

([17]) ANDERSON and BELNAP (1975) contains the most sustained treatment of relevant implication to date. I am using the expression 'relevant implication' in a broad sense to mean that sort of implication which requires content-sharing between implicans and implicandum.

([18]) Cf., PARRY (1933) for an example of a system of analytic implication which demands that the conclusion of a valid argument contain no more than what is contained in the premises. A metatheorem of this system is that if *B* is analytically implied by *A* then all the variables occurring in *B* also occur in *A*. The system fails, thus, to contain the law $p \rightarrow (p \vee q)$.

(P8) For any propositions A and B, A logically implies B *if and only if* A has_i a logical form F_1, B *has_i* a logical form F_2, and the form $\supset(F_1, F_2)/A \supset B$ is valid.

Though other forms of $A \supset B$ are constructible from forms F_1 and F_2, e.g., that whose intensional array is the minimal synthesis of F_2 to F_1, forms which also might be valid, the form $\supset(F_1, F_2)/A \supset B$ has been described in such a way to insure that these are among its approximations and, so, by principle (1) of section II, it is sufficient to determine implications in its terms. By (P8), a statement with a valid form is implied by any statement, and one with an unsatisfiable form implies every statement. But though (P8) supports this broader concept of implication, it, like principle (3), requires the truth of both (4) and (5), and this is further evidence that (P8) is a genuine rival to (3) within the programmatic confines of (LF).

Finally, it is not possible to present here a precise derivation of principles (6) and (7) of section III from the other claims without a more detailed scrutiny of refinement. Still, they seem warranted by the account of correspondence, form-necessitation, and implication that has been offered. In rough terms, refinement reflects deeper levels of statement composition and yields more exact forms, structurings, and c-levels from less exact counterparts. As such, refinement may be said to rest upon an analysis of various g-components. If the g-components are shared by distinct statements A and B then their analysis as regards A can be paralleled by a like analysis as concerns B, and, therby, the more refined form produced for A will be matched by a more refined form of B. If the analyzed components of A are not shared by B then the form F_4, posited by (6) and (7), is simply F_2, remembering that refinement is reflexive.

VII. *Concluding Remarks*

The foregoing account has been designed to clear away one stumbling block in the way of a systematic elucidation of (LF). In a sense, the problems of correspondence may be relatively minor, hardly requiring the machinery of sections IV-VI above. On the other hand, the concept of *logical form* remains relatively vague, despite the attention given to it, and some efforts towards precision seem necessary to fend off the slightest counterexamples to one of the more important theses of logical theory.

Certains strengths of the account should be emphasized. For one thing, the proposals are broad enough to adjust to different perspectives on the nature of statements, specifically, to views which construe the bearers of logical properties and relationships as propositions and to those which remain content with sentences. In either case, statements are c-structures containing g-components which can be classified under various logical categories in a manner required by principles (P4)-(P6). Secondly, the account is flexible enough to admit different interpretations of implication, as has been indicated in section VI. Thirdly, the analyses offered can be readily extended to other notions of interest to the logician, specifically, to consistency, incompatibility, logical truth, and logical necessity, since all interrelate and can be reduced to questions of the validity, satisfiability and unsatisfiability of logical forms.

The extension of the account to the one-placed alethic modalities is of independent philosophical interest. If statements are identified with *propositions* whose structural attributes include logical forms, then we have the makings of an "applied semantics" that, for at least *de dicto* logical necessity, would rival the familiar possible worlds approach.([19]) With some imagination, the contents of formal couplings could be altered in such a way to incorporate extra-logical content, so giving rise to a conception of extra-logical a-structures of propositions or *material forms* which would make it possible to interpret languages that embody extra-logical modalities and relationships of material consequence.([20]) Whether such a program could,

([19]) PLANTINGA (1974), pp. 125-128 and 248-251, contrasts *applied* with *pure* semantics, where the former seems concerned with supplying meaning to expressions of a language in terms of "real" structures so that its models are purported to reflect ontological commitment in a way that the models of pure semantics do not. See also HAACK (1978), pp. 187-194.

([20]) A discussion of extra-logical or material consequence in relation to material forms is contained in KAPITAN (1982). Other philosophers have employed the terms 'form' and 'propositional form' in this broader fashion as well, to allow that a matrix like 'x is red' specifies a form, for example, RUSSELL (1964) pp. 85-88 and (1919), p. 158; LEWIS and LANGFORD (1959), p. 264; and CASTAÑEDA (1975), chp. 3. A number in turn, be applied to quantified modal logics is yet another matter, but here the connection between form and generally, accented by Russell, is pregnant with possibilities.

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of writers, it may be added, have observed that the distinction between "logical" and "extra-logical" is problematic, e.g., TARSKI, von WRIGHT, PAP and QUINE. More recent discussions of this matter can be found in PEACOCKE (1976), HACKING (1979) and HAACK (1978). Cf., also, the treatment of the issue in KNEALE and KNEALE (1962), pp. 737-742. Our concern here, however, is not so much with *which* logical forms there are but with how (LF) is to be grounded in a theory of logical form.(*)

(*) While developing the ideas of this paper the author was a fellow in a National Endowment for the Humanities Seminar (1980-81) offered at Indiana University under the direction of Professor Hector-Neri CASTAÑEDA. Much of the inspiration for the present work derives from the theories set forth in CASTAÑEDA (1975) and (1977).

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