GLIMPSES OF THE DISASTROUS INVASION OF PHILOSOPHY BY LOGIC

Wulf REHDER

0. Introduction

Imagine the following ingenious method for solving philosophical problems. Let P denote the problem under investigation. Assume that P admits a logical representation in a sufficiently rich language L, state a set A_P of axioms plus some rules of transformation and inference, including the modus ponens. Call the triplet S = (P, L, A_P) our formalized philosophical problem. Now! Let f(S) be a function of S whole value T = f(S) is the (or one) solution of S. Properties of this "solving function" f are studied in the rest of our imagined paper. Also some generalizations are suggested, and the philosophical impact of f and T is discussed and related to recent publications on the same subject matter.

A caricature? Of course it is. But we'll see that this caricature has some striking, if exaggerated, similarities with a very popular modern approach to philosophy. This approach is adorned with a respectable name: "Philosophical Logic", although quite a few of its aspects should rather remind us of a variation of a famous Wittgensteinian theme, the disastrous invasion of philosophy by logic.

In sustaining the claim of this paper's polemic title, I do not expect to refute all of Philosophical Logic, and certainly this very lively offspring of the old Maid Philosophy who fell (for want of other lovers?) for this dashing young parvenu Mathematical Logic – this very lively half–breed, I say, will survive our weak attack. As well it should. So, in the end, this is just some old-fashioned advice for Philosophical Logic to avoid triviality during the upcoming formative years of growing up.

What is it then that we intend to do and hope to achieve here? It's this: we'll take three prominent examples from the established body of published work in philosophical logic, and show that all three follow the pattern of procedure caricatured above. The examples are the following:

1. Stalnaker's theory of conditionals and the so-called Stalnaker thesis.
2. G. H. von Wright's tense logic T.
3. Fads and fallacies in quantum logic.

1. Stalnaker's theory of conditionals.

Let it be confessed right away, lest I am accused of ignorance or worse, of bad philosophical taste: Stalnaker's "A Theory of Conditionals" [17] and the subsequent very influential paper on "Probability and conditionals" [18] are exciting and original contributions adressing a new field of research, the study of counterfactuals and conditionals, in a fresh way.

In "A Theory of Conditionals", Stalnaker's principal concern is the *logical problem of conditionals*:

> "This is the task of describing the formal properties of the *conditional function*: a function, usually represented in English by the words "if...then", taking ordered pairs of propositions into propositions." ([17], p. 41).

After discussing the shortcomings of truth-functional analysis, which assigns a truth value to the if-then proposition, treating it like an ordinary material implication and after dismissing as "not a necessary condition" the idea of a logical or causal *connection* between antecedent (if) and consequent (then), Stalnaker offers his own solution. Based on an idea by F. P. Ramsey, this is how to evaluate a conditional:

> "First, add the antecedent (hypothetically) to your stock of beliefs; second, make whatever adjustments are required to maintain consistency (without modifying the hypothetical belief in the antecedent); finally, consider whether or not the consequent is then true." ([17], p. 44).

How do I climb this 3-step analysis technically? The ready made ladder is Kripke's semantical system for modal logics:

> "An analysis in terms of possible worlds also has the advantage of providing a ready made apparatus on which to build a formal semantical theory." ([17], p. 45).

It is here that we enter shallow waters: "providing a ready made apparatus". Very quickly the translation of the problem of counterfactual conditionals into Kripke's logical model structure is performed, and this is done not without intuitively convincing features. Immediately thereafter, however, the devil's foot shows: enter the solving function f.

> "In addition to a model structure, our semantical apparatus includes a *selection function*, *f*, which takes a proposition and a possible world as arguments and [has] a possible world as its value. The s-function selects, for each antecedent A, a particular possible world in which A is true. The *assertion* which the conditional makes, then, is that the consequent is true in the world selected. A conditional is true in the actual world when its consequent is true in the selected world." ([17], p. 45).

This is quite a task and responsibility for a selection function! But not enough, the s-function must meet some more (at least four, see [17], pp. 46, 47) conditions, which have a strong impact upon the selected worlds (they establish an ordening, which is of no interest for our purpose here). All these restrictions on f

> "are necessary in order that this account be recognizable as an explication of the conditional, but they are of course far from sufficient to determine the function uniquely". ([17], p. 47).

That's bad enough, but let's aim at Stalnaker's weakest point; he forgets *the crucial question*: *is there such a s-function f at all?* Stalnaker bypasses the problem of the *existence of a "solving function"*, as I called it at the beginning. An affirmative answer as to the existence of f is vital, otherwise the entire system operates in very thin air, or, technically speaking, on the empty set.

Yet not enough that the existence of a selection function is in limbo. Stalnaker proceeds to coaxing us into "the class of valid formulas of conditional logic according to the definitions sketched" by offering a formal system, C2, which is to impress us through being

> "proved sound and semantically complete with respect to the interpretation sketched in the present paper. That is, it is shown that a formula is a consequence of a class of formulas if and only if it is derivable from the class in the formal system, C2". ([17], p. 54).

We accept this pleasing statement of "goodness" of C2 with satifaction, and, nevertheless, take a closer look at it. We find, that C2 is rather rich in its expressive power: it contains, besides the usual propositional logic, as a further primitive a conditional connective > (called the corner), upon which the modal notions of necessity and possibility are based. Furthermore, seven axiom schemata regulate how all the introduced notions are "compatible" with each other; e.g. it is stipulated that the corner is intermediate in implicative strength between strict implication and the material conditional.

And again, there is the crucial question: *is there such a conditional connective as the corner, with its purported properties?* Stalnaker does not raise the *problem of the existence* of the "solving function", which is, in this case, a binary relation: the corner. The air is getting even thinner.

In the remainder of that section, Stalnaker studies, instead, properties of the corner, which behaves, it turns out, unlike both the material and strict implication: (a) the corner is not a transitive relation, and consequently the "rule of strengthening the antecedent" is not valid: from $A > B$ we cannot generally infer $(A \& C) > B$; (b) the denial (negation) of a conditional for a possible antecedent A is equivalent to a conditional with the same antecedent and opposite (negated) consequent, that is: $\sim(A > B) = (A > \sim B)$; (c) thirdly, the law of contraposition does not hold: $A > B$ is not equivalent to $\sim B > \sim A$.

These findings "have been noted by philosophers in the past", i.e. Stalnaker is now relating his theory to publications of his predecessors (Goodman and Chisholm), following exactly the pattern outlined in caricature at the beginning of section 0.

I have mentioned the three points (a), (b), and (c) for a different reason, which is important here. Namely, there has been an attempt to supply the missing existence proof for the selection function and for the corner. In his paper "The Conditional in Abstract and Concrete Quantum Logic" [7], G. Hardegree claims to have found, in a Hilbert space setting, concrete representatives for the s-function and for the corner. The former is, essentially, an orthogonal projection onto a subspace (which represents a yes-no proposition, e.g., the above A, B, C,...), the latter is identified as the so-called Sasaki arrow, a connective which is the closest possible analogue to material implication in the non-boolean lattice of closed subspaces in a Hilbert space.

Unfortunately, however, the Sasaki arrow does not behave like the corner with respect to negation, see (b) above, whereas it displays non-transitivity (a) and contradicts the law of contraposition (c) as the corner does. There may be (and there *are*) other shortcomings of Hardegree's modelling C2 by a quantum-logic inspired setting in Hilbert space. It suffices to say that he did not succeed in proving the existence of a conditional connective as required by Stalnaker's theory.

On the other hand, Hardegree's approach is methodically the correct and needed one. In order to avoid theories about the empty set, we have to be able to rely on at least one *concrete model*, whose existence can be proved (e.g., by constructing it from sets of numbers, functions, etc., whose existence – like that of N, R, $L^2(R)$ – is not questioned).

In other words, one *instantiation*, one *example* of the theory in a tangible *model* is mandatory for it to be taken seriously. This model must at least reflect, by its intrinsic structure, the principle features of the purported theory, in Stalnaker's case, e.g., we must be able to find the corner in the object language of the model. It need not, perhaps, be of necessity intuitively and immediately clear to everybody; why should, at first sight, the Hilbert space model of square integrable functions represent the basic features of quantum theory? This is not immediate, and requires training and sophistication, not common sense. The proposed model also need not be unique or canonical, although, if these properties *can* be had, a unique canonical (mathematical) representation of a physical theory (say) is always looked for, and sometimes found. (Famous examples are the "equivalent pictures" of quantum theory given by Heisenberg, Schrödinger, Dirac). Back to Stalnaker's theory.

In "Probability and Conditionals", Stalnaker employs the interpretation of a conditional probability statement as a "semantic or pragmatic relation between two propositions" ([18], p. 107), representing a measure for conditional or hypothetical knowledge, "the degree to which he *would* have a right to believe certain proposition *if* he knew something which in fact he does not know" ([18], p. 114). Taking a conditional probability function as primitive, Stalnaker has the means of interpreting the absolute probability of his conditional connective, the corner, in terms of conditional probabilities, stating in his well-known thesis (or hypothesis):

> "The absolute probability of a conditional proposition – a proposition of the form $A > B$ – must be equal to the conditional probability of the consequent on the condition of the antecedent,
>
> $$\Pr(A > B) = \Pr(B \mid A)\text{''}$$
>
> ([18], p. 120).

Indeed, in v. Fraassen's suggestive phrasing of the last equality: "What is the probability that I throw a six if I throw an even number, if not the probability that: if I throw an even number, it will be a six".

No doubt, Stalnaker's thesis agrees with our linguistic intuition, and it sounds attrative. The sad reason why we need not pursue the matter any further, is D. Lewis's triviality result, which was "a devastating result" ([18], p. 13) for Stalnaker's thesis, in fact to his whole theory. Lewis's result states that any probability function Pr which satisfies the Stalnaker thesis and the identity

$$(*) \qquad \Pr(A > B \mid C) = \Pr(B \mid A \wedge C)$$

if $\Pr(A \wedge C) \neq 0$, is "trivial" in the sense that Pr has at most four different values ([10], p. 134). The blame of triviality can also be laid upon the language in which we form the sentences A, B, C,...: "any language having a universal probability conditional [or one satisfying (*)] is a trivial language", i.e., it cannot provide three possible but pairwise incompatible sentences ([10], p. 132). I prefer to put the blame upon the corner: Lewis' triviality result shows that *there is no conditional connective such as Stalnaker's corner*, more precisely, the Stalnaker corner can be added to the list of primitive symbols in the object language of C2 only at the cost of ousting Stalnaker's thesis.

I wish to quickly add that even without this contradiction the existence of the corner would not have been assured at all.

Lewis' proof of his result is very elementary. It may in this context be of interest to mention that the proof not only uses the five axioms on p. 130 of [10], but also the distributive law for propositions: (11), p. 132 of [10]. This is essential to his argument. Propositions in Lewis' paper therefore are assumed to form a Boolean algebra (this is later acknowledged explicity on p. 136 of [10]).

2. *G. H. von Wright's tense logic T.*

Stalnaker's theory of conditionals was meant as a logical analysis of "if...then" sentences expressing propositions not explicitly depending upon time. Even though the "if... then" proposition usually assumes a succession in time (the if-clause generally containing the earlier condition or cause, whereas the then-clause asserts the later consequent or effect), time does not enter the formulation as an extra variable. This is only done, in many variations, in the existing tense logics. We choose as our paradigm v. Wright's tense logical calculus T, which he developed in his paper "And next" [20]. This tense logic is supposed to be the basis underlying a general theory of action which naturally presupposes a logic of change. A change in time takes place when the present state of affairs p is transformed into a state of affairs q, where $q = p$ is not excluded: p then continues to obtain. In order to capture the formal properties of "p changes into q", v. Wright introduces a tense-logical binary constant T which is to play the role of a non-commutative connective, and p T q is read 'p and next q". The resulting T-calculus is presented as a formalized axiomatic structure which embraces classical two-valued propositional logic, has four axioms regulating how the T-relation fares with respect to "or", "and", and negation; and there are three harmless rules of inference, including modus ponens.

We see that v. Wright goes about his task of translating the philosophical problem of time-dependent propositions into a logical framework in the same pattern as did Stalnaker: a binary connective T is "introduced" (what an innocent word!) *in order to do the job*, that is, TO SOLVE THE PROBLEM. Some intuitively satisfying axioms for T are set down, and again the remainder of the work is devoted to establishing soundness and completeness of the calculus thus created (see v. Wright's [20] and Aqvist [2]). As if by this demonstration the T-calculus were well established as a working tense-logical model, it is then applied to problems in deontic logic [2], a general theory of action, and complemented, in the same fashion, by a logic of the past [4].

I have shown elsewhere [15] that v. Wright's relation T is just one instance of infinitively many possible homomorphisms on the Boolean algebra of propositions, and have offered an alternative which is not quite as arbitrary but shows, in contrast to T, some uniqueness properties. I have also discussed (see section 4 of [15]) conditions under which tense-logical functions may be used to define an S4 necessity operator. And again, I arrived at a certain "triviality result", which is, perhaps, even counterintuitive and a fatal consequence for setting up a tense logic in the naive and intuitive way of v. Wright: there is, as can be shown, always at least one proposition which does not change at all, it is a fixpoint of the time operator derived from T (see [15], end of section 4).

To sum up: v. Wright's and other existing tense logics do but "introduce" a new variable t, or a corresponding time operator, or a relation T, into an enlarged system of propositional logic which is *claimed* to represent out intuitive sense of tense, time, and change.

I repeat, as with Stalnaker's theory, the duty of constructing *concrete* mathematical models, including existence proofs for the alleged solving function. Otherwise, the logical "theory" is nothing but a fancy abbreviation by logical symbols of the philosophical problem, and "time", in particular, is nothing but a dummy variable.

3. *Fads and fallacies in quantum logic.*

As I have mentioned at the close of section 1 above, Lewis' triviality result rests upon the additivity of probability functions *and* on the distributivity for propositions. This means that any proposition A can be decomposed equivalently into the "sum" of two propositions "A and B" and "A and non-B". Indeed, if A stands for "throwing an even number" and B for "throwing a number smaller or equal to three", non-B "throwing a number four, five, or six", then asserting A is obviously equivalent to asserting "$(A \wedge B) \vee (A \wedge \text{non-B})$" where $\wedge$ and $\vee$ denote Boolean conjunction ("and") and disjunction ("or"), respectively.

The first of Lewis' assumptions, viz. additivity of probability functions, is also intuitively clear, and is in no probability theory disputed. It is, therefore, a surprising and unique feature of quantum mechanical probabilities that distributively of propositions is questioned upon good reasons, and additivity of probability functions does not seem to be attainable for mathematical reasons. The cause for both of these anomalies is often said to be Heisenberg's uncertainty relation, and often quoted paradigms are the canonical observables q (position) and p (momentum) and Young's two slit experiment; and for remedy a new logic, quantum logic, is proposed, which abandons the law of distributively:

> "Because of the indeterminacy principle standard logic, which is distributive, does not hold. Consider a particle with momentum p confined in a box of volume V, which may be conceptually subdivided into n subvolumes v_i of such a size $pv_i = h/10$, where h is Planck's constant. Through the use of projection operators one could also let p (or v_i) stand for the statement "The particle has momentum p (or is in the volume v_i)". Then the distributive law proper to classical logic suggests:
>
> $$p \,\&\, V = (p \,\&\, v_i) \vee (p \,\&\, v_2) \vee \ldots \vee (p \,\&\, v_n).$$
>
> The left side of this supposed identity is simply a restatement of the boundary conditions. Each disjunct on the right is forbidden by Heisenberg's indeterminacy principle. With the standard definition of '&' and '$\vee$' a true statement comes out logically equivalent to a disjunction of statements, each of which is false" ([12] pp. 523-524).

Let us have a second look at the structure of this argument: the author describes a quantum mechanical situation and states, using logical symbols and abbreviations ("through the use of projection operators"), the disputed identity, which is then rejected on the ground that the minimum value for the product of p and v_i is, in quantum logic, given by h/2 (Heisenberg), and thus cannot possibly be as small as h/10.

The upshot of this kind of argument is: distributivity has to go because empirical evidence contradicts it. The underlying philosophy is, therefore: Logic is empirical.

Well, it turns out that you don't have to be so dramatic as to revolutionize logic. Occam's razor cuts earlier. The whole thought-experimental set-up described in the above quote from MacKinnon cannot obtain! He assumes a situation which is already impossible for theoretical reasons, and you don't need Heisenberg's authority. It can be proved with a little mathematics that *it is not possible to restrict position and momentum simultaneously in the way described*. Moreover, both sides of the distributivity-identity are zero, and hence *distributivity does hold* – vacuously! So, at least in this case, there is no need to abandon it. And certainly, the uncertainty principle cannot be blamed. For a proof of *this* "triviality result" see Gibbins [5] and Rehder[16], although the argument goes back to Jauch[9], Berthier and Jauch[3], Lenard[11], Amrein and Berthier[1].

As to the two-slit experiment, distributivity, and quantum logic, our second proponent is H. Putnam[14]. He argues as follows. Let A_1 be the proposition that the particle goes through slit 1, A_2 be the proposition that the particle goes through slit 2, and B be the proposition that the particle strikes the screen at some localized region of the screen behind the slits. The paradox of the two-slit experiment then arises, according to Putnam, because of a quantum-logically illegitimate use of the distributive expansion

$$B \wedge (A_1 \vee A_2) = (B \wedge A_1) \vee (B \wedge A_2)$$

which would enable us to derive the conclusion that the two-slit diffraction pattern is essentially the sum of the two single-slit patterns. This would in fact follow, using an appropriate additive (conditional) probability function.

Again, the *empirical fact* of the observed interference seen on the screen contradicts the classical distributive expansion, and the paradox can only be blocked by rejecting it and proposing a weaker quantum logic. And again, it can be shown that this radical cure is premature. The above situation in the two-slit experiment cannot refute distributivity which can again be shown to hold trivially: both sides are equally impossible, see Gibbins and Pearson [6]. Gibbins and Pearson also show how the time evolution of the particle when travelling from the slits to the screen effects the state of the particle. This is an example of a working tense mathematics in the model of the Heisenberg picture of quantum mechanics.

So far we have seen two examples of premature condemnation of classical logic by fashionable logic. Quantum logic was proffered as a medicine for an empirical situation, which, at the discriminated point at least, did not need remedy.

On the other hand, quantum logic has helped explaining theoretical (and maybe even empirical?) facts in quantum theory. It must, in any case, be successfully blended into an appropriate probability theory, since all quantum events are of an intrinsically statistical nature. This program runs into some notorious difficulties, however: either you have to accept negative probabilities (cf, Mittelstaedt [13]), or the probabilities are not additive (Wigner [19]), or most of the probabilities are zero (Jauch [9]). Quantum logic, that I hold to be true, will only live – or survive – if a working quantum probability theory can be found.

4. Conclusion.

The fads and fallacies in fashionable philosophical logic, of which this paper gave three examples, arise from one common source: their use of the attractive tools and machinery of logic is premature and deceptively staightforward. The employment of logic superimposes structures upon the philosophical problems which are too often alien to them. At the very least, more care is needed. And more knowledge. If you want to enjoy the elegance of logic, you had better prepare yourself to take the pains of the ensuing mathematics as well. Otherwise the air might get very thin, and the elaborate construction very void.

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