THE REDUCTION OF DEONTIC LOGIC TO THE FIRST-ORDER PREDICATE CALCULUS

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Concluding [18] G. H. Von Wright presents a very concise sketch of a deontic logic «...which departs radically from the traditional models and which seems to possess some advantages over them». (1) Von Wright's nth «New System of Deontic Logic» is, properly speaking, a reduction of traditional deontic logic to the first-order one-place predicate calculus. In this paper I shall take the «Old System» (2) of Von Wright as traditional model of deontic logic. The symbolism of [18] will be adapted to that of the «Old System». The advantages of the reduction-system with respect to the «Old System» are considerable. In this paper we shall emphasize the interpretative value of the reduction-system. A lot of interesting features of the «Old System» are retained in the reduction-system. Moreover the reduction-system will be expanded in order to formalize conditional obligations (commitment), «strong» permission (free-choice permission), and the distinction between a forbearance and a «mere» not-doing. These expansions of the original reduction-system are intended to meet some of the criticisms of the «Old System» and at the same time show the superiority of this reduction-system with regard to other reduction-systems.

A. *The reduction of the «Old System» to the first-order one-place predicate calculus.*

1. A *reduction-system* for a logical system x is an interpretation of

(1) [18], p. 418.
(2) [9]; [10], pp. 36-41; [12], pp. 173-177.

«x» and consists in translating the well formed formulae of «x» by means of *reduction-definitions* in terms of a logical system y. The formulae of «x» translated in this manner will be called *reduction-formulae*. A reduction-system is *adequate* with regard to a logical system x iff the reduction-formulae of all valid formulae in «x» are valid in «y» and the reduction-formulae of all invalid formulae in «x» are invalid in «y». This may necessitate the assumption of special *reduction-axioms*.

Reduction-systems are not a novelty in deontic logic. In [1] A. R. Anderson presents his well-known reduction-system of deontic logic into alethic-modal logic. The Von Wright reduction-system in [15] links up with the former. The «Dawson-models» in [2] and [6] also reduce deontic logic into alethic-modal logic.

In [9] H. Silverstein introduces a reduction-system for Von Wright's dyadic deontic logics (3). His system reduces the dyadic deontic logics to the first-order *two-place* predicate calculus.

2. In this paper we first reduce the «Old System» of deontic logic to the first-order one-place predicate calculus. Since the predicative variables have a special meaning they will be replaced by the following symbols:

- the capitals A, B, C, ... are used as variables for *act qualifying properties* (4);
- the capital-cursive $\mathscr{P}$ is used as a predicative constant. As in [18] «$\mathscr{P}$» shall stand for the *property of being permitted* (5).

The replacement of predicative variables in the one-place predicate logic by the letters A, B, C, ..., and $\mathscr{P}$ results in a limitation of the range of the individual variables to *variables for individual actions*. In the reduction-system the variables x, y, z, ... shall therefore stand for individual actions, over which can be quantified.

(3) [13]; [14].
(4) [9], p. 2.
(5) About the possibility to treat «permittedness» as a property see [18], p. 420.

3. *The atomic formula PA in the «Old System».*

In the «Old System» the atomic deontic formula PA expresses «the proposition that the act named by A is permitted» [6]. This reading of «PA» as a *categorical* suggests considering an act qualifying property (represented in the «Old System» by the capitals A, B, C, ...) and the notion of permittedness as properties of individual actions. Since «PA» can be read as a categorical, how should we quantify this reading of «PA»?

The *universal* reading reduces the formula PA to the formula $\forall x(Ax \supset \mathscr{P}x)$ [7]. Doing so however the reduction-system would not satisfy the principle of deontic distribution of the «Old System» [8]. This principle warrants the validity of the distribution-formula:

$$\text{i. } P(A \vee B) \equiv (PA \vee PB).$$

The reduction-formula of i., i.e. the formula:

$$\text{ii. } \forall x((Ax \vee Bx) \supset \mathscr{P}x) \equiv (\forall x(Ax \supset \mathscr{P}x) \vee \forall x(Bx \supset \mathscr{P}x))$$

however is invalid in the predicate calculus.
A reduction-system using the reduction-definition:

$$PA =_{df} \forall x(Ax \supset \mathscr{P}x)$$

cannot be adequate with regard to the «Old System».

The *existential* reading of «PA» amounts to the reduction-definition:

$$PA =_{df} \exists x(Ax \,\&\, \mathscr{P}x).$$

This reduction-definition guarantees the distribution properties of the deontic operator P in the «Old System», i.e. the reduction-formulae of all valid «Old System»-distribution-formulae, containing the operator P, are also valid in the reduction-system; the translations of invalid distributions remain invalid in the predicate calculus. The fact that «PA» must be interpreted existentially and not universally illustrates the very weak meaning of the deontic operator P in the «Old System». This contrasts with the strong «free-choice» permission obeying the distribution-axiom:

(6) [9], p. 4.
(7) [18], p. 418.
(8) [9], p. 7.

iii. $P(A \vee B) \equiv (PA \& PB)$ (9).

The existential reduction-formula for «PA» however does not satisfy the principle of permission of the «Old System» (10). This principle warrants the validity of:

iv. $PA \vee P{\sim}A$

in the «Old System». The reduction-formula of iv., i.e. the formula:

v. $\exists x(Ax \& \mathscr{P}x) \vee \exists x({\sim}Ax \& \mathscr{P}x)$

is not valid in the predicate calculus.

Von Wright suggests assuming this principle axiomatically (11).

In the reduction-system it is however not necessary to assume the whole formula v.. Since v. is equivalent to «$\exists x\mathscr{P}x$», it suffices to assume this last formula. «$\exists x\mathscr{P}x$» exactly expresses the content of the principle of permission viz. that at least one action-type shall be permitted.

It is not even necessary to assume a special reduction-axiom. One obtains the same results by weakening the particular reading of «PA» into its non-existential form and defining:

$PA =_{df} \exists x\mathscr{P}x \supset \exists x(Ax \& \mathscr{P}x)$.

The adapted reduction-definition for «PA» still guaranties the distribution properties of the P-operator in the «Old System». The reduction-system now also satisfies the principle of permission. The formula $(\exists x\mathscr{P}x \supset \exists x(Ax \& \mathscr{P}x)) \vee (\exists x\mathscr{P}x \supset \exists x({\sim}Ax \& \mathscr{P}x))$ is valid in the predicate calculus.

4. *The atomic deontic formulae OA and FA in the «Old System».*

In the «Old System» obligation and prohibition are defined in terms of the operator P as follows (12):

$OA =_{df} {\sim}P{\sim}A$
$FA =_{df} {\sim}PA$.

(9) [18], p. 419. This «strong» permission will be discussed later.
(10) [9], p. 9.
(11) [18], p. 419.

Because of this definition-set and the foregoing we suggest as reduction-definitions:

$$OA =_{df} \exists x\mathscr{P}x \;\&\; \forall x(\sim Ax \supset \sim\mathscr{P}x)$$
$$FA =_{df} \exists x\mathscr{P}x \;\&\; \forall x(Ax \supset \sim\mathscr{P}x).$$

These existential reduction-definitions satisfy the distribution properties of the O- and the F-operator in the «Old System». The reduction-system moreover satisfies the principle of deontic contingency (13). By virtue of this principle of the «Old System» the formula Ot (t = an instance of an PL-tautology) must be considered as a contingent formula. And indeed in the reduction-system the formula:

$$\exists x\mathscr{P}x \;\&\; \forall x(\sim(Ax \vee \sim Ax) \supset \sim\mathscr{P}x)$$

can be shown to be neither a contradiction nor a tautology.

The reduction-system in its present state of development satisfies all the principles of the «Old System». Therefore one may assume that the reduction-system is adequate with reference to the «Old System». This however will not be proved here.

The reduction-system sketched out in the foregoing paragraphs does not require special axioms. Neither is it necessary to introduce a logic of actions using two different negations as Von Wright does (14). The reduction-system satisfies the principle of deontic contingency without a special action-logic.

5. *The interpretative value of the reduction-system.*

The reduction-system we have outlined above is very economic and the use of first-order one-place predicate logic makes the system easy to grasp for students in (deontic) logic. The system moreover has a great interpretative value. Firstly, the reduction-system shows the very weak sense in which the P-operator of the «Old System» is to be understood in contrast with the strong meaning of the F- and the O-operators. Secondly, the system shows that the negation-sign in front of a deontic operator of the «Old System» must be understood as *propositional negation* and not as the «negation» of the deontic word only (15). The reduction-system further points out that the validity of

(13) [9], p. 11.

(14) [18], p, 420.

(15) Treating the negation as negation of the deontic constant would result in a

«Pt» and the invalidity of «Ot» are not anomaly in the «Old System» (16).

As in the «Old System», formulae with iterated or nested deontic modalities cannot be well-formed in the reduction-system. By means of the «Old System» one cannot express the difference between «to do not-A» and «not to do A». The same holds true for the reduction-system; this follows from the simple fact that first-order predicate calculus does not allow one to point out the difference between «to be not a» and «not to be a».

Finally, the reduction-system reveals the very important but under-estimated fact that deontic logic from the very beginning was not a logic of norms. The atomic deontic formulae of the «Old System» do not express norms. Von Wright states it in this way: «We are thus not suggesting, which would be quite wrong, that norms are logically equivalent with general statements about the deontic character of individual actions of certain kinds.» (17)

B. *Extensions of the reduction-system*

6. *Commitment*

The formalization of «commitment» has always been a favourite topic in deontic logic. Many attempts have been made (18). Many of them have failed, lacking an adequate symbolism and even lacking a sufficient insight in the conditions for a «good» formalization of commitment. What is the problem with commitment?

If one consults the literature on this subject, one will read that commitment is to be so conceived that the following arguments are valid:

reduction-system which no longer satisfies the distribution-properties of the operators F and O.

(16) Many logicians prefer a deontic calculus in which both formulae are valid. This calculus, often called standard deontic calculus has a very simple modelsemantics.

(17) [19], p. 27.

(18) See e.g. [7].

I. A (is done)

(Doing) A commits one to do B

B is obligatory

II. A is obligatory

(Doing) A commits one to do B

B is obligatory

The following argument however must be invalid:

III. A is forbidden

(Doing) A commits one to do B.

Commitment thus is to be formalized in such a way that at least the conditions I, II, and III are fulfilled.

In the «Old System» Von Wright suggested «$O(A \supset B)$» as a formalization of «(doing) A commits one to do B». Since «$FA \supset O(A \supset B)$» is valid in the «Old System», the proposed formalization of commitment is not adequate (19). Condition III. cannot be satisfied. Moreover by means of the language of the «Old System» one cannot adequately express the argument I.

«$A \supset OB$» is another candidate for the formalization of commitment. This formula however can only function in a deontic logic allowing so called mixed modalities. The formalization must also be rejected. Although III. is invalid and I. valid, the formalization of II. results in an invalid argument. One does not change this situation if one strengthens the material implicator «$\supset$» in «$A \supset OB$» into a strict implication. This in fact happens in the reduction-systems of Anderson and Von Wright [15].

The dyadic deontic forula O(B/A) in Von Wright's dyadic deontic calculi also fails as a candidate for the formalization of commitment. Firstly, several dyadic deontic calculi of Von Wright are highly problematic (20). In the remaining non-problematic calculi the argument I. is invalid. For these reasons «O(B/A)» cannot be used as a formalization of a «contrary-to-duty imperative» (21) i.e. a special case of commitment.

(19) [12], p. 175.
(20) See [8].
(21) [3], p. 366; [5].

In this essay the proposition that A commits one to do B shall be shortened to «A comm B» and be defined as:

$$\text{A comm B} =_{df} \forall x(Ax \supset (\exists y \mathscr{P} y \,\&\, \forall y(\sim By \supset \sim \mathscr{P} y))).$$

One recognizes in the consequence of the first implicator the reduction formula for «OB». «A comm B» thus amounts to the proposition that for all actions of the type A all actions of the type ~B are forbidden. This expresses a very strong sense of commitment. The reduction-formula for «A comm B» indeed is logically stronger than the reduction-formula for «A ⊃ OB» and «O(A ⊃ B)». The reduction-formula for «FA ⊃ A comm B» is invalid. The reduction-formulae for «(OA & A comm B) ⊃ OB» *and* «(A & A comm B) ⊃ OB» are both valid. And so «A comm B» satisfies the requirements I, II, and III.

7. *Strong permission*

In 3. of this paper we pointed out that «$\forall x(Ax \supset \mathscr{P}x)$» is not the adequate reduction-formula for «PA». The definition:

$$PA =_{df} \forall x(Ax \supset \mathscr{P}x)$$

does not satisfy the principle of deontic distribution of the «Old System».

This definition however satisfies the distribution-axiom iii.. Von Wright calls the deontic operator «obeying» this axiom a strong permission operator [22]. This operator must be distinguised from the P-operator of the «Old System» and therfore will be represented as «P*». For the strong free-choice permission the following axiom holds:

$$\text{Ax1} \quad P^*(A \vee B) \equiv (P^*A \,\&\, P^*B).$$

P. Bailhache showed in this journal [23] that Ax1 alone does not suffice for a strong «unconditional» permission.

The following axiom:

$$\text{Ax2} \quad \sim(P^*A \,\&\, P^*\sim A).$$

(22) [14], p. 22 and 31.
(23) [4], p. 292.

must be assumed too.

Defining «P*A» as «$\forall x(Ax \supset \mathscr{P}x)$» however the reduction-system would not satisfy the axiom Ax2. The reduction-formula for Ax2 i.e. the formula:

$$\sim(\forall x(Ax \supset \mathscr{P}x) \,\&\, \forall x(\sim Ax \supset \mathscr{P}x))$$

is not valid in the predicate calculus.

Therefore the reduction-formula for «P*A» must be adapted. This adaptation will be analogue to the adaptation of the reduction-formula of «PA» in the «Old System». We suggest the following definition:

$$P^*A =_{df} \exists x \sim \mathscr{P}x \,\&\, \forall x(Ax \supset \mathscr{P}x).$$

This reduction-definition satisfies both Ax1 and Ax2.

8. The fusion of both reduction-systems for the «Old System» and for the strong P*-operator does not present any problem. The reduction-formulae of both systems are formulated in terms of precisely the same predicative constant $\mathscr{P}$.

The fusion of both systems into one axiomatic system presents some difficulties (24). One evident «bridge-axiom» is:

$$\text{Ax3} \quad P^*A \supset PA.$$

It states that a strong permission entails a weak one. In the reduction-system however Ax3 only becomes valid on the assumption that «$\exists x Ax$» is true, i.e. the formula:

$$\exists x Ax \supset ((\exists x \sim \mathscr{P}x \,\&\, \forall x(Ax \supset \mathscr{P}x)) \supset (\exists x \mathscr{P}x \supset \exists x(Ax \,\&\, \mathscr{P}x)))$$

is valid in the predicate calculus.

The reduction-system thus suggests that strong permission entails weak permission only on the condition that the permitted action is logically possible (the act-qualifying property is not empty). This suggestion can be translated axiomatically by putting a restriction on the applicability of substitution in Ax3. The arguments of the deontic operators in Ax3 should not be substituted by logical contradictions. Without this restriction one could prove «$P^*A \supset PB$» and this formula cannot be accepted as a deontic theorem. The axiom Ax3, in its

(24) [4], pp. 293-294.

restricted form, can be accepted as a bridge-axiom. The reduction-system shows us why.

9. *The difference between «forbearing» and «mere not doing».*

A deontic logic of the ought-to-do type (Tunsollen-Typus) must be able to formalize the difference between «forbearing (omitting) A» and «mere not doing A»; i.e. the difference between «to do not-A» and «not to do A» (25). The «Old System» does not allow one to formalize this difference. This is one of the reasons why Von Wright criticizes his own system. In one of his new systems of deontic logic Von Wright bases the distinction between «to do not-A» and «not to do A» on an action-logic, using two different negations (26). This action-logic can of course be built into the reduction-system of this paper. We suggest, however, proceeding in a different manner. One reason is our concern for the «economicity» of the reduction-system. Moreover we suspect this action-logic with two negations of being ad-hoc. It seems to be an attempt of Von Wright to take away validity from some controversial formulae in deontic logic (so-called paradoxes). One of this formulae is «Ot». This formula is not valid in the deontic calculus with two negations (27). Our reduction-system for the «Old System» however does the job in a more elegant way.

The so-called Ross's Paradox, i.e. the formula $Op \supset O(p \vee q)$ (28), is also no longer valid in Von Wright's two-negation deontic logic. However another formula, just as «paradoxical» as Ross's paradox, i.e. the formula $(Op \& Fq) \supset O(p \vee q)$ (28) is valid in Von Wright's new system. This shows why the two-negation deontic logic does not even solve the problem of the «paradoxes» in deontic logic.

Is it possible to introduce in our reduction-system a distinction between «to do not-A» and «not to do A» without using two different negations? We think it is.

«To do A» and «not to do A» are *contradictory* action-types, being mutually exclusive *and* jointly exhaustive.

«To do A» and «to do not-A», i.e. «to forbear A», are *contrary*

(25) [18], p. 410.

(26) [11], pp. 26-27; [16], pp. 38-42, and [17].

(27) [18], p. 420.

(28) More precisely the formula $0[p]x \supset 0[p \vee q]x$.

action-types, being mutually exclusive *but not* jointly exhaustive. For them the *tertium non datur* does not hold.

Contrary action-types are of course different action-types and therefore shall be represented by different variables.

Since they are mutually exclusive one can say that:

«A» and «B» are contrary if «$\forall x(Ax \supset \sim Bx)$» holds good.

This universal proposition can be used as antecedent clause each time we want to express in a deontic formula two contrary action-types A and B.

A simple application.

Von Wright claims [30] that the prohibition to do A does not mean «the mere not doing of A is obligatory» but «the omission of A is obligatory». In the reduction-system outlined in this paper this strong sense of prohibition (represented by the operator F*) can be defined as:

$$F^*A =_{df} \forall x(Ax \supset \sim Bx) \,\&\, \exists x \mathscr{P}x \,\&\, \forall x(\sim Bx \supset \sim \mathscr{P}x).$$

That «A», is (strongly) forbidden means that an action-type, contrary to «A», is obligatory. The fact that «F*A» is logically stronger than the formula FA of the «Old System» can easily been shown. The entailment of the reduction-formulae is nothing but a special instance of the existential formalization of the mode *Cesare* of the 2nd figure.

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([29]) More precisely the formula $(0[p]x \,\&\, F[q]x) \supset 0[p \vee q]x$.

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