IDENTITY STATEMENTS IN THE SEMANTICS OF SENSE AND REFERENCE

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By a semantic theory of sense and reference we understand an ordered quintuple $<E, S, R, <s_i>_{i \in I}, <r_j>_{j \in J}>$, where E is a set of syntactically well formed expressions of a given language – possibly divided in syntactic categories –, S is a set of senses, R is a set of (possible) referents, the s_i are partial functions that assign to (at least some) members of E members of S, and the r_j are partial functions that assign to members of S members of R.[1]

Frege's semantic theory falls in a natural way under the definition just given. In Frege's semantics we have essentially two categories of expressions: the category of proper names and the category of functional expressions. The category of proper names can be further subdivided in two subcategories: the category of sentential proper names (or statements) and the category of non-sentential proper names (which includes both proper names in the strict sense and definite descriptions). The category of functional expressions, on the other hand, contains as a special case the conceptual (and relational) words. According to Frege, all (meaningful) expressions express a sense and through this sense refer to something. Proper names, e.g. have a sense and through it they refer to an object. In the particular case of sentential proper names, the sense expressed is a thought and the object referred to is a truth-value, i.e. either the true or the false. Functional expressions also express a sense, but they refer to a function, not to an object. In particular, a conceptual word expresses a sense and through this it refers to a concept. This concept – that by being a function belongs to an ontological category different from that of objects – determines an (possibly empty) extension – that belongs

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[1] A more exact definition could be given, but it is not necessary for our purposes.

to the ontological category of objects. (²)

§ 1 Frege's semantics is not the only possible semantic theory of sense and reference, (³) and our discussion in this article is, thus, not restricted to it. We assume with Frege, however, that in identity statements the expressions at each side of the identity sign are proper names (in Frege's wide sense that includes both definite descriptions and strict proper names) and not conceptual words. Under this general assumption, we will try to show that in a semantic theory of sense and reference there is only one sound interpretation of identity statements. In our discussion we will benefit from some particularly interesting remarks of Frege in *Über Sinn und Bedeutung* on this issue, but we will not try to establish that our interpretation coincides with Frege's, since neither in the days when he wrote *Begriffsschrift* nor in his mature years did Frege have a clear notion of identity statements. We think, however, that our interpretation is the only one that does justice to those remarks of Frege in *Über Sinn und Bedeutung*, and that it is, thus, implicit in that article.

§ 2 Let us consider an identity statement of the form "a = a", e.g. "Venus = Venus", and a true identity statement of the form "a = b", e.g. "the morning star = the evening star" or "Tully = Cicero". Frege has clearly stated (⁴) that there is an important difference in cognitive value between statements of the form "a = a" and true statements of the form "a = b". Sometimes complicated mathematical calculations or extense empirical investigations are needed for establishing that a statement of the form "a = b" is true (or that it is false), whereas for knowing that a statement of the form "a = a" is true we do not need any such investigation. An interpretation of identity statements has to do justice to this difference in cognitive value between a statement of the form "a = a" and a true statement of the form "a = b", if it is to be considered an acceptable interpretation.

On the other hand, it seems that not all statements of the form

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(²) For a detailed and critical exposition of Frege's semantics vide either D. Shwayder's article 'On the Determination of Reference by Sense' or our 'Remarks on Sense and Reference in Frege and Husserl'.

(³) Vide our article cited in the foregoing note.

(⁴) Vide '*Über Sinn und Bedeutung*', pp. 143, 162.

"a = b" are empirical (or even synthetic), since, e.g. the statement "the least even number = the only even prime number" seems to express an analytic truth. But it seems that some of the statements of the form "a = b" are empirical, e.g. "the teacher of Alexander the Great = Aristotle" or "the morning star = the evening star". An acceptable interpretation of identity statements cannot, thus, exclude neither the possibility that a true identity statement expresses an empirical truth nor the possibility that it expresses a logical or mathematical necessity.

We arrive, thus, to the following two adequacy criteria for interpretations of identity statements.

Criterion I: An interpretation of identity statements must do justice to the difference in cognitive value between statements of the form "a = a" and true statements of the form "a = b".

Criterion II: An interpretation of identity statements cannot imply neither that all identity statements of the form "a = b" are empirical nor that they are necessary (where – to fix concepts – we understand 'necessity' in Kripke's sense, i.e. a statement S is necessary if it is true in every possible world in which exist the objects referred to by denoting expressions in S). (⁵)

§ 3 Now, how are we to interpret identity statements? First of all, we must distinguish two questions that are frequently taken as one and the same, namely: (1) what is expressed by an identity statement of the form "a = b"? and (2) under what conditions is an identity statement of the form "a = b" true? The second question can be answered without difficulties by saying that in case that the expressions "a" and "b" have the same object as referent. This, however, does not answer the first question, which is the one under discussion when we try to interpret identity statements.

We shall consider the following six interpretations of identity statements: (I) They express a relation of identity between the two objects that are the referents of the expressions at each side of the identity sign. (II) They express the relation of identity with itself of an object that is the common reference of the expressions at each side of the identity sign. (III) They express the relation of identity between

(⁵) Vide Kripke's 'Naming and Necessity' and 'Identity and Necessity'.

the expressions at each side of the identity sign. (IV) They express the
congruence relation, determined by sameness of reference, between
the expressions at each side of the identity sign. (V) They express the
relation of identity between the senses of the expressions at each side
of the identity sign. (VI) They express the congruence relation,
determined by sameness of reference, between the senses of the
expressions at each side of the identity sign.

In *Begriffsschrift* Frege seems to have fluctuated between inter-
pretations (III) and (IV). When he introduces the identity sign, Frege
states that identity statements express a relation between names or
signs, what seems to indicate that he accepts interpretation (III). [6]
However, when he tries to explain his conception in detail, Frege
seems to favour interpretation (IV). [7] Caton's [8] and Schirn's [9]
opinion that after all Frege's conception of identity statements in
Begriffsschrift is not radically different from his conception in *Über
Sinn und Bedeutung* seems to receive support therein. We will later
see, however, that interpretation (IV) is essentially different from
interpretation (VI), which, in our opinion, is the one implicit in *Über
Sinn und Bedeutung*. Dummett [10] and Khatchadourian [11] seem to
interpret Frege's mature position in the sense of interpretation (II).
Although – as remarked by Schirn – [12] some statements of Frege in
Grundgesetze der Arithmetik [13] seem to give support to that inter-
pretation, we think, however, with Schirn, that such an interpretation
does not do justice to Frege's analysis of identity statements in *Über
Sinn und Bedeutung*. Some remarks of Frege in *Ausführungen über
Sinn und Bedeutung* [14] as well as some remarks of Dummett [15] seem
to favour interpretation (I). It seems, however, that in both cases such
isolated remarks do not faithfully represent the opinions of their
authors.

[6] *Begriffsschrift*, p. 13.
[7] *Ibid.*, p. 14.
[8] Vide Ch. E. CATON'S 'The Idea of Sameness Challenges Reflection'.
[9] Vide M. SCHIRN'S 'Identität und Identitätsaussage bei Frege', pp. 181-182.
[10] *Frege: The Philosophy of Language*.
[11] Vide his 'Kripke and Frege on Identity Statements'.
[12] *Op. Cit.*, p. 188.
[13] *Grundgesetze der Arithmetik* Vol. I, p. 8.
[14] 'Ausführungen über Sinn und Bedeutung', pp. 130-131.
[15] *Op. cit.*, p. 544.

Let us consider now some difficulties that present themselves to interpretations (I)-(V).

§ 4 First of all, if what is expressed by an identity statement were a relation of identity between expressions – i.e. if interpretation (III) were the case – then all statements of the form "a = b" would be false, since "a" and "b" are different expressions. Hence, e.g. the statements "Londres = London", "Hesperus = Phosphorus" and "the morning star = the evening star" would all be false. Only statements of identity of the form "a = a", like e.g. "London = London", could be true under such an interpretation (and even this only under the assumption that by an expression we mean an expression-type and not an expression-token).

On the other hand, if identity statements were to express a relation of identity between senses – as is stated by interpretation (V) –, then all statements of the form "a = b", where "a" and "b" are expressions with different senses, would be false. Hence, e.g. the statements "London = London" and "Londres = London" would be true, but the statements "Hesperus = Phosphorus" and "the morning star = the evening star" would be false.

Let us consider now interpretation (I), according to which identity statements express a relation of identity between two objects that are the referents of the expressions at each side of the identity sign. But if they are two objects, then one cannot properly speak of identity. They could not have exactly the same properties since each one of them would have the property of not being the other, and this property could not be shared by this other, since an object cannot be different from itself. All that we could have under interpretation (I) would be a so-called relative identity, or, more precisely, an identity in a certain aspect. In such a case, however, one would have to establish in which aspect are the objects under discussion identical, and to establish it the corresponding identity statement is of no help, if we want to avoid an infinite regress. Moreover, it could happen that the object that is the referent of "a" were identical in a certain aspect to the object that is the referent of "b", and that it were identical in a completely different aspect to the object that is the referent of "c". In such a case, the statements "a = b" and "c = a" would both be true, but the statement "c = b" would possibly be false, since the referents of "b" and "c" could be non-identical in every aspect. Hence, under

interpretation (I), the transitivity of identity would not be valid. Moreover, under such an interpretation, the identity sign would be equivocal. We would either (1) have to abandon the transitivity of identity, or (2) have so many relations of identity as there are aspects of reality in which two objects can be identical. Both (1) and (2) are inadmissible. Interpretation (I) is, thus as unsound as interpretations (III) and (V).

According to interpretation (II), identity statements express the relation of identity of an object with itself. Under such an interpretation there would not be any difference in cognitive value between an identity statement of the form "a = a" and a true identity statement of the form "a = b". More precisely, under this interpretation every true identity statement of the form "a = b" is necessary, since it is true in every possible world that every object is always identical with itself. Hence, under this interpretation not only "London = London" and "Londres = London" would be necessarily true, but also "Hesperus = Phosphorus" and "the morning star = the evening star". Interpretation (II) violates both Criterion I and Criterion II. Moreover, one could ask the proponents of interpretation (II) if a false statement of the form "a = b" also expresses the relation of identity of an object with itself or something else. If it expresses the relation of identity of an object with itself, it could not be false, and, therefore, all statements of the form "a = b" would be necessarily true. If it expresses something else, then the sense of a statement of the form "a = b" would be a function of its truth-value, but this is clearly non-sense. Interpretation (II) is, thus, unacceptable.

Finally, let us consider interpretation (IV). According to this interpretation, an identity statement expresses a congruence relation, determined by sameness of reference, between the expressions at each side of the identity sign. This interpretation clearly satisfies Criterion I, since it allows us to establish an essential difference between an identity statement of the form "a = a" and a true identity statement of the form "a = b". As remarked by Schirn[16] and Kienzle[17] however, the relation between an expression and its referent is arbitrary. Therefore, we could neither establish the truth

[16] *Op. cit.*, p. 184.
[17] Vide his 'Notiz zu Freges Theorien der Identität', p. 218.

nor the falshood of a statement of the form "a = b" solely by the analysis of its component expressions. Hence, with the exception of linguistic conventions, all identity statements of the form "a = b" would be, as observed by Kienzle,([18]) synthetic, and even a posteriori, under this interpretation. Thus, even statements like "$2^2 = 4$" or "Londres = London" would be a posteriori under interpretation (IV). Such an interpretation clearly violates Criterion II.

§ 5 On the other hand, the relation between a sense and its reference is not an arbitrary one. That the sense of, e.g. words such as "the smallest even number", or "the teacher of Alexander the Great", or "the morning star" have the referent they have (i.e. the number 2, Aristotle and Venus, respectively) is not the result of an arbitrary stipulation. It is an objective fact that two senses have the same referent, and, thus, that according to interpretation (VI) they belong to the same equivalence class of senses determined by sameness of reference. Moreover, interpretation (VI) – according to which identity statements express a congruence relation, determined by sameness of reference, between the senses of the expressions at each side of the identity sign – is the only one that: (1) satisfies Criteria I and II; (2) avoids the difficulties that present themselves to interpretations (I)-(V); and (3) does justice to Frege's remarks in *Über Sinn and Bedeutung*,([19]) according to which not only the reference, but also the sense plays a decisive role in identity statements.

First of all, interpretation (VI) does justice to the difference in cognitive value that exists between identity statements of the form "a = a" and true identity statements of the form "a = b". Whereas in an identity statement of the form "a = a", not only the reference but also the sense of the expressions at each side of the identity sign is the same, in a true identity statement of the form "a = b" the expressions "a" and "b" have the same reference, but usually have different senses. The cognitive value is in the two cases completely different. Whereas "2 = 2" and "the teacher of Alexander the Great = the teacher of Alexander the Great" are true identity statements that do not add any new knowledge, the true identity statements "the smallest even number = the only even prime number" and "the

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([18]) *Ibid.*, p. 218.
([19]) 'Über Sinn und Bedeutung', pp. 143-144.

teacher of Alexander the Great = the most famous disciple of Plato''
do add new knowledge, and have, therefore, a greater cognitive value
than the first two. According to interpretation (VI) both ''a = a'' and
''a = b'' express that the senses of the expressions at each side of the
identity sign have the same reference, i.e. belong to the same
equivalence class of senses, determined by sameness of reference.
But whereas in the first case this is obvious, in the second case, it not
only is not obvious, but could even be not the case, since ''a = b''
could be false. If, however, ''a = b'' is true, then we have learnt
something that is not obvious, or, more exactly: that although the
senses of ''a'' and ''b'' are different, they belong to the same
equivalence class determined by sameness of reference, i.e. they have
the same reference. Thus, interpretation (VI) satisfies Criterion I.

Moreover, in the first of the last two examples the surplus in
cognitive value is obtained 'analitically', whereas in the second
example the surplus in cognitive value is obtained by empirical means.
Hence, one should clearly distinguish between the extensions of the
concepts (and a fortiori between the concepts) 'statement with
cognitive value' and 'synthetic statement' and, thus, between the
extensions of the concepts 'statement with cognitive value' and
'empirical statement'. (More exactly, to have a cognitive value greater
than zero is a mark /Merkmal/ of, the concept 'true synthetic
statement' and a fortiori of the concept 'true empirical statement'.) To
know the truth of the statement ''the smallest even number = the only
even prime number'' one needs some elementary knowledge of
arithmetic, whereas to know the truth of the statement ''the teacher of
Alexander the Great = the most famous disciple of Plato'' one needs
some knowledge of history of Ancient Greece. Moreover, the first
statement is not only an a priori truth, but also a necessary one,
whereas the second statement is empirically true and contingent.
Interpretation (VI) does not exclude any such possibility, and, thus,
satisfies Criterion II.

It should by now be clear that interpretation (VI) not only does
justice to the difference in cognitive value between an identity
statement of the form ''a = a'' and a true identity statement of the
form ''a = b'' (where the expressions ''a'' and ''b'' have different
senses), and to the fact that some true identity statements are
necessary and a priori, whereas others are contingent and a posteriori,

but also avoids all the difficulties that present themselves to interpretations (I)-(V). Moreover, it is the only interpretation that does justice to each of the following theses more or less implicit in Frege's discussion of identity statements in *Über Sinn und Bedeutung* : (1) An identity statement is true if and only if the expressions at each side of the identity sign have the same referent. (2) In a true identity statement the expressions at each side of the identity sign can either have the same sense or different senses. (3) True identity statements in which the expressions at each side of the identity sign have different senses possess a greater cognitive value than those in which the expressions at each side of the identity sign have the same sense. (4) Sameness of reference is not sufficient to do justice to the difference in cognitive value between identity statements of the form "a = a" and true identity statements of the form "a = b". (5) That which is decisive for the difference in cognitive value existing between statements of the form "a = a" and true statements of the form "a = b" is the sense. [20]

§ 6 Recently[21] Saul Kripke has offered an account of identity statements that has received wide acceptance in some circles of analytical philosophy. Although we have discussed this account somewhere else,[22] we would like to finish this article with some remarks about it.

Kripke divides the class of refering expressions in two disjoint subclasses, namely, the class of rigid designators and that of non-rigid ones. A designator is called rigid if it refers to the same object in every possible world, and is called non-rigid if that is not the case, i.e. if it refers to different objects in different possible worlds. According to Kripke, proper names in the strict sense (like 'Socrates' or 'Napoleon') are rigid designators – although not all rigid designators are strict proper names (e.g. 'the square root of 2' is a definite description and seems to be rigid). On the other hand, definite descriptions usually are non-rigid designators. Kripke follows Frege in attributing a sense to definite descriptions (like e.g. 'the teacher of Alexander the Great' or 'the evening star') and in considering that through this sense

[20] *Ibid.*

[21] In 'Naming and Necessity' and 'Identity and Necessity'.

[22] In our 'Necessità a posteriori e contingenze a priori in Kripke: alcune note critiche'.

they refer to an object in the actual world. He adds, however, that in other possible worlds definite descriptions could have a different referent from the one they have in the actual world. Hence, in another possible world the expression 'the teacher of Alexander the Great' could have had Socrates or Napoleon as referent. Strict proper names, however, do not express any sense, but only refer to some object, and this object, if it exists, is the referent of that strict proper name in every possible world. Therefore, Kripke concludes that a true identity statement of the form "a = b", where both "a" and "b" are strict proper names, is necessarily true, i.e. true in any possible world in which both "a" and "b" have a referent. On the other hand, if "a" or "b" or both are definite descriptions, the statement "a = b", if true, would probably be only contingently true. (A possible exception would be a statement like "the smallest even number = the only prime even number".)

Kripke acknowkedges the possibility that a strict proper name have had a different referent from the one it has in the actual world. ([23]) Thus, e.g. "Hesperus" and "Phosphorus" could have had different referents. But "Hesperus = Phosphorus" is, after all, necessarily true. According to Kripke, this is based on the identity with itself in every possible world of the object Venus, which is the common referent of "Hesperus" and "Phosphorus" in the actual world. Khatchadourian, ([24]) who coincides with Kripke in believing in the rigidity of strict proper names, observes correctly that Kripke confuses the problem of the rigidity of designators with the completely different issue of the identity of an object with itself in every possible world. According to Khatchadourian, the sole (or at least principal) ground for the rigidity of strict proper names is that this is a condition for the intelligibility of any counterfactual statement in which they occur.

But this is not a distinct trait of strict proper names. In ordinary (or usual) discourse one assumes not only that strict proper names, but also that definite descriptions always refer to the same object, and this assumption is a condition for the intelligibility of such a discourse. Both when someone says "Tully could have been not a senator" ([25])

([23]) Vide 'Naming and Necessity', pp. 270 and 276-277.

([24]) Op. cit., p. 276.

([25]) Syntactically this and the following examples are instances of very bad English, but they express exactly what we mean.

and when someone says "The man who denounced Catiline could have been not a senator" usually the intelligibility of the statements presupposses that every strict proper name and every definite description that occurs in them refer to the object which is its referent in the actual world. Hence, in this aspect strict proper names are not more rigid than definite descriptions. Certainly, however, there exist unusual situations in which someone intends to express by a statement like "The person who denounced Catiline could have been not a senator" not that the man Cicero could have been not a senator, but that the person who denounced Catiline could have been a person different from the man Cicero and not a senator. E.g. it could have been a former collaborator of Catiline. In such a case the statement would express the possibility that the definite description have a different referent from the one it has in the actual world. But the statement "Tully could have been not a senator" and even the statement "Tully could have been not Ciero" admit similar interpretations. The first one could be expressing the possibility that the proper name "Tully" have a different referent from the one it has in the actual world, i.e. the man Cicero. The second statement could be expressing the possibility that the proper names "Tully" and "Cicero" have different referents, although in the actual world they have the same referent. There is no difference in rigidity between strict proper names and definite descriptions.

Moreover, Kripke's thesis that, e.g. "Hesperus = Phosphorus" is necessarily true, whereas "the morning star = the evening star" is only contingently true, is based not on any difference in rigidity between strict proper names and definite descriptions, but on the following assumptions of Kripke: (1) Strict proper names do not have any (Fregean) sense, whereas definite descriptions do have. (2) Identity statements between definite descriptions are interpreted according to interpretation (VI) above, i.e. as expressing a congruence relation, determined by sameness of reference, between the senses of the two definite descriptions. (3) Identity statements between strict proper names are interpreted according to interpretation (II), i.e. as expressing – if true – the identity of an object with itself. Now, if Kripke were to interpret identity statements between definite descriptions according to interpretation (II), then, as we have seen, all true identity statements between definite descriptions would

be necessary, and there would be no modal difference between such identity statements and those between strict proper names. If, on the other hand, Kripke were to assume that strict proper names have a sense and were to interpret identity statements between strict proper names according to interpretation (VI), then there would be no modal difference neither between such identity statements and identity statements between definite descriptions. Moreover, on the basis of (2) and (3), Kripke would have some difficulty in interpreting an identity statement between a strict proper name and a definite description. Such a statement would express a strange relation of identity between the sense of the definite description and the object referred to by the strict proper name. One would have either to consider all such logical monstruosities as semantical nonsense, or one would have to consider them all as false, since even in a case like ''Aristotle = the teacher of Alexander the Great''– where both expressions have the same referent – the sense of the definite description does not coincide with the referent of the strict proper name.

Finally, we would like to observe that since the relation between an expression and its referent is arbitrary, if we do not assume that strict proper names have a sense, then there is no way of showing that there exist rigid strict proper names. Moreover, we suspect that only a definite description that refers to a mathematical or other sort of nonreal (or ideal) entity could be a possible candidate for rigid designator. A discussion of this problem, however, would take us too far afield.

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