

THE OMNIPOTENCE PUZZLE

Igal KVART

1. Can an omnipotent God create a stone so heavy he will not be able to lift? ⁽¹⁾ On the one hand, being omnipotent, he should be able to do so; on the other hand, if he does, he will not be omnipotent.

Let the statement *J* be:

J: God can create a stone so heavy he will not be able to lift.

We can translate «*x* can ...» into «*x* can bring it about that ---»

Thus, *J* becomes:

J: God can bring it about that there be a stone so heavy he will not be able to lift.

Abbreviations:

G: God

xLy: *x* lifts *y*

Ly: *y* is lifted

Sx: *x* is a stone

xBp: *x* brings it about that *p*

$\Diamond xBp$: *x* can bring it about that *p*.

With these abbreviations, *J* becomes:

$$J \equiv \Diamond GB[(\exists x)(Sx \ \& \ \neg(G \text{ can lift } x))].$$

In accordance with the above, «God can lift *x*» is tantamount to «God can bring it about that *x* is lifted», i.e. to $\Diamond GB(Lx)$.

J therefore becomes:

$$J \equiv \Diamond GB[(\exists x)(Sx \ \& \ \neg \Diamond GB(Lx))].$$

By omnipotence we mean the following (where « \Diamond » is the logical possibility operator):

$$x \text{ is omnipotent iff for any } p, \Diamond xBp \equiv \Diamond p.$$

We shall now *assume* that God is omnipotent, and even that he is *necessarily* omnipotent; i.e.: $\Box \Diamond xBp \equiv \Diamond p$.

The condition on his omnipotence renders J into:

$$J_1: \Diamond[(\exists x)(Sx \ \& \ -\Diamond GB(Lx))].$$

Since God is necessarily omnipotent, we can substitute « $\Diamond(Lx)$ » in the scope of the operator « \Diamond » for « $\Diamond GB(Lx)$ » and get:

$$J_2: \Diamond[(\exists x)(Sx \ \& \ -\Diamond(Lx))].$$

We shall now make the following most obvious assumption concerning logical possibility:

Assumption: For any stone, no matter how heavy, its being lifted is a logical possibility in any possible world.

From this assumption it follows that for any possible world w , « $(x)(Sx \supset \Diamond(Lx))$ » is true in w .

Hence: $\Box[(x)(Sx \supset \Diamond(Lx))]$.

Hence: $-\Diamond[(\exists x)(Sx \ \& \ -\Diamond(Lx))]$,

which is $-J_2$. But J_2 followed from J (under the assumption and definition used). Hence $-J$.

Conclusion: J is false if God is necessarily omnipotent. Thus, if God is necessarily omnipotent, he cannot create a stone so heavy he will not be able to lift.

We saw that (symbolizing «omnipotent» by «Om» and «necessarily omnipotent» by «NecOm»):

$$\text{NecOm}(G) \rightarrow -J.$$

Hence $\{\text{«NecOm}(G)\text{»}, J\}$ is an inconsistent set.

The temptation in this puzzle is to feel that $-J$ is inconsistent with God being omnipotent. This, however, as we saw, is false: It is perfectly consistent with the necessary omnipotence of God, since it is a consequence of it. Thus:

$$\{\text{«NecOm}(G)\text{»}, -J\} \text{ is consistent;}$$

and since $\text{NecOm}(G) \rightarrow \text{Om}(G)$:

$$\{\text{«Om}(G)\text{»}, -J\} \text{ is consistent.}$$

The key point is that it is not logically possible that there be a stone so heavy that a necessarily omnipotent God could not lift (see proof

below). And since omnipotence requires only being able to bring about what is logically possible, a necessarily omnipotent God need not be able to bring that about. Hence: that *J* is false is not incompatible with God's necessary omnipotence.

Proof: (This proof repeats steps of the main analysis).

Let *K* be: it is logically possible that there is a stone which God cannot lift.

Hence: $K \equiv \Diamond[(\exists x)(Sx \ \&\ \neg \Diamond GB(Lx))]$.

Due to God's necessary omnipotence:

$$\Box(\Diamond GB(Lx) \equiv \Diamond(Lx)).$$

Hence by substitution we get:

$$\Diamond[(\exists x)(Sx \ \&\ \neg \Diamond(Lx))].$$

But, on the assumption that in any world it is logically possible that any stone be lifted, this is false. Hence *K* is false.

Notice where the assumption of necessary omnipotence comes in. If God were *merely* omnipotent, but not necessarily so, then it is logically possible that he would not be omnipotent, and thus it is possible that he would bring an end to his own omnipotence in some way. To create such a stone will have this effect. To require that he is necessarily omnipotent is to curb his power of curtailing his own omnipotence. Thus, being merely omnipotent, but not necessarily so, *J* need *not* be false: even though he is omnipotent, it is logically possible that he not be so; and as an omnipotent God he should be able to bring that about. In particular, he can make a stone so heavy he will not be able to lift, which will render him thereby non-omnipotent. But if he is necessarily omnipotent, he cannot make himself non-omnipotent. Thus, just omnipotence does not decide the question we started with. An omnipotent God who is *not* necessarily omnipotent can create such a stone; but a necessarily omnipotent God cannot. Being necessarily omnipotent may be more of a «perfection» than being merely omnipotent; but with this added perfection a limitation sneaks in too.

2. But we may want a finer treatment in order to show more specifically what is wrong with the counter-argument to the effect that:

If God creates such a stone and he cannot lift it, then he is not omnipotent; but we assumed he is.

For such a finer treatment, we should recognize that what one is able to bring about at one time, he need not be able to bring about at another, and that what is logically possible at time t need not be logically possible at another time (i.e., one cannot at a given time change a past event, and relative to a given time, the past is logically determined while the future is not). Thus, in order to preserve the viability of the omnipotence condition $\Diamond xBp \equiv \Diamond p$ after having introduced temporal parameters, we had better adopt an interpretation of temporally-relativized logical possibility, for which at a given time t any statement which is logically consistent with the history of the world prior to t is logically possible at t ⁽²⁾. Thus, we shall temporally relativize our operators in « $\Diamond p$ » and « $\Diamond xBp$ » so as to get « $\Diamond_t p$ » and « $\Diamond_t xBp$ », which will read as « p is logically possible at t » and « x can bring it about at t that p » respectively.

We need therefore also to adjust our definition of omnipotence. It should now read as:

x is omnipotent at t ($Om(x, t)$) iff for any p , $\Diamond_t xBp \equiv \Diamond_t p$.

Now, if we assume further that the effect must temporally succeed, the cause as a matter of logical necessity (though we shall not essentially use this assumption below), then surely even an omnipotent agent cannot bring it about at t that a certain state or event will take place at t – only at some later time $t + \Delta t$. Thus, for any $p(t)$ (describing an event or state occurring exactly at t),

$$\Diamond_t xBp(t)$$

must be false. (This is particularly clear if $p(t)$ is false). More specifically,

$$\Diamond_t GB[(\exists x)(Sx(t) \ \& \ \neg \Diamond_t GB(Lx))]$$

must be false. But whether

$$\Diamond_t GB[(\exists x)(Sx(t + \Delta t) \ \& \ \neg \Diamond_{t + \Delta t} GB(Lx))]$$

is true is another matter. This is the question that we face.

Suppose at t God brings it about that at $t + \Delta t$ there is a stone which he cannot lift at $t + \Delta t$. (We have just seen that he cannot at t bring it

about that there is a stone at t , which he cannot lift at t , based on our assumption above). This would indeed make him non-omnipotent at $t + \Delta t$.

Of course, we cannot say that it makes him non-omnipotent at t . It is a matter of logical impossibility to lift a stone which does not exist, or before it existed. We ask whether at t , when he is omnipotent, he can create such a stone. Such a stone cannot of course, already exist at t , since he is to create it at t , and hence we must be asking, in order to have a *prima facie* point, whether at t he can bring it about that there be a stone at $t + \Delta t$ which he cannot lift at $t + \Delta t$. This is so since it is not logically possible to change the present, only to affect the future, and hence an omnipotent God can't be expected to create at t any stone so that it will exist at t , whether heavy or light. So we must be asking whether he can, at t , create a stone (which will exist at $t + \Delta t$, but not at t) that he will not be able to lift at $t + \Delta t$.⁽³⁾ Also notice that if he is omnipotent at t , a stone which he cannot lift at t does not already exist at t . To allow that it does is for

$$(\dagger) (\exists x)(Sx(t) \& \neg \Diamond_t GB(Lx))$$

to be true. But, according to our assumption, now temporally refined, it is logically possible for any stone which exists at t to be lifted at t , thus:

$$(x)(Sx(t) \supset \Diamond_t(Lx)),$$

which is logically equivalent to:

$$\neg (\exists x)(Sx(t) \& \neg \Diamond_t(Lx)),$$

which contradicts God's omnipotence at t and (\dagger) .

Thus, if God creates such a stone at t , it does not follow that he is not omnipotent at t ; but if he creates such a stone at t , then he will not be omnipotent at some later $t + \Delta t$.

But this is just to say that an omnipotent God at t can make himself non-omnipotent at $t + \Delta t$. There is no problem here: the misleading aspect of the puzzle here results from an unscrutinized attention to the times involved.

But if God is necessarily omnipotent *at all times*, this is impossible, since here we would have a situation in which he can become (though not in the actual course of events) non-omnipotent at $t + \Delta t$.

Thus, being necessarily omnipotent (at all times), rather than allowing *more* power to God, in fact curtails his power to render himself non-omnipotent⁽⁴⁾.

Notice that his being able to create such a stone is compatible with his being omnipotent at all times (though not with his having necessary omnipotence at all times). If God is omnipotent at all times, then, of course, he *will not* create such stone at any time, since it will render him *then* non-omnipotent. Thus, in a non-actual⁽⁵⁾ course of events in which he creates such a stone, he will be non-omnipotent (at some time); but this is consistent with his being omnipotent in the actual course of events in which he creates no such stone. Thus, if he is omnipotent at all times, he still *can* create such a stone (even though he *will not* in the actual course of event); but in such a world in which he will, which is not the actual course of events, he will not be omnipotent at all times.

Let us make the last point clearer. If there is at t a stone that God cannot lift at t , then, as shown above, he is non-omnipotent at t (since lifting a stone is a logical possibility). Hence, if God is omnipotent at all times, at no time t is there a stone which he cannot lift at t .

Suppose God is omnipotent at all times (in the actual world, but does not possess necessary omnipotence at all times). For him to be able to create such a stone at t amounts to

$$\Diamond_t GB[(\exists x)(Sx(t + \Delta t) \& \neg \Diamond_{t + \Delta t} GB(Lx))],$$

which amounts to (by definition of omnipotence at t):

$$\Diamond_t [(\exists x)(Sx(t + \Delta t) \& \neg \Diamond_{t + \Delta t} GB(Lx))],$$

which amounts to:

for some appropriate possible world w_1 .

$$(*) \quad \langle (\exists x)(Sx(t + \Delta t) \& \neg \Diamond_{t + \Delta t} GB(Lx)) \rangle \text{ is true in } w_1.$$

However, by our assumption that any stone in any possible world can be lifted at any time:

$$\Box(x)(Sx(t) \supset \Diamond_t(Lx)),$$

hence: $\Box(x)(Sx(t + \Delta t) \supset \Diamond_{t + \Delta t}(Lx))$,

and hence, in every possible world w :

$$(x)(Sx(t + \Delta t) \supset \Diamond_{t+\Delta t}(Lx)),$$

hence, in particular in w_1 ; $-(\exists x)(Sx(t + \Delta t) \& -\Diamond_{t+\Delta t}(Lx))$.

Hence, due to (*), in w_1 : $\Diamond_{t+\Delta t}GB(Lx) \not\equiv \Diamond_{t+\Delta t}(Lx)$.

Thus, the assumption that God is able to create such a stone at t makes him non-omnipotent in w_1 at $t + \Delta t$, and is thus incompatible with his having necessary omnipotence at all times; but is compatible with his omnipotence at all times in the actual world.

Conclusion:

If God is omnipotent at t , there is no stone at t which he cannot lift at t . But his omnipotence at t does not rule out the existence of such a stone at some other time.

If God is omnipotent at all times (but not necessarily so), then there is at no time a stone which he cannot lift; but he can create a stone which he will not be able to lift thereby being non-omnipotent at some possible world at some other time.

If God is necessarily omnipotent at all times, he cannot create, nor can there be, at any time, any stone which he cannot lift.

Hebrew University, Jerusalem

Igal KVART

FOOTNOTES

(¹) I am indebted to Avishy Margalit for the stimulation to deal with this question.

(²) Of course, since we deal with logical possibility, laws of nature play no role, and thus the past is barely any constraint on the future. Thus, just about any non-contradictory statement pertaining to times later than t is logically possible at t .

(³) Of course, the puzzle could be different. We could have asked: can God make himself at t unable at $t + \Delta t$ to lift a stone which already exists at t . This will eliminate the element of God *creating* such a stone at t . But apart from this minor difference, this version is essentially the same as the one we are dealing with.

(⁴) I assume, of course, that in a world in which God exists, he belongs to that world, an assumption which is necessary in order to interpret modal talk about God in terms of possible worlds.

(⁵) I am proceeding here by assuming God's omnipotence at all times in the actual-course-of-events; but the same point can be made with respect to any world in which he is omnipotent at all times *mutatis mutandis*: the interest of the puzzle is not predicated upon the acceptance of the *actual* existence of God.