INDUCTIVE DEFINITIONS

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I

«Definition, in the clearest sense, is what occurs when a new notation is introduced as short for an old one.» (Quine)

Inductive definition is not definition. A paradigm inductive definition is that of natural number in terms of 0 and successor:

(i) (1) 0 is a natural number.
(2) For all y, if y is a natural number then y′ is a natural number.
(3) The only natural numbers are those given by (1) and (2).

This does not tell us how to rewrite «x is a natural number» in terms only of 0 and successor (and logic). The «defined» predicate must be brought in as an addition to the stock of primitive predicates in the language. The inductive definition may, indeed, be converted into a genuine eliminative, or direct, definition, by a familiar technique due to Frege:

(ii) x is a natural number $=_{df}$ x belongs to every class which contains 0 and the successor of every member.

But this is a direct definition in terms of 0, successor and class membership – the latter predicate being one which does not appear in the inductive definition.

Then what does an inductive definition do? Like a direct definition, it lays down truth conditions for the newly introduced predicate. In a systematic context, it may specify new axioms, linking the new predicate with others.

Now both of these are in general risky undertakings. It might be impossible for a predicate to have the truth conditions I lay down. For example, even if I can *say* (in the meta-language) that a new predicate (of the object language) is to be true of just those predicates not true of themselves, my statement is necessarily false. And the addition to a theory of a new axiom, containing a new predicate, may of course make it inconsistent.

These risks are absent when the new predicate is introduced by a direct definition. An hypothesis I want to explore is that they are also absent in inductive definition: that it is always possible for a predicate to have the truth conditions laid down in an inductive definition; and that the addition of a new predicate with axioms provided by an inductive definition can never make a system inconsistent. This hypothesis has, I think, some intuitive support: an inductive definition does not *seem* to introduce any new ideas or assumptions; and we are happy to *call* it «definition». But a proper assessment of the hypothesis requires some clarification of the concept of an inductive definition, and that is the main topic of this paper.

Although everyone can recognize an inductive definition, it seems to be surprisingly difficult to give a moderately precise general characterisation. The ideas that spring most readily to mind (on the evidence of discussion with colleagues) are open to pretty obvious criticisms. The only thorough published account I know of is in Kleene's *Introduction to Metamathematics* (1), and I will argue that it, too, is quite unsatisfactory.

Not that I can offer a definitive account myself – at any rate not one which quite matches the intuitive concept. In the end I will suggest an account of inductive definition which does, apparently, satisfy the requirement that inductive definition not be «risky» in the ways mentioned above; but it diverges somewhat from the intuitive conception, in the direction of liberality – it counts as inductive definitions everything we would intuitively, but lets in extras.

II

The present paper bears on my «Sets as Open Sentences», *American Philosophical Quarterly*, vol. 14, No. 3, July 1977. There, I was able to derive a quite powerful set theory (adequate to classical mathematics short of transfinite arithmetic) from a theory called SF, which was interpreted with a domain of open sentences. In arguing that this interpretation does make SF come out true (the consistency of SF is not in doubt), I relied on a theory of open sentences in which a crucial part was played by an assumption of closure under inductive definition: I assumed that whenever there is a language with predicates having specified truth conditions, there is also a language meeting the additional requirement of closure under inductive definition – i.e. it contains predicates having the specified truth conditions, plus all the predicates inductively definable in terms of them, plus all the predicates inductively definable in terms of *them*, and so on. The present paper is intended to clarify the concept of inductive definition there used, and to provide the basis for a defence of the closure assumption. (For if the introduction of a new predicate inductively defined in terms of previously introduced ones is always possible, so is the successive introduction of all such predicates.)

III

Let me begin by mentioning a troublesome matter of terminology. Some writers equate «inductive definition» with «recursive definition» (2); others equate «recursive definition» with «definition by induction (not to be confused with 'inductive definition')» (3); others would equate «inductive definition» with «definition by induction». This instability of terminology may suggest too close a connection between inductive definition and recursive function theory.

The recursive predicates of elementary number theory are just those whose characteristic functions are recursive. There are ways of defining new predicates which yield only recursive predicates, when applied to recursive predicates. But such definitions do not exhaust the inductive definitions of elementary number theory. A predicate may be inductively definable in terms of recursive predicates, without itself being recursive.

For example, let S be some standard axiom system for arithmetic, with Modus Ponens and Generalisation as its only rules of inference. Then the predicates «x is an axiom [i.e. the Godel number of an axiom]», «x follows from y and z by Modus Ponens», and «x follows from y by Generalisation» are all recursive. Now the predicate «x is a theorem» is inductively definable in terms of the others:

(iii) (1) If x is an axiom, then x is a theorem.
(2) If x and y are theorems and z follows from them by Modus Ponens, then z is a theorem.
(3) If x is a theorem and y follows from it by Generalisation, then y is a theorem.
(4) The only theorems are those given by (1) - (3).

But «x is a theorem» is not recursive.

There may turn out to be some less direct connection between inductive definition and recursive predicates, but we should not look to recursive function theory for an elucidation of the concept of inductive definition. (4)

IV

As a representative of the «Naive View» of inductive definition, I will consider the following suggestion: (5)

An inductive definition introducing a new predicate ⌜F⌝ into a given language L has the following form:

Base Clauses

Fa_1
Fa_2
. where each a_i is a closed term of L
.
.
Fa_m

Generating Clause

$(x_1)\ldots(x_n)(y)(Fx_1 . \ldots . Fx_n . R(x_1,\ldots,x_n,y) .\supset Fy)$ where ⌜$R(x_1,\ldots,x_n,y)$⌝ is any n + 1 – place predicate of L

Extremal Clause

$(y)(Fy \supset. (y = a_1 \vee \ldots \vee y = a_m)$
$\vee (Ex_1)\ldots(Ex_n)(Fx_1 . \ldots . Fx_n . R(x_1, \ldots,x_n,y)))$

Inductive definitions of this sort are, as Frege in effect showed, a legitimate way of introducing a predicate ⌜F⌝ into L in the following sense: given any model $\mathcal{M}$ for L, there is a unique model $\mathcal{M}^+$ which extends $\mathcal{M}$ in such a way as to provide an extension for ⌜F⌝ satisfying the clauses of the definition.

The form suggested here for the direct clauses («Base» and «Generating») is no good, since the direct clauses in some well-known inductive definitions can't be put into that form; but I will leave this point till we discuss Kleene's account, which shares the same defect. Here let me criticise the suggestion as to the form of the extremal clause.

As applied to our paradigm of «natural number» (where the direct clauses *are* in the suggested form), the Naive View says that the inductive definition is just the statement of three axioms, which determine the extension of «natural number», for any given interpretation of 0 and successor. The three axioms would be:

$$N(0)$$
$$(x)(y)(N(x) \, . \, y = x' \, . \supset N(y))$$
$$(y)(N(y) \supset . \, y = 0 \vee (Ex)(N(x) \, . \, y = x'))$$

The trouble is that these three axioms *don't* determine the extension of N(x): they admit non-standard models, as well as the standard one.

Moreover, these three axioms don't even limit the interpretation of N(x) as closely as the Peano axioms: the induction schema is not deducible from these three – or from any other finite bundle of axioms. If one sees the inductive definition as specifying axioms for the new predicate, it would be better to take the extremal clause as corresponding to an infinite bundle of axioms – the induction schema, in this case. But that would still not be good enough: not even with infinitely many axioms can non-standard models of arithmetic be excluded.

On the other hand, the inductive definition of «natural number» *does* completely determine the meaning of «natural number», when the definition is understood in the intuitively obvious way. That inductive definition is just what you would use to *explain the meaning* of «natural number» to someone. And it would tell him enough about the meaning of «natural number» to let him distinguish, as we do, between standard and non-standard models for a given axiom system of arithmetic.

An inductive definition does determine the extension of the new predicate, relative to the interpretation of the predicates in terms of which it is defined. But it does not do so simply by specifying axioms for the new predicate.

V

Kleene is aware of this. He says (6)

> The *direct clauses* tell us certain objects for which the predicate takes the value t. The *extremal clause* says that those are the only objects for which the value is t, so that we can attribute the value f whenever we are able to see that the direct clauses do not require the value to be t.

If we eliminate the talk about what people can do and are able to see, Kleene is saying that the extremal clauses states that the predicate is true of only those objects which the direct clauses «require» it to be true of, hence false of all other objects. «Requirement», I take it, is a matter of the restriction of possible interpretations: the direct clauses in the inductive definition of a predicate F *require* it to be true of an object x iff every interpretation of F which makes the direct clauses true (given the standard interpretation of the predicates in terms of which F is being defined) makes F true of x.

Construing the extremal clause in this way is, I think, a great advance over the Naive View. For Kleene the direct clauses are still axioms for the new predicate, but the extremal clause is a meta-language statement, being about the truth of the direct clauses on various interpretations. And this is just what is required to explain the meaning of «natural number»: the distincion between the genuine natural numbers and the extras in a non-standard model for arithmetic is that only the genuine ones are *required* to be natural numbers by the assumption that 0 and the successors of natural numbers are natural numbers.

VI

So I think this is a good way of construing the extremal clause. But Kleene's account of the direct clauses is no good. They tell us, he says (see quote above), that the predicate being defined is true of certain objects, but not what it is false of. More particularly, he says (7) that they

> generally include *basic clauses*, each of which tells us outright (or under hypotheses involving only previously defined predicates) that the value is t for a certain object, and *inductive clauses*, each of which tells us that, if the value is t for certain objects (and possibly under hypotheses involving previously defined predicates), then the value is t for the object related to those in a given way.

The exceptions indicated by the «generally» are degenerate cases in which either basic or inductive clauses are absent. Kleene does not admit any *other* kind of direct clause than basic or inductive, as described.

This account of the direct clauses, like the Naive View's, is too restrictive. One well-known inductive definition which cannot be put into this form is Tarski's inductive definition of «satisfies». (8) To make this mesh with our discussion, which has for convenience focused on inductive definitions of one-place predicates, let us regard it as defining a one-place predicate «Sat», true of ordered pairs of sequences and sentential functions. Then Tarski's definition may be put as follows (remember that Tarski's object-language has only one predicate, class inclusion, and $\iota_{k,l}$ is the object-language sentence in which this predicate links the k^{th} and l^{th} variables; $\cap_k y$ is the universal quantification of y by the k^{th} variable):

(iv) Sat (<f,x>) $\equiv$ f is an infinite sequence of classes, and x is a sentential function, and
either (α) there exist natural numbers k and l such that $x = \iota_{k,l}$ and $f_k \subseteq f_l$;
or (β) there is a sentential function y

such that $x = \bar{y}$ and not Sat (<f,y>);

or (γ) [disjunction]

or (δ) there is a natural number k and a sentential function y such that $x = \cap_k y$ and, for every infinite sequence g which differs from f in at most the k^{th} place, Sat (<g, y>).

As it stands, this consists of just one long direct clause. We can easily tack on an extremal clause:

> Sat (<f,x>) for no ordered pairs <f,x> for which this is not required by the direct clause.

But an extremal clause would be redundant: the direct clause itself settles not only what «Sat» is true of, but also what it is false of. Because of this, and because the direct clause is neither basic nor inductive (as described by Kleene), the inductive definition of «Sat» does not conform to Kleene's account.

Can we reformulate the definition to fit the Kleenean mould? As far as I can see, no. Suppose we try as follows:

(v) (α) If f is an infinite sequence of classes, and x is a sentential function, and
there exist natural numbers k an l such that $x = \iota_{k,l}$
then Sat (<f,x>) iff $f_k \subseteq f_l$.

(β) If f is an infinite sequence of classes, and x is a sentential function, and
there is a sentential function y such that $x = \bar{y}$,
then Sat (<f,x>) iff not Sat (<f,y>).

(γ) [disjunction]

(δ) If f is an infinite sequence of classes, and x is a sentential function, and
there is a natural number k and a sentential function y such that $x = \cap_k y$,
then Sat (<f,x>) iff, for every infinite sequence g

which differs from f in at most the k^{th} place,
Sat (<g,y>).

(ε) Sat (<f,x>) for no ordered pairs <f,x> for which this is not required by (α) - (δ).

Here we have a number of separate clauses, and the extremal clause is not entirely redundant: the direct clauses do not by themselves imply that only sequences satisfy, or that only sentential functions are satisfied. But the direct clauses still imply that «Sat» is false of all those ordered pairs of sequences and sentential functions that it is false of. And the direct clauses are not all either basic or inductive. Clause (α) for instance, implies that if not $f_k \subseteq f_l$ then *not* Sat (<f,x>); so it is not a Kleenean basic clause. We could make (α) Kleenean by weakening it to.

(α′) If f is an infinite sequence of classes, and x is a sentential function, and
there exist natural numbers k and l such that $x = \iota_{k,l}$, and $f_k \subseteq f_l$,
then Sat (<f,x>).

But then (β) could not get a footing: the direct clauses collectively would not imply that any sequences satisfied *negations*. And a similar weakening of (β) - (δ) would still not make then Kleenean inductive clauses: (β′), for instance, would still tell us that, if the value is *f* for a certain object, then it is t for a related object.

These difficulties seem insurmountable. There seems to be no way of formulating the inductive definition of «Sat» to make the direct clauses all either basic or inductive (Kleenean), nor to make the direct clauses as a whole imply only that the predicate is *true* of certain objects, leaving it to the extremal clause to say that it's false of everything else. I conclude that Kleene has not given a satisfactory general account of inductive definition: we must reject his suggestion as to the form which sentences containing the new predicate must take, to make them acceptable as direct clauses in an inductive definition.

VII

Do we need to place *any* restrictions on the direct clauses? Could we describe an inductive definition as simply some collection of axioms for the new predicate, plus a Kleenean extremal clause ? This would let in everything we would want to call an inductive definition, certainly, but it would let in too much. In particular, it would prevent us saying that inductive definition was *safe*. For it would let in inductive definitions with inconsistent direct clauses, and, since on Kleene's account an inductive definition consists of the conjunction of the direct and extremal clauses, such definitions would themselves be inconsistent.

Could we require merely that the direct clauses be consistent? This would be to abandon any hope of a *formal* account of inductive definition. But, worse than that, it would not ensure the consistency of the inductive definition as a whole. The following case shows this. It also shows that an even stronger condition on the direct clauses, namely the Kleenean condition that they not imply that the predicate being defined is *false* of anything, is insufficient to ensure the consistency of the inductive definition as a whole. (We have already seen that this latter condition is too *strong*, in that it implies that the definition of «Sat» above is not inductive.)

Consider the following inductive definition (or purported inductive definition) of «Sgr» («Smith friendship-group member»):

(vi) (1) Smith is a Sgr.
(2) Anyone who loves all other Sgrs is a Sgr.
(3) No one is a Sgr unless his being so is required by (1) and (2).

Suppose that Jones and Robinson both love Smith, but not each other; there are no other people. Who are the Sgrs? Note first that the direct clauses do not imply that anyone is *not* a Sgr: everyone could be a Sgr, consistently with (1) and (2). So the direct clauses are consistent. But, secondly, the definition as a whole is not consistent; this may be shown as follows. The direct clauses do not require that Robinson be a Sgr: (1) and (2) would be true if Smith and Jones were the only Sgrs. So, by (3), Robinson is not a Sgr. Similarly, Jones is not a Sgr. So, by (1), Smith is the only Sgr. But then (2) is false: Robinson loves all other Sgrs but is not one himself.

VIII

If the basic format of direct clauses plus (Kleenean) extremal clause is accepted, some restriction has to be placed on what can go into the direct clauses; otherwise things with inconsistent direct clauses, and things like (vi), will count as inductive definitions, and inductive definition will not be safe. Restrictions are needed, and Kleene's won't do; what restriction will do?

Perhaps a clue may be got by examining our counterexample to Kleene's suggestion, the inductive definition of «Sat». It will be observed that in this definition, unlike that of «Sgr», we proceed by «starting with simple cases and building up». ([9]) The ordered pairs of sequences and sentential functions may be ranked according to the complexity of the sentential function, and the various clauses of the definition give the truth conditions for the application of «Sat» in terms of its application to ordered pairs *earlier* in that ranking.

There is some analogy here to the way in which a recursive predicate or function, in elementary number theory, is defined by showing how its values for later arguments are determined by earlier values. But the analogy is not very close. A recursive predicate is computable; its later values are each determined by finitely many earlier ones. But in the ranking of the ordered pairs of sequences and sentential functions a pair may have infinitely many predecessors; whether a sequence satisfies a quantified sentential function is not in general computable. (Tarski's inductive definition of satisfaction can be modified so as to require only *finite* sequences ([10]). These, like sentential functions, can be coded by natural numbers; hence so can the ordered pairs of sequences and sentential functions. The inductive definition of «Sat» above then comes to define a predicate of natural numbers. Still, «Sat» is not then a recursive predicate. It is not even arithmetical: if it were, so would be «true», contrary to Tarski's Theorem. ([11]))

At any rate, what establishes the ranking of the ordered pairs to which «Sat» applies is the ranking (in terms of complexity) of the sentential functions, and what establishes *that* ordering is the inductive definition of «sentential function»; and that inductive definition *does* conform to Kleene's account. The idea suggests itself, then, that we might distinguish two kinds of inductive definition: fundamental ones, which establish a ranking of certain objects, and which conform to Kleene's account; and non-fundamental ones, which make use of such a ranking to define a predicate, perhaps in the quasi-recursive way of «Sat».

Such a suggestion leaves the problem of the form of non-fundamental inductive definitions unsolved, of course. But, even in its account of fundamental inductive definitions, it needs supplementation (at least). For not everything conforming to Kleene's account of inductive definition could be a fundamental one: the usual inductive definition of «theorem», for instance, does not rank the objects satisfying that predicate in a unique order.

Something like this distinction between fundamental and non-fundamental inductive definitions is made by Kleene. ([12]) But he does not apply his account of inductive definition only to fundamental ones. (In fact, he only *says* it applies to non-fundamental ones. But he offers no separate account of fundamental ones. And he thinks that the distinction is not a formal one: «To which category a given inductive definition belongs may vary with the context or theory in which it is being used.» That is why I suppressed the distinction earlier, and took Kleene's account to be of inductive definition in general. At any rate, «Sat» could hardly rate as a fundamental inductive definition, so my use of it as a counterexample is fair enough, even if Kleene's account is restricted in scope to non-fundamental inductive definitions.)

IX

In summary, Kleene's account places too great a restriction on what the direct clauses can be like; *some* restriction is needed if the possibility of inconsistent inductive definitions is to be excluded; but it is not obvious how to formulate the required restriction. I now want to suggest another view of inductive definition, which avoids these problems altogether, by giving up the whole idea that an inductive definition is a *conjunction* of direct and extremal clauses.

I will say that an inductive definition is anything of the form:

> (vii) Something is an F iff it is required to be so by the following premise:
> «...»

This is an inductive definition of F in terms of whatever other predicates appear in the quoted clauses. No restriction at all is placed on the quoted clauses.

Let's see how this works for our paradigm, first of all. I would formulate the inductive definition of «natural number» as:

> (viii) Something is a natural number iff it is required to be so by these premises:
> (1) 0 is a natural number.
> (2) For all y, if y is a natural number then y′ is a natural number.»

Half of this biconditional («Something is a natural number *only if* ...») is simply the extremal clause of the Kleene definition. But the other half is new. Instead of *asserting* the direct clauses, we say that something is a natural number if it is required to be so by them. Although the direct clauses are now *only* quoted, and no longer asserted, it is evident, in this kind of case, that the truth of the direct clauses follows from the definition: The direct clauses require 0 to be a natural number, so it is one. Similarly, if the direct clauses require any object to be a natural number, they require its successor to be one too; so if any object *is* a natural number, so is its successor. So the definition entails that all the usual things are natural numbers, and – since it also entails the Kleenean extremal clause – that only they are: the definition has the usual extension.

Now suppose that we have an inductive definition with inconsistent direct clauses K. The inconsistency is not asserted, but only quoted. The definition asserts that F is true of whatever it is required to be true of by K, which is everything. The definition is not inconsistent: it just makes the defined predicate true of everything.

Returning, finally, to the problem of the Sgrs, the definition becomes:

> (ix) Someone is a Sgr iff his being so is required by these premises:

«(1) Smith is a Sgr.
(2) Anyone who loves all other Sgrs is a Sgr.»

This statement is not inconsistent with our assumptions: it merely turns out that Smith is the only Sgr. The truth of (all) the direct clauses does not follow from the definition.

This account of inductive definition seems to ensure, then, that inductive definition is safe, and it will not reject, or alter the extension of, anything we would intuitively regard as an inductive definition. It does, however, let in some things we would not intuitively regard as inductive definitions. For one thing, it seems to be part of the intuitive conception of inductive definition that the direct clauses should be *true*; but our account allows inductive definitions with inconsistent direct clauses, and things like the definition of «Sgr». Perhaps our explication is of a *generalisation* of the intuitive concept of inductive definition. Within this wider class of inductive definitions, it might prove interesting to find a formal condition on the direct clauses which ensures that they come out true, and see whether the narrower class of inductive definitions conforming to this condition matches the intuitive concept more closely. But the simplicity of our explication, the fact that it confirms our intuitions about the safety of inductive definition, and the fact that it makes the safety of inductive definition independent of whether the direct clauses come out true, seem to show that the wider concept is of some interest. ([13])

X

There is another point in favour of our account of inductive definition, which comes up in connection with Frege's method for obtaining direct definitions from inductive ones. Frege's method is quite mechanical. Starting with an inductive definition of some predicate F, you just take the conjunction of the direct clauses, replace F(x) by $x \varepsilon z$ throughout (and similarly for other variables), put $\supset x \varepsilon z$ at the end and (z) in front of the whole; this yields the direct definition of F(x). Now the point is that this method works quite normally when applied to the «counterintuitive» cases of inductive definition admitted by our account. Applied to our inductive definition of «Sgr», for instance, it yields for «Sgr(x)»:

$$(z)(\text{Smith } \varepsilon\, z.\ (w)((y)(y\varepsilon z\ .\ w\neq y\ .\supset L(w,y)) \supset w\varepsilon z)\ .\supset x\varepsilon z)$$

And this is true of Smith, and Smith alone (on our assumptions) – just like «Sgr» as defined by (ix). The Frege directification technique is generally applicable to the wider class of inductive definitions allowed by our account.

And this is no accident: the Frege technique fits in much better with our account of inductive definition, even in application to paradigms of inductive definition. On our account of inductive definition (and construing «requires» as above) the inductive definition of F says that F is true of x iff every interpretation of F which makes the direct clauses true makes F true of x. If we now construe this talk of interpretations in the usual model-theoretic way, in terms of assignments of sets as extensions, the truth conditions for F(x) come out to be that x belongs to every set whose assignment as extension to F in the direct clauses makes them true, i.e. every set which has as members the things F is predicated of in the direct clauses.

There is thus a direct equivalence between the truth conditions given by the inductive definition of F(x), on our account, and those given by the Frege directification. A further advantage of our account, then, is that it explains the general applicability to inductive definitions of the Frege directification technique.

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NOTES

(1) Amsterdam and Groningen, 1964; § 53.

(2) E.g., QUINE, *Philosophy of Logic* (Englewood Cliffs, 1970), p. 41.

(3) KLEENE, *op. cit.*, p. 217.

(4) Not all predicates inductively definable in terms of recursive predicates can be expressed in the form (Ey)R(x,y), where R(x,y) is recursive, although «x is a theorem» can be. See note 11 below.

(5) In the words of an anonymous referee for another journal.

(6) *Op. cit.*, p. 259.

(7) *Ibid.*

(8) A. TARSKI, «The Concept of Truth in Formalised Languages», in *Logic, Semantics, Metamathematics* (Oxford, 1956); p. 193.

(9) QUINE, *ibid.*

(10) QUINE, *op. cit.*, p. 37 ff.

(11) Incidentally, this seems to provide a counterexample to Kleene's claim (*op. cit.*, p. 261) that «When the form of an inductive definition (with elementary direct clauses) is specified in a natural way, the predicates $P(x_1, ..., x_n)$ definable by the use of inductive definitions in the natural number arithmetic are exactly expressible in the form $(Ey)R(x_1, ..., x_n, y)$ with R primitive recursive.»

(12) *Op. cit.*, p. 258 ff.

(13) Thus, consider the following objection:

> How can the view that the following is an inductive definition even be taken seriously?
>
> «x is an F iff x is required to be so by this premise: '3 is an F'.»

At most the objection would show that the term 'inductive definition' was being unreasonably stretched. I would not attach much importance to that. Our wider concept is important enough in its own right if it is true that inductive definition in this wide sense is safe (that would also imply the safety of inductive definition in the ordinary sense).

But, anyway, ability to exclude such «degenerate cases» is not a reasonable requirement on a formal account of a concept. Kleene's account, for instance, admits inductive definitions in which the inductive clause is absent or vacuous; thus «3 is an F; nothing else is an F» would count as an inductive definition for him, too.