ALMOST SKOLEM FORMS FOR RELEVANT (AND OTHER) LOGICS

R.K. MEYER

In [1], it was established that relevant logics are second-degree reducible. In [2], this result was generalized to a number of additional sentential and first-order calculi, including the first-order versions of the relevant logics. In this paper, I explore some consequences of this reduction, especially as they involve finding *normal forms* for formulas of relevant logics. Acquaintance with [2] will be helpful.

For definiteness, consider the first-order version R^x of R introduced in [3].[1] According to [2], every formula of R^x may be given a second-degree normal form which looks like this:

(1) $AX(A) \supset Fx_1 \ldots x_n$

In (1), $\supset$ is material implication, and F is an atomic predicate. $x_1,\ldots,x_n$ are just the variables that occur free in A itself. And AX(A) is a conjunction of exact *equivalences* (universally quantified coimplications), whose conjuncts are determined univocally by the *form* of A itself. Leaving the reader to check [2] for details, we note merely that the following are typical formulas that *might* be conjuncts of AX(A).[2]

(2) $\forall x\,(Gx \leftrightarrow \sim Hx)$
(3) $\forall x \forall y\,(Rxy \leftrightarrow Gx \vee Hy)$
(4) $\forall x \forall y\,(Rxy \leftrightarrow Gx \rightarrow Syx)$
(5) $\forall x \forall y\,(Rxy \leftrightarrow \forall z\,(Txyz))$
(6) $\forall x\,(Gx \leftrightarrow \exists z Hx)$
(7) $\forall x\,(Gx \leftrightarrow Hx \& Sxx)$
(8) $p \leftrightarrow t$ (where t is a sentential constant)
(9) $p \leftrightarrow \forall x Gx$

This suffices, I trust, to convey the flavor of the reduction. Every conjunct in AX (A) is the closure of a biconditional, with an atomic formula on the left and the result of applying a single logical operation to one or two atomic formulas on the right. There are further specifications on AX (A), but we shall ignore these here. But we note that AX (A), may contain predicate letters which need not occur in A itself.

Let us now turn to the normal form question for R^x and related logics. R^x, like the intuitionist predicate calculus H^x and unlike classical logic TV^x, resists straightforward normal forming techniques. For example, for a given formula A we cannot necessarily find a relevantly (or even materially) equivalent formula A′ of R^x in prenex normal form. The reason is as in the intuitionist case. Some of the classical theorems that are used to *justify* transformation into normal form are *not* relevantly valid. In fact, as [3] observes, the *reasons* that intuitionists and relevantists have for holding certain quantifier formulas invalid differ, but there is significant (though not inevitable) agreement on the *particular* formulas to be held valid and invalid. In R^x, the classical move to prenex normal form is blocked by the relevant invalidity of the following pair of lemons.

(10) $p \rightarrow \exists xFx \rightarrow \exists x\,(p \rightarrow Fx)$

(11) $\forall xFx \rightarrow p \rightarrow \exists x(Fx \rightarrow p)$

Since the principles are dual, by the classical quantifier interchange and contraposition properties that do remain relevantly valid, we may identify them as embodying a single rejected principle. The reason that it is rejected is clear from the similar rejection of the sentential analogue $p \rightarrow q \vee r \rightarrow (p \rightarrow q) \vee (p \rightarrow r)$, which would do terrible things to relevant insights.(3) Also interesting is the form of the rejected principle when expressed with the handy fusion connective o, which is R-definable by $\sim(p \vee \sim q)$. This is

(12) $\forall x\,(Fxop) \vee \forall xFxop.$

Nor do any of (10) - (12) hold in R^x when their main $\rightarrow$ is weakened to a material $\supset$, defined as usual by $\sim p \vee q$. This justifies our assertion above that there is not even a materially available way acceptable in R^x to transform arbitrary formulas A into prenex normal form.

Not having prenex normal forms available is a nuisance. It blocks, for example, straightforward efforts to prove R^x semantically complete. For quantifiers may be deeply buried structurally in formulas, within nested arrows within quantifiers within nested arrows and so forth, and there is no immediately evident way to extract these quantifiers from their nests, so that we can get a clearer picture of what a formula intends to say. (There are different sorts of opacity, and one of them occurs when it is difficult to make out what is being said.)

Perhaps in attempted recompense for its justifiable but annoying recalcitrance to pronounce (10)-(12) valid, R^x (unlike H^x) does pronounce valid all the *other* principles that one needs for prenex normal form, including rewriting laws for bound variables. In particular, the *converses* of each of (10)-(12) hold as true-blue relevant implications. This is locking the barn door after the horse has had half his legs chopped off. (He won't run away, and it is *equivalences*, not one way implications, that most facilitate normal forming arguments.)

More useful is the fact that all of the *other* formulas needed for prenex normal form arguments do hold in R^x as genuine relevant equivalences. Some specimens follow.

(13) $\forall x\,(p \rightarrow Fx) \leftrightarrow p \rightarrow \forall x Fx$
(14) $\exists x Fx \rightarrow p \leftrightarrow \forall x\,(Fx \rightarrow p)$
(15) $\forall x\,(p \& Fx) \leftrightarrow p \& \forall x Fx$
(16) $\forall x\,(p \vee Fx) \leftrightarrow p \vee \forall x Fx$
(17) $\exists x\,(p \& Fx) \leftrightarrow p \& \exists x Fx$
(18) $\exists x\,(p \vee Fx) \leftrightarrow p \vee \exists x Fx$
(19) $\exists x\,(p \circ Fx) \leftrightarrow p \circ \exists x Fx$
(20) $\forall x\,(p \supset Fx) \leftrightarrow p \supset \forall x Fx$
(21) $\exists x\,(p \supset Fx) \leftrightarrow p \supset \exists x Fx$
(22) $\sim \forall x Fx \leftrightarrow \exists x \sim Fx$
(23) $\sim \exists x Fx \leftrightarrow \forall x \sim Fx$
(24) $p \supset \exists x Fx \leftrightarrow \exists x\,(p \supset Fx)$
(25) $\forall x Fx \supset p \leftrightarrow \exists x\,(Fx \supset p)$

(24) and (25) are the exact analogues of (10) and (11), and they are interesting as evidence that the failures of exact equivalence for $\rightarrow$ statements do not infect the corresponding $\supset$ statements, where the steps that lead to normal form are concerned. Thus, for example, every $\rightarrow$ free formula of R^x has an equivalent prenex normal form, determined as in the classical case. But arrow-free R^x is just TV^x anyway, so the fact is not *that* interesting.

If we cannot get prenex normal forms for R^x, perhaps we can get something like Skolem normal forms, which, although not provably equivalent to their parent formulas, are at least deductively equivalent in the sense that, if A′ is the Skolem normal form of a formula of TV^x, namely A, then A and A′ are at least deductively equivalent in the sense that A is a theorem of TV^x iff A′ is a theorem of TV^x, and where A′ also simplifies A in some conceptual respects (say by putting all existential quantifiers *first*). This would be particulary useful for R^x, which lacks the convenience of ordinary prenex normal forms.

We cannot *quite* achieve this goal, on present techniques. But we can *almost* achieve it by applying the reduction methods of [2], which prompts the title of this paper. For let us consider again the formula A* associated with A by (1). A is always deductively equivalent in R^x to a closed formula, namely its universal closure, and so we may without loss of generality think of A itself as already closed. Since A in this case contains no free variables, we shall henceforth think of A* simply as a formula of the following form.

(26) $AX\,(A) \supset p$,

where AX (A) is a formula satisfying the general specifications laid down at the outset, and p is a sentential variable.

Let us now transform A* into an «almost Skolem form» A^+. The only quantifiers that occur in A* occur in the conjuncts of AX (A), either as universal quantifiers prefaced to a biconditional or as either universal or existential quantifiers occuring immediately to the right of the biconditional. The following is near enough to a typical instance of (26) for our purposes.

(27) $\forall x\,(Fx \leftrightarrow Px \vee Qx) \,\&\, \forall x\,(Gx \leftrightarrow .\, Rx \rightarrow Sx) \,\&\ldots\&\, \forall x \forall y$
$(Hxy \leftrightarrow \exists z\,(Txyz)) \,\&\, \forall x\,(Jx \leftrightarrow \forall y Uxy) \supset p$

We note first that the universal quantifiers *prefaced* to biconditionals are immediately extractable, applying (15) to make their scope the entire antecedent of (27), rewriting variables as necessary, and then applying (25) to trade in these universal quantifiers governing the antecedent of (27) for existential quantifiers governing the whole formula. While it isn't necessary for the method, we can ease the picture visually, and save ourselves some rewriting, by appealing to the fact that ∀ distributes *equivalently* over & in R^x (and in most other logics). Making these moves we get the following from (27).

(28) $\exists x\, \exists y\ [(Fx \leftrightarrow Px \vee Qx) \,\&\, (Gx \leftrightarrow .Rx \rightarrow Sx) \,\&\ldots\&$
$(Hxy \leftrightarrow \exists a Txya) \,\&\, (Jx \leftrightarrow \forall b Uxb) \supset p]$

We assume that analogous moves have been made to rid the omitted conjuncts presented simply as «...» of their prefaced universal quantifiers also. Note that the move which puts the prefaced existential quantifiers on the whole formula is available only because the *main* connective of A* is always ⊃. Were it an →, as it might be on some applications of the method of [2], the equivalent transformation of (27) into (28) would be blocked by the failure of (11).

We have now removed quantifiers from the first two conjuncts of the antecedent of (28), which for the time being we assimilate to the «...». Remaining, however, are the quantifiers buried within the last two conjuncts. They are no longer very deeply buried, and we may partly unbury them further by trading in ↔ for the conjunction of → statements in terms of which it is R-definable. Rearranging conjuncts slightly, here is our next R-equivalent form of (27).

$$(29)\quad \exists x\, \exists y\ [(\exists a Txya \rightarrow Hxy) \,\&\, (Jx \rightarrow \forall b Uxb) \,\&\, \ldots \,\&\, (Hxy \rightarrow \exists a Txya) \,\&\, (\forall b Uxb \rightarrow Jx) \supset p\,]$$

On our rearrangement, the first two conjuncts now contain *extractable* quantifiers, in virtue of the R^x-valid principles (13) and (14). The ∃a of the first conjunct governs only the antecedent, but it may be brought out by (14) as a ∀a whose scope is the whole first conjunct. Using (15) as before, make the scope of this ∀a the whole antecedent of the long ⊃ statement, and then apply (25) to bring it out front as a ∃a again. A similar move, using (13), will extract the ∀b of the second conjunct as, eventually, a prefaced ∃b. Rewriting the variables that remain bound by interior quantifiers, we get this formula.

$$(30)\quad \exists x\, \exists y\, \exists a\, \exists b\ [(Txya \rightarrow Hxy) \,\&\, (Jx \rightarrow Uxb) \,\&\, \ldots \,\&\, (Hxy \rightarrow \exists z Txyz) \,\&\, (\forall w Uxw \rightarrow Jx) \supset p\,]$$

We have now reached the limit of straightforward application of these methods, though we can pretty them up in some further respects. Meanwhile, let us say that a formula A^+ of R^x is in *almost Skolem normal form* provided that it can be expressed in the way suggested by (30). That is, A^+ must be a closed formula of the following form.

$$(31)\quad \exists x_1 \ldots \exists x_n\ [(A_1 \rightarrow B_1) \,\&\ldots\&\, (A_m \rightarrow B_m) \supset p\,],$$

where p is a sentential variable, and the A_i and B_i may be subjected disjunctively to the following further restricions, for each i such that $1 \leq i \leq n$:

(i) A_i and B_i are both atomic formulas, or
(ii) A_i is atomic and B_i is $\sim C$, where C is atomic, or
(iii) B_i is atomic and A_i is $\sim D$, where D is atomic, or
(iv) One of A_i, B_i is atomic, and the other is of the form $C \rightarrow D$, where C and D are atomic, or
(v) A_i is atomic, and B_i is of the form $\exists xC$, where C is atomic, or
(vi) B_i is atomic, and A_i is of the form $\forall xD$, where D is atomic, or
(vii) One of A_i, B_i is atomic, and the other is of one of the forms C&D, $C \vee D$, where C and D are atomic.

Of course it is unnecessaryto specify the A_i and B_i as carefully as we have, the main point lying in the conditions (v) and (vi) which limit the quantificational complexity of *non-prefixed* quantifiers of a formula A^+ in almost Skolem normal form.

We can now state and prove for relevant logics the theorem toward which our informal considerations have tended.

Theorem 1. Let S^x be any of the first-order relevant logics E^x, R^x, T^x, RM^x, EM^x of [3]. Then for each formula A of S^x, we can effectively determine an associated formula A^+ in almost Skolem normal form which is deductively equivalent to A, in the sense that A is a theorem of S^x iff A^+ is.

Proof. We may assume A closed, whence the principal corollary of [2] delivers a sentence A^* of S^x, of the form (26), which is deductively equivalent to A. Using the method just suggested illustratively, and breaking up all $\leftrightarrow$ statements into their conjoined $\rightarrow$ statements, we arrive eventually at a formula A^+ provably equivalent to A^* in S^x, with all quantifiers prefaced existential ones except for the exceptions allowed under (v) and (vi) above, and otherwise satisfying the conditions laid down, by an easy verification. Since A^* and A^+ are provably equivalent in S^x, *a fortiori* they are deductively equivalent in that system. By transitivity of deductive equivalence, A and A^+ are deductively equivalent in S^x, ending the proof of the theorem.

The title of the paper has suggested that our results hold for relevant *other* logics. The reader is at this point entitled to know, «Which other logics ?» He is hereby given leave to find out. The methods of [2] were very general, and produced second-degree (or, at worst, third-degree) formulas deductively equivalent to a given formula over a wide spectrum of predicate logics (such as, for example, the intuitionist predicate calculus H^x). The methods of this paper are equally general, depending as they do mainly on the quantifier exchange principles (13)-(25), and in fact on only some of these principles. These principles may be expected to hold over a wide choice of logics. And even where they fail (as for example (13) fails for some but not all quantified modal logics, for Barcan-formula-related reasons), we can still press the method as far as it will go, since it will always yield a bound on quantificational complexity (and on other kinds of complexity), wherever the devices of [2] itself are applicable.

We return now to relevant logics, and to R^x in particular. It would be delightful to find a way to unbury the last buried quantifiers, which, on the evident relations of duality, may always be taken to occur in contexts like

(32) $Fx \leftrightarrow \exists yGxy$,

with both F and G atomic. (As a degenerate case, these contexts can be sometimes of forms like $Fx \leftrightarrow \exists yGx$, but then the quantifier can be immediately eliminated as vacuous.) And, of course, our final elimination steps rid us of half the problem, leaving the following as the tough context.

(33) $Fx \rightarrow \exists yGxy$

As a first stage in thinking about the problem (and, by the way, uncovering some further implications which [1] has at the sentential level for R itself), let us go back to our sample A*, which is (27). One point that makes (27) and its various reductions difficult to think about is that all the work is going on in the *antecedent* of material $\supset$, a particle which does not interact too smoothly with the other notions that are primitive in R and R^x. So let us apply the definition of $\supset$ to turn (27) into a disjunction instead. When everything is dualized, and the reduction steps which led to (30) are simultaneously carried out, breaking up all the biconditionals and applying DeMorgan laws and the definition of fusion o, what (27) becomes, under provably R-equivalent transformation, is the following:

(34) $\exists x\, \exists y\, \exists a\, \exists b$ [(Fxo.~Px&~Qx) ∨ ((Px ∨Qx) o~Fx) ∨
∨ (GxoRxo~Sx) ∨ ((Rx→Sx) o~Gx) ∨... ∨
... ∨ (Txyao~Hxy) ∨ (Jxo~Uxb) ∨
∨ (HxyoVz~Txyz) ∨ (∀wUxwo~Jx) ∨ p]

The reader, if he is up to it, is now invited to survey the *form* of the disjuncts of (34). It is a form that, near enough, is perfectly general for R^x, and it is one which illustrates all of the simplifications to which our reductions lead. Note that each disjunct, save the last, is now of the form AoB. The second disjunct, (Px ∨Qx) o~Fx, remains a little too complicated, since o distributes in R over ∨. It should be replaced, in thinking about (34), with Pxo~Fx ∨Qxo~Fx. The point is just the finite analogue of the way in which we got the existential quantifiers out front, whose basis, in this context, is the law (19) that gets ∃ out of the interior fusion position, after which one may think truth-functionally.

Otherwise, our fusions turn out to have just five possible distinct forms, given that all of o, &, ∨ are both commutative and associative in R. We shall not fuss much over the distinction between atomic formulas and their negates, identifying a formula A as a *littoral* if A is either itself atomic or if A is the negation of an atomic formula. This provides an alternative characterization of almost Skolem normal form for R^x, which in order to prevent confusion, we shall call *almost Skolem standard form*. A formula A# shall be said to be of this form provided that it is the following sort of closed formula:

(35) $\exists x_1 \ldots \exists x_n\ [A_1 o B_1 \vee \ldots \vee A_m o B_m \vee p]$,

where p is a sentential variable, and the disjuncts $A_i o B_i$ are the subject to the following conditions, for each i from 1 to n:

(a) A_i and B_i are both littorals, or
(b) A_i is a littoral and B_i is of the form CoD, where C and D are littorals, or
(c) A_i is a littoral and B_i is of the form C→D, where C and D are lirrotrals, or
(d) A_i is a littoral and B_i is of the form C&D, where C and D are littorals, or

(e) A_i is a littoral and B_i is of the form $\forall yC$, where C is a littoral and the variable y occurs free in C but not in A_i.

In addition, we may impose the condition that exactly the same variables shall occur free in each of A_i, B_i, under any of the clauses from (b) to (e). (Under clause (a), we may get an extra free variable on one side as the result of exporting a $\exists a$ to the preface.) We pause for a theorem.

Theorem 2. For each formula A of R_x, we can effectively determine an associated second-degree closed formula A# of R_x such that (i) A# is in almost Skolem standard form and (ii) A# is deductively equivalent to A in R_x.

Proof. Use theorem 1 and properties of R_x, checking that the moves (illustrated above) that transform (31) into (35) and (i) - (vii) into (a) - (e) do not leave anything out. This is busy work, which may be safely omitted, ending the proof of theorem 2.

As I hinted above, theorem 2 has significance not merely for R_x but also for its quantifier-free part R itself. References to Skolem are now inappropriate, so let us say simply that a formula A# of R itself is in *standard form* provided that it is so in virtue of the applicable parts of the specifications just above. In this restricted context, *littorals* are just sentential variables and their negates. And A# is in standard form provided that it is of the form

(36) $A_1 o B_1 \vee \ldots \vee A_m o B_m \vee p$,

where p is a sentential variable, each of the A_i are littorals, and each of the B_i is one of the forms CoD, $C \rightarrow D$, C&D, where C and D are littorals. While it is clear enough from [1] anyway, we draw without proof from theorem 2 the following evident corollary.

Corollary. Every formula A of R is deductively equivalent in R to a second degree formula A# in standard form.

Thinking about standard form in the sentential case offers insight into why we have difficulty unburying the last buried quantifiers in expressions like (33). After dualizing, we can think of (33) as having the form

(37) Fxo∀yGxy,

which might appear as a disjunct in an expression (35). The quantifier-free analogue is

(38) qo.r&s,

which might appear as a disjunct in an expression (36) (or even in (35), for that matter). (36) provides (38) with a pretty simple context; the only enclosing layer is truth-functional disjunction. And, just as the conceptual problem posed by (37) is that of manipulating this formula, in some way that will preserve deductive equivalence in context so that the ∀y can be pulled *outside* the fusion in which it is nested, we have a similar problem even with (38). Note that, in (36), all occurrences of ∨ have been pulled *out* to the enclosing layer. It would be nice to do the same thing with &, which would yield a pretty picture of simple intensional expressions in →, o, and ~ (the latter always having been counted in relevant logics, as Belnap remarks, as an *intensional* negation) being enclosed in an extensional picture supplied by & and ∨. For this is just the picture that we are looking for in trying to find a way to prenex quantifiers, at least so far as we can.

However, theorem 2 has already gone as it is possible on present knowledge to go in the quantifier case, and, even in the sentential case, the corollary to the theorem is the best result. The problem, which has been familiar to students of relevant logic for a long time, is that o and & do not interact very well, even as the failure of (12) limits analogously the interaction of ∀ and o.(4) Interestingly, if one proceeds to the stronger system RM, o does distribute over &, yielding a sort of sentential analogue of prenex normal form for the formulas of this system. (This has been known for a long time). Perhaps not coincidentally, the decision problem for RM has also been long since put away, while the conditions for its semantic modelling are considerably simpler than any that have been discovered for R, as Dunn and I have shown in a variety of ways.

So the problems of understanding R better, do not go away on applications of the techniques surveyed in this paper. It is to be hoped merely that, as I put it in [2], «decreasing the degree of relevant involvement» will help to render more transparent some problems of relevant quantification theory that have hitherto appeared all too opaque. At the very least, quantifiers can be brought *towards* the surface, if not *to* the surface, by putting the formulas of R_x and other relevant logics in one of our *almost Skolem* forms, in which all of the existential quantifiers can be brought out front, on the *standard* recension that is conceptually clearest, and the universal quantifiers are buried under just one layer of relevant involvement. (We pause for a final technical aside on this subject; while we were delighted to bring $\exists$ *all the way* to the front, the obvious alternative is to let $\vee$ furnish the *outermost* layer; since $\exists$ distributes equivalently over $\vee$ in R_x, as $\forall$ does over &, this would leave the occurrences of $\exists$ on the *second outermost* layer; after distribution, we could then drop some of the $\exists$'s as vacuous, allowing us to think semantically about the problem of satisfying universally some existentially closed sentences. This redistribution merely reverses the process by which we got into standard form in the first place.)

There is a price for our normal forming techniques, as Slaney has pointed out in conversation. In the effort to gain technical and semantical control over the relevant logics, it was sensible to concentrate on problems appropriate to formulas of *reduced degree*. Not much thought was given to the *number of variables*, propositional and predicate, appearing in these formulas. For a given formula, this number must be finite, and one naturally expects the validity of a given formula A to depend only on the variables that actually occur in this formula. So it was easy to think of the degree-reducing problem as the hard one, which, once solved, would work for any number of variables.

But the methods of [1] and [2] work typically by adding *new* variables. This is the feature which allows us to choose normal and standard forms that are so simple. Moreover, for the relevant logics, the degree-reducing problem has been permanently put to rest at degree = 2. (This means, for relevant predicate logics also, that any two *distinct* relevant logics must differ in their supply of second-degree tautologies, upsetting any expectations that might have been formed from Belnap's discovery that the first-degree tautologies remain the same on substantial variation of underlying logic. In particular, E_x, R_x, and T_x all have the *same* first degree.) But, as

Slaney observes, it might be more accurate to say that the degree-reducing problem has itself been reduced to the variable-number problem. The latter problem, which has hitherto masqueraded as trivial, is now to be seen as part of a trade-off. If one is allowed more variables, one can decrease degree. But, to make real progress, the extra variables must be brought under control at the reduced degree. Perhaps this is possible; perhaps not. At any rate, where the truly hard problems are concerned, more effort is needed, if the methods of this paper and its predecessors are to bear real fruit.

Our survey of almost Skolem forms has a surprising denouement. While it was hardly the purpose of this paper to think about *classical* problems (if only because, as [23] also will observe, somebody else is sure to have thought about them first), we have by the way proved that every formula A of TV_x has a deductive equivalent in Skolem normal form (no longer «almost»). Since a theorem to this effect is scarcely required, we content ourselves with a concluding.

Observation. For every formula A of TV_x, we can effectively find a deductively equivalent A # of the form $\exists x_1 \ldots \exists x_k \forall y_1 \ldots \forall y_m B$, where B is quantifier-free. Moreover, B may be required to satisfy disjunctively (i)-(iv), (vii) attached to (31). (5, 6)

NOTES

(1) Only the *name* R^x will be introduced in [3] (more fully, on Anderson-Belnap notational conventions, $R^{\forall\exists x}$), the system itself being well-known in the relevant literature as RQ. Full axiomatization and discussion of R^x and other first-order relevant logics will appear in [3].

(2) The binary connectives which we introduce below are ranked &, o, ∨, →, ↔ in order of increasing scope.

(3) This is witnessed by the system RM, in which theoremhood of the displayed formula *does* do terrible things to relevant insights. Added to R the formula suffices for the relevantly dubious *simple order* principle (A→B) ∨(B→A), which in turn yields fallacies of relevance like p&~p→q∨~q. Interestingly, all of these principles are properly weaker, when added to R than the RM axiom A→.A→A itself.

(4) The lack of interaction between o and & in R is most strikingly illustrated by Dunn's Gentzen formulation of R_+ (a similar formulation is due independently to Minc), which introduces *two* sorts of sequences, intensional and extensional ones, to correspond respectively to binding of premisses by o and binding them by &. Belnap had earlier made a similar proposal, but Dunn's succeeded because he allowed interlacing of the two

sorts of sequence to arbitrary depth, precisely to take account of the lack of interaction between & and o. Were some interaction after all discovered which would put effective bounds on this interlacing, Dunn's methods would presumtively lead to a decision method for R_+. But this problem is also open, and, infuriatingly, the Gentzenization doesn't seem to help.

(5) To complete the proof of the observation, note that all steps involved in the proof of theorem 1 go through in the same fashion for TV^x. Buried quantifiers are then restricted to conjuncts of the antecedent of non-prenex part of (31), in contexts like $Gy \rightarrow \exists zHyz$ and $\forall xFx \rightarrow q$. Since (10) and (11) hold without restricion as biconditionals in TV^x, these quantifiers also may be unburied, whence they will eventually show up at the tail end of the prefix as $\forall$'s.

(6) Thanks are due to Anderson, Belnap, Dunn, Routley, and Slaney for helpful discussions of various topics, and (especially) to Mc Robbie for valuable assistance.

REFERENCES

[1] R. K. MEYER, «Career induction ends here (and here = 2)», *Journal of philosophical logic*, 8(1979) 361-371.

[2] R. K. MEYER, «Career induction for quantifiers», *Notre Dame Journal of Formal Logic*, 21(1980), 539-548.

[3] A. R. ANDERSON and N. D. BELNAP, Jr., *Entailment: The Logic of Relevance and Necessity*. Vol. II, Princeton University Press, Princeton, forthcoming.