THE DIALECTICS OF LOGIC

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Logic is a deductive science but it has, or should have a special connection with argument and the conduct of discussion. This special connection is rarely looked at very closely. In the present paper I shall describe a system of dialogue (first described in my [Q]) in which the participants can engage in deductive argumentation, and then examine the differences which arise when they endeavor to establish theorems (not just of some deductive theory but) of the particular logic which governs their own discourse. In the clarification of these differences, it is found useful to describe a further system of dialogue, arising out of the first. This further system takes us beyond the familiar distinction between internal and external properties of a logical system, and enables us to make some steps towards an account of the dialectics or pragmatics of a logical system; that is, the way in which a logic is related to the conduct of discussion.

A dialogue occurs when people say things (sentences, locutions) to each other, one at a time. The locutions are grammatically complete utterances, types rather than tokens, forming a set L. The people or other participants (fictional characters, organisations, perhaps machines) form a set P.

Given these, a *locution act* is a member of $P \times L$. A member of a dialogue is to be a locution act at a stage, and hence of the form $<n, p, \ell>$, $n \varepsilon N$, $p \varepsilon P$, $\ell \varepsilon L$. Such a triple is a *locution event* $e \varepsilon E$. Its first member is its *stage*, its second member is its *speaker*, and its third member is its *locution*. [1] For convenience we shall consider only dialogues with just two participants, $\overline{\overline{P}} = 2$; this enables us to refer also to the *hearer* of a locution event, namely the participant other than its speaker. To avail ourselves of the two third-person singular pronouns in English, we shall suppose that the participants differ in gender, though the same rules apply to one or the other indifferently. The set D_n of *dialogues of length n* is $\{d \subseteq E: \overline{\overline{d}} = n \;\&\; (\forall k \varepsilon N)\ (k < n \supset (\exists e \varepsilon E)\ (e \varepsilon d \;\&\; \exists! p \varepsilon P)\ (\exists \ell \varepsilon L)\ (e = <k, p, \ell>)))\}$. The set D of *dialogues is:*

$$D = \bigcup_n D_n$$

The *last* and *next* events to $<n, p, \ell>$ in a dialogue are those whose stages are $n-1$ and $n+1$ respectively. By *henceforward* with reference to an event $<n, p, \ell>$ I shall mean in any event whose stage is $k > n$.

The System DC

The dialogue is conducted in a formal language, consisting of the set L of locutions. L is generated from a given set S of statements (eternal declarative sentences). Statements are the only kind of sentences normally considered by logicians, and a suitable set S would be that used in propositional logic, consisting of sentence letters together with statements produced from them by statement connectives, though it is not required that the connectives be truth-functional. I assume that the members of S can be alphabetically ordered, and that the following are all members of S:

(i) The *negation*, N 's, of any statements $s \varepsilon S$. (Reading: 'It is not the case that p', or briefly 'Not p', is the negation of the statement 'p'.)

(ii) The *conditional*, C '$<s, t>$, of any ordered pair of statements $s, t \varepsilon S$. (reading: 'If p then q' is the conditional whose antecedent is 'p' and whose consequent is 'q'.)

(iii) The (alphabetically ordered, left-associating) *conjunction*, K 'T, of any finite, non-empty set of statements $T \subseteq S$. Where $T = \{s\}$, K ' $T = s$. (Reading: 'Both p and q' is the conjunction of {'p', 'q'}.)

In giving readings, the letters 'p' and 'q' are used as schematic letters holding place for statements throughout this paper. Quotation marks containing schematic letters are to be understood as quasi-quotation in the sense of Quine, [M) 33f. By the *denial*, D 's, of $s \varepsilon S$, I shall mean N 's unless $(\exists t \varepsilon S)$ ($s = N$ 't), in which case D ' $s = t$. The negation of 'Not p' is 'Not not p', but the denial of 'Not p' is 'p'.

We extend the language by the use of locution modifiers. By a locution modifier in DC I shall mean an expression which, with a statement, forms a locution other than a statement. Since locution modifiers cannot grammatically occur except with statements, no problem of their iteration or interaction arises. Locution modifiers connect each locution neatly with a statement, providing a ready-made syntactically defined one-one function to the set of statements from each other class of locutions. Locutions other than statements cannot grammatically be combined by use of statement connectives, though the statement to which a locution modifier is applied may itself be formed from simpler statements by the use of those connectives.

Specification of the set L of locutions

Name	*Reading*	*Function to S*
Statements S	'p'	*I*
Questions	'Is it the case that p?'	*Q*
Withdrawals	'No commitment p'	*W*
Challenges	'Why is it to be supposed that p?'	*Y*
Resolution demands	'Resolve whether p'	*R*

$$L =_{df} S \cup \{\ell : (\exists s \varepsilon S)\ (\ell Q s \vee \ell W s \vee \ell Y s \vee \ell R s)\}$$

Challenges will also be written briefly as 'Why p?'. Since 'why' in English has several senses, it should be noted that a challenge is a demand for evidence, not for an explanation whether causal or teleological.

As well as typographically obvious relationships between locutions (such as that 'No commitment p' is the withdrawal of a conjunct of the antecedent of 'If both p and q then r'), we need four other syntactically specified notions. The first is that of being an *allowable answer* to a question: a statement, its withdrawal, and its denial each bears this relation to the question of that statement, i.e.

$$\ell \text{ ans } Q\text{‘}s \equiv \ell\ \varepsilon\ \{s, W\text{‘}s, D\text{‘}s\}$$

The other three notions are specified with the aid of a list V of *preferred valid argument schemata.* The schemata to be included in this list will be discussed later. At least the schema for modus ponens should be included, thus:

If p then q; p/q

Using this list, we define the syntactic relationship which holds between a set of statements which exemplify the schemata to the left of a slash on a liné of V and that statement which exemplifies the schema to the right of that slash. This relationship is that of *immediate consequence.* Since modus ponens is in V, any statement is an immediate consequence of the set consisting of a conditional of which it is the consequent, together with the statement which is the antecedent of that conditional:

$$\{C`\langle s,t\rangle, s\} \text{ Imc } \{t\}$$

A set of statements is *immediately inconsistent* if it consists either of a statement together with its denial, or of some finite set $Z \subseteq S$ together with the denial of an immediate consequence of Z:

$$\text{Imn } I \equiv (\exists s \varepsilon S)\,(\exists Z \subseteq S)\,(T = Z \cup D`s\ \&\ (Z \text{ Imc } \{s\} \vee Z = \{s\})$$

Immediate consequence conditionals, λ, are conditionals whose consequent is an immediate consequence of the conjuncts of the antecedent:

$$C`\langle K`T, s\rangle \varepsilon \lambda \equiv T \text{ Imc } \{s\}$$

The claims that these notions are specified syntactically, given the list V, are meant strictly. For example, for T, U, $Z \subseteq S$, it may be that T Imc U but not $T \cup Z$ Imc U. Equally it may be that Imn T but not Imn $(T \cup Z)$. This allows for the fact that the members of T can be so «buried» among others that the relation is not immediate. Given the list V, the immediate relationships are just those which exemplify the schemata of V. It should also be noticed that Imc is not transitive. Only the schemata in V, and not those they entail, form sets between which Imc holds. This is realistic when we consider complex but truth-functionally valid argument schemata. Even students of logic sometimes fail to recognise these as valid, as their teachers well know.

Commitment

In addition to syntactically specified properties among locutions, we need also the distinctive feature of dialectic, the notion of *commitment.* The term is perhaps unfortunate, in that it suggests a long-term, firmly held belief. A commitment in the sense in which we are interested may be momentary and it need not be a belief at all.

Beliefs, whatever they are, can be kept private. Commitment, since dialectic is an empirical science, must be public. Participants need neither believe their commitments, nor commit themselves to their beliefs. The commitment store of a participant may be visualised as a slate on which tokens of locutions may be written and from which they may be erased; it serves as an indication of the state of play in the dialogue at each stage. Formally, there is a commitment function from $N \times P$ to the power set of L, which assigns a set of locutions to each participant A at each stage n as his commitment, $C_n(A)$. This function is specified inductively. A is whichever participant is the speaker of the locution event concerned, B is the other participant.

The initial commitment of each participant is null.

CR_0: $C_0(A) = \Lambda;\ C_0(B) = \Lambda$

Questions and resolution demands do not affect commitment.

CR_Q: After $\langle n, A, Q\text{‘}s\rangle$
$C_n+_1(A) = C_n(A);\ C_n+_1(B) = C_n(B)$

CR_R: After $\langle n, A, R\text{‘}s\rangle$
$C_n+_1(A) = C_n(A);\ C_n+_1(B) = C_n(B)$

After a withdrawal, the statement withdrawn is excluded from the speaker's store; the hearer's store is unchanged.

CR_W: After $\langle n, A, W\text{‘}s\rangle$
$C_n+_1(A) = C_n(A) - \{s\};\ C_n+_1(B) = C_n(B)$

A statement which does not occur as the reply to a challenge is included in both participants' stores. This means that one's interlocutor can place a statement in one's store. It is not observed in all real-life dialogues, but that something like it is observed in some is indicated by the point of order ‘If you didn't agree, why didn't you say so?'.

CR_S: After $\langle n, A, s\rangle$, where the event at $n - 1$ is not
not $\langle n - 1, B, Y\text{‘}t\rangle$,
$C_n+_1(A) = C_n(A) \cup \{s\}$;
$C_n+_1(B) = C_n(B) \cup \{s\}$

After a challenge, the statement challenged is included in the hearer's store, and excluded from the speaker's store. The challenge itself is included in the speaker's store. If one is challenged to produce evidence for a statement with which one disagrees, one feels the need to say that one disagrees (to remove explicitly the commitment incurred by the challenge), and this is what is reflected by including the statement challenged in the hearer's store. The challenger's commitment to the challenge may make intuitive sense as his having declared the statement of which it is the challenge to be in doubt or problematic.

CR_Y: After $<n, A, Y\text{'}s>$
$C_{n+1}(A) = C_n(A) \cup \{Y\text{'}s\} - \{s\}$;
$C_{n+1}(B) = C_n(B) \cup \{s\}$

A statement 'p' which occurs as the reply to the challenge 'Why q?' commits both participants to the reply (or *defence*) 'p', and also to the conditional (or *argument-step*) 'If p then q'.

CR_{YS}: After $<n, A, s>$, where the event at $n-1$ is $<n-1, B, Y\text{'}t>$,
$C_{n+1}(A) = C_n(A) \cup \{s, C\text{'}<s, t>\}$;
$C_{n+1}(B) = C_n(B) \cup \{s, C\text{'}<s, t>\}$

The rules R of the dialectical system DC are formulated in terms of commitment and syntactically specified characteristics. The rules should enable us to decide given a legal dialogue of length n, $d_n \varepsilon K$, whether the addition of a particular event renders the dialogue d_{n+1} illegal, considering only the preceding event at stage $n-1$, the commitment $C_n(A)$ at n of the speaker A of the event, the commitment $C_n(B)$ at n of the hearer B of the event, and syntactic properties of and relations between locutions. The same rules apply to both participants – 'A' may be either participant at any stage since either may be the speaker of the next except where this is excluded by the rules.

Rules of Dialogue for DC

Each participant contributes a locution at a time, in turn; and each locution must be either a statement or the question, withdrawal, challenge or resolution demand of a statement:

R_{Form}: No legal dialogue contains an event $\langle n, A, \ell \rangle$ if it also contains an event $\langle n-1, A, \ell' \rangle$; or if $\ell \; \varepsilon \; L$

No statement may occur if it is a commitment of both speaker and hearer at that stage:

$R_{Repstat}$: No legal dialogue contains an event $\langle n, A, s \rangle$ where $s \subseteq C_n(A) \cap C_n(B)$

Immediate consequence conditionals may not be withdrawn:

R_{Imcon}: No legal dialogue contains an event $\langle n, A, W\text{‘}s \rangle$ where $s \, \varepsilon \, \lambda$

After ‘Is it the case that p?’ the next event must be either ‘p’, ‘No commitment p’ or the denial of ‘p’:

R_{Quest}: No legal dialogue of length $n+1$ contains an event $\langle n-1, B, Q\text{‘}s \rangle$ unless it also contains an event $\langle n, A, \ell \rangle$ and ℓ ans Q‘s

Immediate consequence conditionals may not be challenged:

$R_{LogChall}$: No legal dialogue contains an event $\langle n, A, Y\text{‘}s \rangle$ where $s \, \varepsilon \, \lambda$

The reply to the challenge ‘Why p?’ must take one of three forms. It may be the withdrawal of ‘p’. Secondly, it may be the resolution demand of an immediate consequence conditional whose consequent is ‘p’ and whose antecedent is a conjunction of statements to which the challenger is committed. This reply enables a participant to call to order one who challenges a statement which is an immediate consequence of his current commitments. Thirdly, it may be a statement to whose challenge the challenger is not committed. The point of excluding defences to whose challenge the challenger is committed is to prevent defences which beg (i.e., ask to be granted as a premiss) the (statement which is in) question, as is argued in [Q].

R_{Chall}: No legal dialogue of length $n+1$ contains an event $\langle n-1, B, Y\text{‘}s \rangle$ unless it also contains an event $\langle n, A, \ell \rangle$ and either

(i) $\ell W s$; or
(ii) $\ell R\text{‘}C\text{‘}\langle K\text{‘}T, s \rangle$ where $T \subseteq c_n(B)$

and T Imc $\{s\}$; or
(iii) $\ell \varepsilon$ S and $\{Y`\ell\} \not\subset C_n(B)$

The final two rules provide a means for Ann to bring Bob to order if he either (i) becomes committed to immediately inconsistent statements or (ii) withdraws or challenges an immediate consequent of his commitments. They require him appropriately to adjust (resolve) his commitments.

$R_{Resolve}$: No legal dialogue contains an event $<n, A, R`s>$ unless either

(i) $s = K`T$; Imn T; and $T \subseteq C_n(B)$; or
(ii) $s = C`<K`T, u> \varepsilon \lambda$; $T \subseteq C_n(B)$; and either $<n-1, B, W`u> \varepsilon\, d$ or $<n-1, B, Y`u> \varepsilon\, d$

$R_{Resolution}$: No legal dialogue of length $n+1$ contains an event $<n-1, B, R`t>$ unless it also contains an event $<n, A, \ell>$ where ℓ is either

(i) $W`s$, and s is one of the conjuncts of t; or
(ii) $W`s$, and s is one of the conjuncts of the antecedent of t; or
(iii) s, and s is the consequent of t

A fuller discussion of $R_{Resolve}$ and $R_{Resolution}$ may be found in my [T].
Where the statement of the rule is of the form:

No legal dialogue contains an event such that Φ

the rule itself is the set

$$r = \{d \varepsilon D: (\exists e \varepsilon E)(e \varepsilon d \ \&\ e = <n, A, \ell> \text{ such that } \Phi)\}$$

The set R of rules is

$$R = \cup r$$

The set K of legal dialogues is

$$K = D - R$$

The system DC is $<L, P, R>$.

Deduction

Consider the dialogue fragment:

n	Ann:	Why p_0?
n + 1	Bob:	p_1
n + 2	Ann:	Why p_1?
n + 3	Bob:	p_2
n + 4	Ann:	Why p_2?
. . .		
n + 2k − 2	Ann:	Why p_{k-1}?
n + 2k − 1	Bob:	p_k

At n + 2k Ann is committed to 'p_k' and (since she didn't challenge any of Bob's argument-steps, i.e. the conditionals to which he committed her under CR_{YS}) also to a chain of conditionals back to 'If p_1 then p_0'. Thus if at n + 2k she breaks off the succession of challenges she will be in trouble if she later denies, withdraws or challenges any of 'p_0', ... 'p_{k-1}'. When I say she will be in trouble, I mean that Bob will have a strategy culminating in him demanding resolution of her. The proof of this depends on the facts that any point in such a chain can be reached in a finite number of modus ponens steps and that modus ponens is in V. The only statement in the sequence of defences she can challenge without risking an eventual resolution demand is 'p_k'. She may also challenge Bob's argument-steps, unless these are immediate consequence conditionals and immune from challenge under $R_{LogChall}$. In his defences Bob cannot use any statement which she has already challenged, under R_{Chall} (iii). Otherwise he would beg the question (see [Q]). If one writes Bob's successive statement commitments in reverse order, 'p_k', 'p_{k-1}', ...' 'p_1', 'p_0', they constitute a sequence of sentences each member of which follows (by his argument-steps) from earlier ones, and hence a *derivation* of 'p_0' from 'p_k'. The dialogue models the conduct of a teacher defending a theorem of a deductive system by reference to earlier theorems, and ultimately by reference to the axioms. Nonetheless, there is no need for Ann to accept 'p_0' nor to become liable to a resolution demand. She may simply challenge the axiom 'p_k' in its turn. Euclid engaged in dialogues down the centuries, convincing people of his theorems. But though he could convince Lobachewsky of the argument-step that if the parallels postulate was true then the internal angles of a triangle amounted to two right angles, he could not force Lobachewsky to accept commitment to the consequent of that conditional because Euclid had no defence to Lobachewsky's challenge of its antecedent. If someone is satisfied that a given statement is axiomatic or acceptable, then he need not challenge it. (2) But someone else may do so. It is not the role of the empirical theory of dialogue to favor the intuitions of some participants at the expense of others. There are no privileged or unchallengeable statements with which to tie down one end of a line of argument once for all, save as laid down by $R_{LogChall}$.

We have so far been discussing dialogues in which the participants discuss deductive theories of unspecified subject matter. Deductive logic itself, however, is especially related to the conduct of argument, and it is interesting to examine what happens when the participants attempt to defend theorems of the particular logical system which governs their own discourse. (3) The «logic governing» a dialogue is given by the list V of valid argument schemata preferred by the participants. The only requirement made of V, it will be remembered, is that it should contain at least the schema for modus ponens. The significance of V is that it provides syntactic criteria for the properties of immediate consequence, immediate inconsistency, and being an immediate consequence conditional. Since modus ponens is in V, any conditional of the form 'If both p and if p then q, then q' is an immediate consequence one $\varepsilon\lambda$. There may be other immediate consequence conditionals, depending on the list V.

The important thing about immediate consequence conditionals in turn is that they cannot be withdrawn (under R_{Imcon}) nor challenged (under $R_{LogChall}$). Hence they can be established once for all, and one's interlocutor irreversibly committed to them, simply by stating them (CR_S). But other statements can also be established as logical truths. For example, let us suppose that the argument schemata:

Both p and q / q

If p then q / If, if q then r, then if p then r

are both in V, but that

If q then r / If both p and q then r.

is not. A conditional corresponding to this last schema is not, of course, an immediate consequence one. Nevertheless, it can be established thus:

n Bob: Why, if if q then r, then if both p and q then r?
n+1 Ann: If both p and q, then q

Here Bob challenges the conditional in question at n. This is permissible because we have assumed that it is not an immediate consequence one. Ann's reply at n+1 is an immediate consequence conditional. Since it is a defence, it commits both of them under CR_{YS} also to

If, if both p and q then q, then if if q then r then if both p and q then r

as her argument-step. But this is of the form

If, if p then q, then if if q then r then if p then r

And so is itself an immediate consequence conditional. Both the defence and the argument-step are immediate consequence conditionals. Thus Bob's challenge is met with finality. Should he subsequently deny, challenge or withdraw the statement which he challenged at n, he would render himself liable to a resolution demand. If he denies it, then the defence, the argument-step and the denial together form an immediately inconsistent set, and he is liable under $R_{Resolve}$(i). If he withdraws or challenges it while remaining committed to both the defence and the argument-step he is liable under $R_{Resolve}$(ii). Since both the defence and the argument-step are immediate consequence conditionals, he can neither withdraw nor challenge either of them, under R_{Imcon} and $R_{LogChall}$. Thus he will be liable to a resolution demand whenever he denies, challenges or withdraws the statement he challenged at n henceforward. We may describe this by saying that Ann has *established* that statement with him from that stage. Since Ann is herself committed to the two immediate consequence conditionals too, equally Bob has established the same statement with her. Nor is it necessary that a participant first challenge it. Either can establish it at will by simply asserting the two statements from which it follows, under CR_{YS}.

This does not mean that the statement so established is itself an immediate consequence conditional. Indeed it need not be a conditional at all. Certainly it is legal to challenge or withdraw it at any time, though after it is established to do so leaves one open to a resolution demand. A statement which can be established in this way is valid, but it is not *immediately* valid. What has been shown is that Ann can put Bob into a commitment position, willy nilly, in which for him to withdraw, challenge or deny the established statement will leave him liable to a conditional 'If p then q' and also to its antecedent 'p', he can deny, challenge or withdraw 'q' without becoming liable by simply first withdrawing either 'p' or 'If p then q'. But in the present case neither escape route is available because both the conditional and its antecedent are immediate consequence conditionals and cannot be withdrawn (R_{Imcon}) nor challenged ($R_{LogChall}$). Thus an established statement is secure in a way in wich the theorems of other deductive sciences are not secure. No-one may legally play Lobachewsky with it. Its security does not rest upon the participants' failure to challenge an axiom, but on the rules of dialogue and the list V.

A natural further question is whether we can carry out what is in effect the induction step of a completeness proof for the participants. If Ann has established some statement with Bob, can we show that she can also establish any statement which is the consequent of an immediate consequence conditional whose antecedent is a statement she has established? The answer is that we can. Suppose that she has established 'q', and that 'If q then r' is an immediate consequence conditional. Ann then asks 'Is it the case that r?'. Under R_{Quest} he can answer only with 'r', 'No commitment r' or the denial of 'r'. But the denial leaves him liable to a resolution demand under $R_{Resolve}$(i). The reply 'No commitment r' leaves him liable under $R_{Resolve}$(ii). And he will become liable whenever he denies or withdraws 'r' for so long as he is committed to both 'q' and 'If q then r'. He must remain committed to 'If q then r' for it $\varepsilon\lambda$, and he is, as has already been shown, liable to a resolution demand should he withdraw or challenge the established statement 'q'. Thus he can neither deny nor withdraw 'r' henceforward without becoming liable. Additionally he is liable if, when committed to 'q' and to 'If q then r', he challenges 'r', under $R_{Resolve}$(ii). Thus 'r' is itself established. Provided Ann has a chance to ask a question, she has a strategy by which Bob can avoid conceding an establishable statement only at the cost of a resolution demand.

Whether all valid statements of L are establishable depends simply on whether the schemata of V are weakly complete with modus ponens as rule.

Conversely if Bob asserts a counter-valid statement and tries to retain his commitment to it, Ann can force him into a position in which he is liable to a resolution demand. In the simplest case, in which the counter-valid statement is the negation of an immediate consequence conditional, she need only assert that conditional. By R_{Imcon} and $R_{Log\ Chall}$ he cannot remove the commitment he has thus (under CR_S) incurred to it, and so he has immediately inconsistent commitments for as long as such is liable under $R_{RESOLVE}$(i). In more complex cases she should begin by asking the questions of immediate consequence conditionals. He cannot withdraw them, under R_{Imcon}. If he concedes them she goes on to the questions of statements each of which is an immediate consequence of those which he has been forced to concede. If he withdraws or denies any of these he is again liable, for withdrawing or denying an immediate consequence of his commitments. By asking questions she retains the conversational initiative. Her final aim, of course, is to commit him to a valid statement immediately inconsistent with the counter-valid one which he originally asserted. When she does so, he will be liable to a resolution demand for having immediately inconsistent commitments.

The logical system governing the dialogue may be classical propositional logic, or intuitionistic logic, or some relevance or modal logic of one's taste.(4) Quantificational systems are also possible, but require additional machinery. Once one has chosen a system, the schemata to put in V is a familiar matter for orthodox logical inquiry. What is needed is a set of conditional schemata which form with modus ponens a weakly complete set of axiom schemata for the chosen logic. There is no particular advantage to be gained by requiring the axiom schemata to be independent save brevity of the list V. The cost of doing so is that the participants appear logically inept.

The participants of DC can force each other under pain of facing repeated resolution demands to concede any logically valid statement which can be derived by iterated modus ponens steps from the immediate consequence conditionals, which are in turn constructed from the schemata of V. The extra security which logically valid statements enjoy over theorems of other deductive systems consists in no more than the fact that whereas the ultimate defences – the axioms – of other deductive systems may be challenged or withdrawn, by a mere fiat of the person who has set up the rules of dialogue. The dialectical legislator comes under a suspicion of unduly favoring the axioms of logic above those of other deductive sciences; and the certainty of logical truth seems to depend on no more than the dialectical legislator's prejudices.

To suppose these things is to misunderstand significance of the list V and to misconstrue the role of the dialectician. He is no legislator but a humble empirical scientist, formulating law-like statements not in order tyrannically to prescribe them to those who participate in dialogues but merely to systematise his observations of their conduct. The list V he constructs to help him in this task. It contains the schema for modus ponens, he reasons, because if modus ponens does not hold for a connective then the statements formed by that connective can hardly be correctly regarded as conditionals.([5]) Additional argument schemata can be added to V by noticing that the resolution demand of a conditional can occur only if that conditional $\varepsilon\lambda$. A field linguist investigating a language, however, does so most effectively by participating in dialogues in it. Since *ex hypothesi* he does not know the rules he is sure to make mistakes. To accommodate his mistakes it is necessary to consider a more complex dialectical system, which we obtain by extending DC.

Extension to DC⁺

The idea behind this extension is to introduce points of order which can be raised against breaches of the rules of DC; then to amend those rules to accomodate the points of order. We then add a rule, R^J, which provides that if an event occurs which renders the dialogue illegal in the original sense, the dialogue is *legal in an extended sense,* or *legal⁺*, provided that at the next stage the other participant raises the appropriate point of order. In this extension points of order cannot legally⁺ occur unless justified; nor can they be debated.

We begin the extension by introducing a set L_R of *order locutions:*

$$L_R =_{df} \{L_{Form}, L_{Repstat}, L_{Imcon}, L_{Quest}, L_{LogChall}, L_{Chall}, L_{Resolve}\}$$

An order locution for $R_{Resolution}$ would require extra rules to give it the effect of the preceding resolution demand, and is ommitted from this extension. The commitment effect of order locutions is:

$$CR_j\text{: After } \langle n, A, \ell \rangle, \ \ell \, \varepsilon \, L_R, \quad C_{n+1}(A) = C_{n-2}(A); \quad C_{n+1}(B) = C_{n-2}(B)$$

In other words, after a point of order of each participant reverts to what it was before the illegal event which occasioned the point of order occurred.

The set L^+ of locutions of the extended system is:

$$L^+ =_{df} L \cup L_R$$

We also need an amendment to the grammatical rule to permit order locutions to occur:

R^+_{Form}: (Like R_{Form}, but with 'L^+' in place of 'L')

It should be noted that the rules R_{Quest}, R_{Chall} and $R_{Resolution}$, which require some locutions to be replied to in a certain way mention a 'legal' (not a 'legal$^+$') 'dialogue of length n'. If a locution which requires a certain kind of reply is itself illegal, this condition is not fulfilled. Thus a point of order after an illegal question, challenge, or resolution demand is not itself illegal$^+$. (This corrects an error in [T].)

We next define the set R' of *amended rules*:

$$R' =_{df} R \cup \{R^+_{Form}\} - \{R_{Form}\}$$

There is a one-one function J from each order locution $\ell \, \varepsilon \, L_R$ to the corresponding amended rule $r \, \varepsilon \, R'$. With the help of this function we may state the extension rule:

R^J, No legal$^+$ dialogue of length n contains an event $\langle n-1, B, \ell \rangle$ such that $d_n \, \varepsilon \, r \, \varepsilon R'$ unless it also contains an event $\langle n, A, \ell' \rangle$ and $\ell' \, Jr$

The rules R^+ of the extended system are:

$$R^+ =_{df} R' \cup R_J$$

The extended system DC^+ is the triple $<P, L^+, R^+>$; its legal$^+$ dialogue are just the members of $K^+ = D - R^+$.

Field linguists who know that the language they are studying is used only for dialogues in K^+ should use the following features in drawing up a list V of valid argument schemata preferred by native speakers.

(i) If $<n, A, L_{Imcon}> \varepsilon d$, then it may be inferred that the locution ℓ of the last event $<n-1, B, \ell>$ is W's: $s \varepsilon \lambda$.

(ii) If $<n, A, L_{LogChall}> \varepsilon d$, then it may be inferred that the locution ℓ of the last event $<n-1, B, \ell>$ is Y's: $s \varepsilon \lambda$.

(iii) From the events $<n, A, R\text{'}C\text{'}<s, t>$, $<n+1, B, \ell> \varepsilon d$, where $\ell \neq L_{Resolve}$, it may be inferred that $C\text{'}<s, t> \varepsilon \lambda$.

(iv) From the events $<n, A, R\text{'}K\text{'}T>$, $<n+1, B, \ell> \varepsilon d$, where $\ell \neq L_{Resolve}$, the linguist should form conditionals by alphabetically ordering and left-conjoining all but one of the conjuncts of K'T as antecedent, and the denial of the remaining conjunct of K'T as consequent, and withdraw these in the hope of evoking L_{Imcon} from an informant.

(v) From the knowledge that a conditional $s \varepsilon \lambda$, it may be inferred that the expression formed by writing the conjuncts of the antecedent of s, in some order, followed by a slash, followed by the consequent of s, exhibits a schema of V.

The role of the points of order locutions L_{Imcon} and $L_{LogChall}$ in signalling conditionals which exemplify valid argument schemata preferred by the natives suggests that their utterance is one of the «bizarreness reactions» to which W.V. Quine refers, [P] 53.

But if the «rules» which we have mentioned are no more that the empirical generalisations of an observer, in what sense are they rules with prescriptive force? For that they do have prescriptive force can easily be shown by experiment: Find a naive subject who has been arguing for some statement 'p', and ask him why it is to be supposed that if both p and if p then q, then q. His reaction to this breach of $R_{LogChall}$ will be indistinguishable from moral outrage. A parallel problem arises for empirical linguistics as to the status of grammatical rules, and the solution which suffices there suffices here. The empirical generalisations which one has formulated as (conjectural) descriptions of the behavior of speakers of a given language are obeyed as prescriptions when one becomes a speaker and oneself composes sentences in that language. (7) The celebrated gap between 'is' and 'ought' is not thereby overcome, because the step (from 'These people behave thus' to 'I ought to behave thus') is not deductively valid.

The logically valid statements of the dialogue are a priori to the dialogue in the sense in which the rules of a game are a priori to the game: a person who breaks them is subject to a penalty prescribed by the rules, in the case of dialogues repeated resolution demands. As in the case of games, one can refuse to play: notoriously it is pointless to attempt to argue someone into listening to arguments. Despite this a priori character, the rules can be investigated and learnt empirically. The systems DC and DC^+ are, of course, not empirically adequate to cover ordinary conversation: and the only statements which can be established within them are instances of theorems of a preferred propositional logic, and hence trivialities. It would require greater syntactic resources for the participants to achieve more interesting results, such as establishing theorems of quantification theory or further, of pure mathematics. Their language would need to provide the syntactic resources to discuss its own syntax before they could force each other to concede general theses about all schemata of a given form and thus begin to do theoretical logic. There is no provision in DC^+ for debating points of order, much less for debating within the dialogue the propriety of resolution demands. Whether 'Resolve whether if not p then p' is a properly formed resolution demand is not a matter which debating intuitionist and classical logicians would allow to be decided for them by a linguist observing from a nearby tree, and any ruling which he gave would be treated as itself a contribution to their discussion. Like the rules of meeting procedure, the rules of dialogue may be changed by the participants themselves.

Despite its limitations, DC^+ does provide a formalisation for the rules of debate in which the simplest kinds of logical discourse, such as establishing valid statements and raising points of order against illegal utterances, can occur. It provides a way of distinguishing between (low level) discussion of the logic governing that discussion and discussion of other deductive systems, and criteria for recognising the «axioms» of the former. It thereby illuminates the connection between deductive logic and the study of argument.

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NOTES

(1) In all this I follow Hamblin's pioneering [D], but the system DC differs from any of his various ways. The system is neutral between propositional logics. Given a logic, the semantics for the statements of DC is that appropriate to that logic. The semantics for a class of locutions other than statements should consist of semantics for the statements from which the locutions are produced, together with the commitment rules and rules of dialogue governing that class of locutions. We must look not only at the (statement-)meaning, but also at the use. Hamblin provides inversion algorithms for defining semantic concepts in terms of the set of legal dialogues for some of his systems. This can also be done for DC, but the present paper is concerned rather with the proof-theoretic properties of the system.

(2) An axiom is a statement we have been given no reason to believe.

(3) Other deductive systems may include systems which are logical in the sense of governing dialogues. The special case arises only when the theorems discussed are theorems of the logic governing the discussion in which they are discussed.

(4) For example, for intuitionist logic the schema

Not not p / p

would not be in V, and neither 'If not not p, then p' nor 'Either p or not p' would be valid. For modal logics we need an additional syntactically defined relationship, viz. that if 'p' is an immediate consequence conditional then 'p' has as immediate consequence 'It is necessary that p'.

(5) Modus ponens holds for various connectives including not only those for the truth-functional (Philonian) and the various stricter kinds of conditional, but also for connectives forming different kinds of biconditionals and conjunctions.

(6) An extension similar to the one here described for DC is made for a simpler system in my [T].

(7) The case is perhaps claerer when we consider rule-governed activities which are not linguistic, such as games. After watching several matches of soccer (Association football), an observer may formulate the generalisation 'No player other than a goalkeeper may handle the ball except when an opponent has kicked it over the sideline', and indeed a further rule corresponding to our extension rule R^J, to the effect that 'Should a player break the rule about handling, the referee may award a free kick to his oppenents'. If the observer then takes part in a match, he may take his conjectural description of playing behavior as a prescription governing his own conduct. He need not do so. Even people who know quite well what the rules are break them, and do not thereby behave inconsistently or commit any error of logic.

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