AN EXPLICATION OF THE NOTION OF A CONSISTENT EVOLVING THEORY*

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Since the publication of certain of Marx and Engels' writings in 1932, especially Economic and Philosophical Manuscripts of 1844, there has been some disagreement about whether Marx and Engels later abandoned some of the views expressed in their early writings. Some hold that Marx and Engels take certain positions in Capital because of their theory of *menschliches Wesen* set forth in *Economic and Philosophical Manuscripts*, while others deny this, maintaining that Marx and Engels had rejected this theory by the time they were writing *Capital*. Others have alleged that there are inconsistencies between Marx and Engels' views, expressed at different times, about the course of future events. Does the theory expressed in Marx and Engels' later work entail the rejection of some of the theory of earlier works? Or does their total life work constitute a consistent but evolving theory?

In answering such questions, it would not be correct to investigate naively the collection of all the assertions made by Marx and Engels in various works as if these collected assertions were merely an unordered heap. A statement can be true at one time but not at another (e.g., «There is no doubt Russia is on the eve of revolution»), and, just because two works written at different times do not express the same opinions on a changing situation we do not ordinarily proclaim that we have discovered an inconsistency.

Instead, to answer these questions requires the prior explication in Carnap's sense of the notion of a consistent evolving theory. I will first present an overview of the proposed explication and then provide some of the more technical details.

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Understand a *theory* to be a set of sentences in the language of an intensional logic having logical constants for the expression of tense. Think of an *evolving theory* as being an ordered set of indexed theories. The temporal order relation–sometimes called the *accessibility relation*–is typically binary, and the various properties (e.g., transitivity) assigned the relation are determined by one's views on the nature of time. Each theory represents the translation of a work (e.g., *Capital*) into the formal intensional language, and the position of the indexed theory in the order is fixed by the time the work to which it corresponds was written. Now conceive of an *assignment*, which can be precisely described, assigning, for each indexed theory, a truth-value to each sentence in the theory. Take an evolving theory to be *satisfiable* if there is an assignment for it (or an evolving theory exactly like it, except that the «names» have been changed) on which, for each indexed theory, each of its sentences is true. For epistemological as well as metaphysical considerations, it is desirable whenever possible to devise a computational method which can be shown to declare a finite evolving theory (that is one having a finite ordered set) *inconsistent* just in case it is not satisfiable. Given such a computational method, take a finite evolving theory to be *consistent* if the computational method, when supplied the evolving theory as input, does not declare it to be inconsistent. The tree method which I will illustrate below provides such a computational method.

In explicating the concept of an evolving theory, we must specify a grammar, a semantics, and a deductive system. Instead of inventing new logics in which to base evolving theories, thus contributing to the overpopulation of logics, it is preferable to take plausible existing logics and base evolving theories in them. Certain of the presupposition-free intensional logics with equality seem excellent candidates for such a use, as these logics capture a number of the important semantic aspects of natural languages. For example, in these logics the domain of discourse can be empty at one time but not at another, and individual names need not always denote. This makes it possible for the following pairs of sentences (when translated into the language of free tense logic) to «mean» different things:

(1) There will always be something.
(1′) There is something which will always be.

(2) It will always be the case that everyone is a worker.
(2′) Everyone will always be a worker.

More technically, in free intensional logics – but not in quantified intensional logics having standard first-order logic as a quantificational base – neither the Barcan formula nor its converse hold. (1)

The free quantificational versions of some of the normal sentential intensional logics will be used as the bases for evolving theories. Included among these are the sentential modal logics K, M, B, S4, and S5 and their deontic and tense counterparts. The term «evolving theory» is most naturally applied in connection with tense logic, but, since there are analogous features in modal and deontic logics, evolving theories can also be seen as based in these logics. All of the logics considered here share the convenient feature that the closure of their accessibility relation can be readily computed.

Hugues Leblanc has played a major role in the development of free logics, and the evolving theories being investigated are based in his logics. (2) The style of the tree method for evolving theories is modeled on Leblanc and Wisdoms's tree method for standard first-order logic. (3)

For simplicity, I will describe only evolving theories based in tense logics, and, for purposes of illustration, I will use the minimal tense logic. (4) The other free normal intensional logics with equality which have an accessibility relation possessing the closure property mentioned earlier are similar.

(1) On this point, see [6] for modal logic and [8] and [1] for tense logic.

(2) See [6] and [8]. I generally follow the terminology of [6] in which historical references regarding the development of truth-value semantics can be found.

(3) See [7]. In [3], A tree method for Leblanc's free logic with equality [6] is presented with demonstrations of its completeness and soundness and is then used to prove a version of the theorems of Craig, Lyndon, and Robinson. I noticed the need for evolving theories while investigating the possibility of similarly establishing extended joint consistency theorems for the free intensional logics. That the concept of an evolving theory has natural applications which have been neglected struck me somewhat later. Hintikka's concept of a model system in modal logic [4] provided many insights for developing the related notion of a consistent evolving theory.

(4) The logic QK_τ* = described in [1]. The logic QK_τ in [8] has a standard quantificational base. Truth-value semantics for tense logics were developed by McArthur and Leblanc.

The logical constants of one of Leblanc's intensional logics with equality include '$\sim$', '$\supset$', '$\forall$', and '='. A tense logic also has '*G*' («It will always be the case that») and '*H*' («It was always the case that») as logical constants. (A modal logic has the necessity operator '$\Box$' as an additional logical constant.) The other connectives can then be defined in the usual manner. The grammar is also familiar (except possibly the clause for the universal quantifier). (5) There are countable sets of individual, sentence, and predicate parameters, so the language is countable.

Regarding the semantics, Leblanc uses truth-value semantics which can be shown to be equivalent to model theoretic or standard semantics as regards completeness and soundness. I quickly sketch a slightly modified version of Leblanc's semantics. Let α be an identity normal truth-value assignment, let P be a set of individual parameters, and let w be a member of some set. (6) An *assignment* is a triple of the sort $<\alpha, P, w>$. Let Σ be a non-empty set of assignments no two of which have the same index, and let R be a relation on Σ.
Let Pr be a property of relations (e.g., arbitrary, reflexive, transitive, reflexive-symmetric, etc.) such that $Pr_{\Sigma}(R)$ is the Pr-closure of R on Σ. A *Pr-truth-value triple* is a triple of the kind $<\Sigma,<\alpha, P,w>,R>$, where $R = Pr_{\Sigma}(R)$.

A *wff* A *is true on a Pr-truth-value triple* $<\Sigma,<\alpha, P, w>,R>$ if:

(1) in case A is atomic, $\alpha(A)=T$,
(2) in case $A=\sim B$, B is not true on $<\Sigma,<\alpha,P,w>,R>$,
(3) in case $A = B \supset C$, B is not true on $<\Sigma,<\alpha, P, w>,R>$ or C is,
(4) in case $A = \forall x B$, $B(I/x)$ is true on $<\Sigma,<\alpha,P,w>,R>$ for every $I \in P$,
(5) in case $A = GB$, B is true on $<\Sigma,<\alpha',P',w'>,R>$ for every $<\alpha',P',w'> \in \Sigma$ s.t. $<\alpha,P,w> R <\alpha',P',w'>$,

(5) If A(I/x) is a wff, then so is $\forall x A$.

(6) See [6], [8], and [1] for details of Leblanc's semantics. The identity-normal restriction on α prevents, e.g., a referring and a non-referring term from being equal. Intuitively, P consists of the referring terms and w is an index. In [6], Leblanc requires that w be a real number but does not require w to be an index in the usual sense. Changes along the lines indicated here are suggested in [1].

(6) in case $A = HB$, B is true on $<\Sigma,<\alpha',P',w'>,R>$ for every $<\alpha',P',w'> \in \Sigma$ s.t. $<\alpha',P',w'>R<\alpha,P,w>$.

Understand a *theory* to be a set of *wffs*. Leblanc takes a theory S to be *Pr-tv-verifiable* if there is a Pr-truth-value triple on which every *wff* belonging to S is true. Understand a theory S to be *isomorphic to a theory* S′ if there is a one-one function f from the individual parameters of S to those of S′ such that, under the naturally induced replacement, f(S)=S′. Finally, a theory S is said to be *Pr-satisfiable* if some theory isomorphic to S is Pr-tv-verifiable.

Evolving theories are based in Leblanc's logics as follows. Let σ be a non-empty set, let s be a function from σ to the power set of the language, and, as before, let Pr be a property of the accessibility relation and let $Pr_\sigma(r)$ be the Pr-closure of r on σ. An *evolving Pr-theory* is a triple of the sort $<\sigma,s,r>$, where $r = Pr_\sigma(r)$. (7)

Now for the (truth-value) semantics of evolving theories. Say that an evolving Pr-theory $<\sigma,s,r>$ is *Pr-tv-verifiable* if there is a Pr-truth-value triple $<\Sigma,<\alpha_1,P_1,\omega_1>,R>$ such that

(1) for every ω belonging to σ there is an assignment with index ω belonging to Σ,
(2) for every ω,ω' belonging to σ, $\omega r \omega'$ iff $<\alpha,P,\omega>R<\alpha',P',\omega'>$, and
(3) for every ω belonging to σ, every wff A belonging to the theory $s(\omega)$ is true on $<\Sigma,<\alpha,P,\omega>,R>$.

The concept of satisfaction for an evolving theory is developed in a broader manner than the concept of satisfaction for a logic. (8)

(7) To simplify the deductive system presented here, we treat only properties of relations having closures satisfying the following condition:

$$Pr_\sigma(r) = Pr_\sigma + (r \cup \{<w,w^+>\} \cap \sigma^2$$
$$= Pr_\sigma + (r \cup \{<w^+,w>\} \cap \sigma^2$$

for any non-empty set σ, any binary relation r on σ, any $w \in \sigma$, and any $w^+ \notin \sigma$ such that $\sigma^+ = \sigma \cup \{w^+\}$. On a future occasion, we shall treat the more general case and fill in technical details.

(8) Whereas Leblanc requires «isomorphism» (his function f is one-one) in his definition of satisfaction in a logic, the weaker condition of «homomorphism» (neither functions f nor h need be one-one) is used for evolving theories. A «homomorphism»

Say that a function f from the set of individual parameters in the theory S to the set of individual parameters in the theory S′ *embeds* S *in* S′ if $f(S) \subseteq S'$. Moreover, understand an evolving Pr-theory $\langle \sigma_2, s_2, r_2 \rangle$ if there is a function f from the set of individual parameters in $\bigcup_{\omega \in \sigma_1} s_1(\omega)$ to those in $\bigcup_{\omega \in \sigma_2} s_2(\omega)$ and a function h from σ_1 tot σ_2 such that

(1) for every ω belonging to σ_1, f embeds the theory $s_1(\omega)$ in the theory $s_2(h(\omega))$, and
(2) for every ω, ω' belonging to σ_1, $\omega\, r_1\, \omega'$ iff $h(\omega)\, r_2\, h(\omega')$.

Finally, take an evolving Pr-theory $\langle \sigma, s, r \rangle$ to be *Pr-satisfiable* if some evolving Pr-theory in which $\langle \sigma, s, r \rangle$ is homomorphically embedded is Pr-tv-verifiable.

Various other accounts of Pr-satisfiability can be obtained by modifying the above definitions. An extremely weak account can be had by dropping clause (2) in the definition of Pr-tv-verifiability. With the relational structure r on σ lost, the evolving Pr-theory $\langle \sigma, s, r \rangle$ is Pr-tv-verifiable (Pr-satisfiable) iff each theory $s(\omega)$ ($\omega \varepsilon \sigma$) taken separately is Pr-tv-verifiable (Pr-satisfiable) in Leblanc's sense. That is, understanding W to be the singleton $\{\omega\}$, $\langle \sigma, s, r \rangle$ is Pr-tv-verifiable iff $\langle \sigma, s, Pr_\sigma(\varphi) \rangle$ is iff $\langle W, s \upharpoonright W, Pr_W(\varphi) \rangle$ is for every $\omega \varepsilon \sigma$. This weak account might be plausible when an evolving theory consists of theories drawn from distinct cultural traditions which developed independently. In this case, though, it is more perspicuous to speak of a number of theories, one for each culture, rather than a single evolving theory in which the individual theories are not interconnected.

A strong account can be obtained by taking an evolving Pr-theory to be Pr-satisfiable if the theory $\bigcup_{\omega \varepsilon \sigma} s(\omega)$ is Pr-satisfiable in the sense of Leblanc. This account might be plausible when either the evolving theory deals with a static situation which was perfectly understood or the evolving theory was created complete, all at once,

account of satisfaction in a logic can be given which is equivalent to the «isomorphism» account as regards which theories are satisfiable. The «homomorphism» account of truth-value semantics in many respects more closely resembles standard semantics than does the «isomorphism» version. Other semantic notions such as validity are defined in terms of satisfiability as in Hintikka [4].

in a dogmatic mind; the evolving theory was merely articulated at different times. However, in such cases it is not clear that the natural expressive power of standard first-order logic needs supplementing with tense operators; furthermore, it is strange to speak of such theories as «evolving».

Another strong account takes an evolving Pr-theory to be Pr-satisfiable if it is a model system in the sense of Hintikka [4]. Yet the work of a human author or group of authors consists of a finite number of chapters of finite length. Since there are works which are intuitively consistent and which, viewed as evolving theories, can be extended to model systems only by making the number of chapters or some of the chapter lengths infinite, the model system account is too strong.

Acounts of intermediate strength, the more plausible of which are generally at least as strong as the one I adopt, can be given. For example, the definitions of both Pr-verifiability and Pr-satisfiability can be modified so that, so to speak, in determining Pr-satisfiability points in time (worlds) cannot be added «in between» the time points in the evolving theory. However, a thinker might have conceived a chapter in his life work but not written it down or a chapter might have been lost. In neither case would we necessarily want to deny the consistency of his work.

The account which I adopt yields what might be characterized as a «charitable reading» of a thinker's life work. If a life work can possibly be held consistent without overly distorting the temporal order, the evolving theory which represents it is deemed satisfiable. This account has the additional advantage of yielding, when coupled with the tree method to be sketched next, various metalogical results (to be reported on a future occasion).

Intuitively, the tree method for evolving theories can be compared with jumping into the middle of a Kripke construction [5], except that neither the initial structures nor the structures generated need be trees. Given a consistent evolving theory as input, the tree method is designed to extend it into an evolving theory which is a model system in the sense of Hintikka [4].

A complete description of the tree method (deductive system) for evolving theories is more lengthy and will be only briefly outlined here. The method calls for a tree corresponding to each «possible world» – so there is a forest of trees – and a scheme for tree and branch numbering.[9] To illustrate the tree method, two derivations will be given.

Following Leblanc and Meyer, call a theory S *infinitely extendible* if there are countably many individual parameters foreign to S. Take an evolving Pr-theory $<\sigma, s, r>$ to be *amenable* if there is some positive integer *wmax* such that

(1) $\sigma = \{w : w \in N \;\&\; 1 \leq w \leq wmax\}$, and

(2) $\bigcup_{\omega \varepsilon \sigma} s(\omega)$ is infinitely extendible.

The tree method can be applied to any amenable evolving Pr-theory.

There is an intuitive justification for restricting the application of the tree method to amenable evolving theories. Evolving theories can be viewed as being quasi-linguistic entities by noticing the analogy with the organization of a work into chapters. An evolving theory $<\sigma s, R>$ stands to a theory s (w), $w \; \varepsilon \; \sigma$, as a work stands to one of its chapters, and the index w (along with the relation r) serves much the same purpose as the number of a chapter (along with the natural order on the integers). Each theory chronicles a period in the historical development of the evolving theory. Just as the number of chapters in a work is finite, so «should be» the index set σ of the evolving theory. The positive integers are used to «number» both the chapters of a work and the theories of an amenable evolving theory. Of course, there are evolving theories which have, for example, uncountable index sets and cannot be homomorphically embedded in an amenable evolving theory, but these seem to be of little linguistic interest.

The tree and branch numbering is somewhat intricate. In applying the tree method to the amenable evolving Pr-theory $<\sigma, s, r>$, for each $w \in \sigma$, there is a tree w having an initial branch numbered $b = 1$. (If the tree method does not find $<\sigma, s, R>$ to be inconsistent and halt, each *wff* A $\in$ s (w) will eventually be placed on the open branches of tree w.) Initially, *the forest* is σ. The branches on each tree are numbered, and the collection of all the branches numbered b gathered from every tree in the forest are known as the *b-branches of the trees*

(9) Fitting [2] and Smullyan [9] have also developed for a family of modal logics tree methods which are applicable to a set of sentences. There is only one tree for all «possible worlds» taken together in their systems, and they do not treat the *free* modal logics.

in the forest. Roughly, these correspond to a Kripke alternate set.(10) The number of trees in the forest at the beginning of the i-th iteration $(i \geq 1)$ of the tree method is defined recursively as follows:

$$pw(i) = wmax \text{ if } i = 1$$
$$\qquad pw(i-1) + t(i-1) \quad \text{otherwise}$$

where $t\,(i-1)$ is the number of trees introduced in iteration $i-1$. Understand $\sigma(i)$ to be $\{w:\ 1 \leq w \leq pw(i) + t(i)\}$.

The rules of the tree method are schematized in the style of Leblanc and Wisdom supplemented with notation for tree and branch numbering.(11)

(1)

tree w
branch b
$\sim\sim B$
.
B

(2)

tree w
branch b
$B \supset C$

branch $2 \cdot b$ — branch $2 \cdot b + 1$
$\sim B$ — C

tree w'
branch b

branch $2 \cdot b$ — branch $2 \cdot b + 1$
surrogate branches

(3)

tree w
branch b
$\sim (B \supset C)$
.
B
$\sim C$

(4)

tree w
branch b
$\sim \forall x A$
.
$\exists x (X = I)$
$\sim A(I/X)$

(10) See [5] for alternate sets.

(11) See [7] for an explanation of rules (1) - (3) and [3] for rules (4) - (6) and conditions for closing branches and trees. Rule (2) is supplemented here with surrogate branching because there can be more than one tree in the forest. The need for this can be seen by comparing Kripke's device of splitting alternate sets in [5].

(5)

tree w
branch b
$\forall x A$
⋮
$\exists y (y = I)$
.
$A(I/X)$

(6)

tree w
branch b
B
⋮
$T_1 = T_2$
.
$B(T_2/T_1)$

(6′)

tree w
branch b
$\exists x (x = T_1)$

$T_1 = T_2$ or $T_2 = T_1$
.
$\exists x (X = T_2)$

(7)

tree w
branch b
$\sim GB$
└ – – – – → .

tree w'
branch b
$\sim B$

(8)

tree w | tree w'
branch b | branch b
GB
└ – – – – → .
B

(9)

tree w
branch b
$\sim HB$
. ← – – – – ┘

tree w'
branch b
$\sim B$

(10)

tree w' | tree w
branch b | branch b
HB
. ← – – – – ┘
B

In an application of rule (2), when branch b of tree w splits into branches $2 \cdot b$ and $2 \cdot b + 1$, so do all the b-branches of the trees in the forest. The branches $2 \cdot b$ and $2 \cdot b + 1$ into which branch b splits on each other tree w' in the forest having a b-branch are known as *surrogate branches.* In (4), I must be foreign to every $s(w')$ $(1 \leq w' \leq wmax)$ and to all the b-branches of the trees in the forest. In rule (6), B must be an atomic wff or the negation of one. Rule (6′) is needed due to interactions between identity, quantification, and tense. In rule (7) (rule (9)), the number of trees introduced in iteration i, t(i), is set to $t(i)+1$, a new tree $w' = pw(i)+t(i)$ is introduced with b as its one branch, and the relation r is set to $Pr_{\sigma(i)}(r \cup \{<w, w'>\})$ ($Pr_{\sigma(i)}(r \cup \{<w', w>\})$). In rule (8) (rule (10)), B is inserted on branch b of each tree w' having a b-branch such that wrw' ($w'rw$). Redundant wffs are omitted in applying any rule. In particular, as specified in [3], rule (4) is not applied to wffs of the form $\exists x (x = T)$.

Branch b of tree w is *closed* if
(1) an atomic *wff* an its negation both appear on it, or
(2) a *wff* of the form $\sim (I = I)$ appears on it, or
(3) branch b of some tree w' in the forest is closed.
In this case, we also speak of the b-branches of the trees in the forest as being *closed*. A tree is *closed* if each of its branches is closed, and the forest is *closed* if for all b the b-branches of the trees in the forest are closed (equivalently, if all the trees in the forest are closed). Finally, understand an amenable evolving Pr-theory $<\sigma, s, r>$ to be *consistent* if the tree method does not declare it to be inconsistent and halt.

The following two examples illustrate the tree method for evolving theories based in the minimal free tense logic with equality for which there is no restriction on the accessibility relation.

Example 1

Consider the following assertions made at times 1 and 2.

Time 1: $\forall x\, G\, W(x)$
(Simplifying, take this to represent 'Everyone will always be a worker.')

Time 2: $\sim \forall x\, W(x)$
('Not everyone is a worker.')

Here $wmax = 2, \sigma = \{1,2\}$, $r = \{<1,2>\}$, $s(1) = \{\forall x\, G\, W(x)\}$, and $s(2) = \{\sim \forall x\, W(x)\}$.
The line numbers indicate the order in which *wffs* are inserted.

tree 1

line	branch 1	*from*	*rule*
1.	$\forall x G\, W(x)$	s (1)	

tree 2

2.	$\sim \forall x\, W(x)$	s (2)	
3.	$\exists x\, (x = I)$	2	(4)
4.	$\sim W(I)$	2	(4)

No further lines can be generated, the 1-branches of the trees in the forest are open, so this is a consistent evolving theory. The failure of tense versions of the Barcan formula can be seen from this example.

Example 2

Let $wmax = 1$, $\sigma = \{1\}$, $r = \varphi$, and

$$s(1) = \{\sim G \sim H \sim (p \supset q), p \supset q\}$$

tree 1

line	branch 1		*from*	*rule*
3.	$p \supset q$		s (1)	
1.	$\sim G \sim H \sim (p \supset q)$		s (1)	
	branch 2	branch 3		
4.	$\sim p$	q	3	(2)
6.	$\sim (p \supset q)$	$\sim (p \supset q)$	5	(10)
7.	p	p	6	(3)
8.		$\sim q$	6	(3)

Notice that line 3 occurs above line 1 on tree 1 as *wffs* from s (1) are added at the top of the tree. The surrogate branches 2 and 3 on tree 2 are introduced along with line 4 on tree 1 by an application of rule (2) to line 3 on tree 1. With something of a domino effect, the 2-branches close after line 7 and the 3-branches after line 8. Since for all b (b =

tree 2 $r = \{<1,2>\}$

2. $\sim\sim H \sim (p \supset q)$ 1 (7)

surrogate branches

5. $H \sim (p \supset q)$ $H \sim (p \supset q)$ 2 (1)

X X

4,7 4,8

2,3) the *b*-branches of the trees in the forest are closed, the forest is closed. Hence, this evolving theory is inconsistent. The provability of the tense logic characteristic axiom $A \supset G \sim H \sim A$ can be seen from this example.

The tree method for evolving theories perhaps provides a new view of what constitutes a proof(12). It applies to an ordered set of theories and not just a single theory as other deductive systems do. Execution of the tree method for evolving theories can be viewed as carrying out proofs in an interleaved manner in a finite number of different trees (worlds, for the metaphysically minded) which interact. These interacting proofs could also be performed in parallel.

In closing, a word should be said about the appropriateness of using the work of Marx and Engels as an example of an evolving theory. The first point to be made for the suitability of this illustration of an evolving theory is that there is a real dispute about whether the views of Marx and Engels changed with the passage of time. In this respect, though, there are also questions about the consistency of the theories of other great thinkers. Are not Plato's views as expressed in the early dialogues at odds with those presented after the Third Man arose in the *Parmenides* ? Is the Platonism of the younger Bertrand Russell compatible with the logical empiricism of the older man ?

However, there is a second point that can be made for the suitability of the example of Marx and Engels. Their work contains assertions

(12) Though it is clearly foreshadowed in Hintikka [4] and Kripke [5].

about the future and past. This tense character is lacking in the theories of Plato and Russell. Much of the writing in mathematics and philosophy has a «timeless» property. Without a tense or other intensional character, the question of the consistency of an ordered set of indexed theories might degenerate into the question of the consistency of the union of the theories in the ordered set. The world view of the early Christians also might be viewed as an evolving theory since it envisions aspects of the future.

One should not glibly assert – at least without first placing firmly tongue in cheek – that textual disputes such as the one over the proper interpretation of Marx and Engels can be resolved by the historian and the computer on which the tree method has been programmed. The least of the problems is that, for certain consistent finite evolving theories, the tree method does not declare the evolving theory to be consistent and halt. Also, the adequacy of consistency, taken by itself, as a criterion for resolving textual disputes is highly questionable. I am not advocating a coherence theory of truth. Moreover, the work of Marx and Engels did not develop in isolation from a class or social movement. The evolving theory of Marxism consists of more than just the writings of Marx and Engels. Similar remarks could be made concerning the social context in which other scientific theories and world views develop.

The fundamental question, though, for determining the adequacy of our explication of the notion of a consistent evolving theory is a *linguistic* question. From the viewpoint of natural languages we ask: Is one of these free intensional logics – or an extension of one of them in a suitable manner (for example, a conservative extension) – the underlying logic of natural languages? Although the answer to this question might turn out to be in the negative, these logics capture a number of our intuitions about time, necessity, and obligation in natural language discourses. To the extent that one of these logics does correspond with the underlying logic of natural languages, the tree method for evolving theories reveals dim shadows cast by the computational mechanism of the human mind.

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