QUANTUM LOGIC AND MODALITY

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1. *Introduction*

In some sense quantum mechanics has strange logical features, whether or not it actually presupposes a non-standard logic, and this has interesting and perhaps quite broad implications, but what I wish to argue for in this paper is that it is not the case that quantum logic dispenses with the rules of distribution of standard truth-functional logic in any interesting sense, as has often been maintained. My conclusion of course is not unique but I hope to make clear a fairly simple but important reason for its truth alluded to by Hooker and van Fraassen. This also throws some light on the question (first posed by Birkhoff and von Neumann in [1], 1936) of what the term «logic» in «quantum logic» means. (Partly because of von Weizäcker's remarks (1) I do not discuss here those quantum logics that are non-standard in the sense of being many-valued.)

2. *Jauch's Views on the Nature of Quantum Logic*

We should first of all look at Jauch's view that quantum logic is not a logic at all, since were it completely correct (I agree that it is correct in part) then the issue considered in the present paper would hardly be

(1) See VON WEIZÄCKER [12] p. 332, footnote 1. (VON WEIZÄCKER's logic is not a truth-functional multi-valued one as some authors have claimed). For reservations about Reichenbach's multi-valued logic see HAACK [10] and SUPPES [17]. With regard to probability: if one *retains* distribution, allows that a probability can be assigned to (for example) p. (x v ⅂x) and that, classically, one must be able to assign a probability to every element of the probability algebra (as in SUPPES [17] Premis 2 p. 342), then as SUPPES argues ([17]) the probability theory of quantum mechanics will be non-classical since p . x cannot be assigned an exact probability. But this problem seems to be independent of the one considered in this paper. Fine argues ([5]; [6]) that quantum probability can be classical by denying Suppes' Premiss 2 referred to above. One might also argue that it makes no sense to assign a probability to p . (x v ⅂x) as it seems Bohr would claim.

worth discussing. In the following 'p' will be used for descriptions of the momentum of a sub-atomic «particle» and 'x' for statements about its position (any other pairs of statements involving reference to complementary or experimentally incompatible quantum properties would, of course, do just as well – all examples other than those just involving non-classical «oddities» appear to be of this kind. See [6], [9] and [15].)

Jauch makes two philosophical claims concerning the nature of quantum «logic». In his book ([13], pp. 77, 78) he argues that the relations between the elementary propositions of a quantum logic which result in its violation of one of the distribution laws are empirical not purely logical relations (i.e., result from the particular meanings which the propositions have); hence quantum logic is not a logic and little interest attaches to the fact that it is non-Boolean (non-distributive). Although it is certainly true that in some sense quantum logic is «based on» a contingent fact (that conjugate variables are not simultaneously decidable), I think Jauch's account of the relevance of this to the question of the nature of quantum logic cannot be correct. Normally, of course, the atomic propositions in a logical system are assumed to be independent of one another, but the reason that Jauch's position does not make clear what is going on in quantum logic is that it is also true that propositions which are «dependent» on one another, i.e., have richer (probably non-truth-functional) physical or semantical relations to one another still obey ordinary logical laws. All we have is the elimination of certain lines in the truth-table; the meaning relations between atomic propositions affect their logical relations *only* in this respect, that it may render an otherwise invalid inference valid. Although, for example, $p.q, \vdash r$ obviously represents an invalid form of argument, a particular case may be valid because of the relations between the atomic propositions, e.g., in the above example if p contradicts q (i.e. we have to eliminate the line in the truth-table expressing the possibility that both p and q are true). But we do not have this situation, that we can replace the atomic propositions in a valid schema by empirically related propositions so as to produce a *special logic*. But then, of course, Jauch is incorrect in claiming that the formal structure of quantum mechanics does not somehow involve a deviant logic.

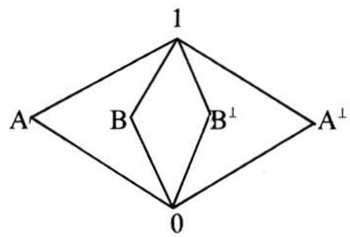
In another publication ([14]) Jauch (and Piron) use a different sort of argument to show that quantum logic is not really logic. They point out that no *conditional* statement is represented by any *element* of the non-Boolean lattice representing quantum logic and, they claim, any formal structure to count as a logic must «contain» inferences which either generate theorems, or do so in effect by saying that some propositions entail others. They conclude that, as a result, quantum logic is not really a logic. Certainly one of the things that is required (in the object language) for a structure to count as a logic are strings of statements the last of which counts as a theorem([2]). But we do have *this* situation with respect to the non-Boolean lattice since some of the elements imply others. Hence although the lattice cannot be said to «contain» inferences, it provides a recipe for constructing them in an object-language that the lattice can be said to partially describe. I mean simply that if the lattice, in effect, says that $p_1 p_2 \dots p_n$ implies p_{n+1} then in an object-language $\ulcorner (p_1 p_2 \dots p_n) \vdash p_{n+1} \urcorner$ can be asserted. But the main point against Jauch here is the following. Whether or not the quantum non-Boolean lattice can be said to represent a complete logic the fact remains that if a proposition of the form $\alpha \leftrightarrow \beta$ would normally hold by a distribution law but fails to in a theory T then surely T employs a non-standard logic even if α and β, but not $\alpha \longleftrightarrow \beta$, are elements of a lattice depicting some of the formal relations of statements in T. Hence at most Jauch's argument might show that the non-Boolean lattice does not depict the complete logic of quanta; it does not seem to provide a means of deciding whether this logic is standard or not.

3. *The Fundamental Argument*

The fundamental argument for the view that the «propositions» of a quantum logic do not satisfy the standard distribution laws or equivalently that the elements or sub-sets corresponding to the observables in

([2]) By a *logic* here I understand, as do most people, a formal language together with a deductive apparatus which enables us to recognize certain strings of symbols as provable (theorems) within the system. (All theorems can be identified in a decidable system. There is no requirement that any atomic proposition of the system be a conditional statement.) See, for example, G. HUNTER, *Metalogic*, London: Macmillan, 1971, Section 3 and 22.

quantum theory form a non-Boolean lattice, is, in a way, very simple. Let A and B be *incompatible* observables and $A^{\perp}$ and $B^{\perp}$ their respective «negations» (orthocomplements) and suppose these are the only four possibilities. I have used A and B (rather than p_1, x_1 etc.) so as to leave undecided for the moment the question of what *type* of statements are involved. These can form a non Boolean lattice like this:



$A \cap (B \cup B^{\perp})$ is equivalent to A since $B \cup B^{\perp}$ is a lowest upper bound of the lattice (corresponding to the universal set) whereas each of $A \cap B$ and $A \cap B^{\perp}$ are the greatest lower bound and thus have the value 0. (The «mathematical» argument here is that quantum «propositions» correspond to the subspaces of a Hilbert space which form such a lattice. See Bub [3]. Chapter IV.)

Perhaps more intuitively (following Putnam) where p represents a statement about momentum and x about position we have p. (x v ⅂x). (I have used '⅂' since the negation is non-standard, on which see below), but not (p . x) V (p. ⅂x) and thus distribution fails.

4. *A Modal Interpretation*

Heelan and Hooker ([11], p. 26; [12], p. 272, footnote 122; see especially Heelan's remarks on Jauch) seem to be the only authors who have remarked on the difficulty of determining precisely what *sort of* propositions various quantum logicians intend as the atomic elements of the non-Boolean lattice (of course the meaning of complex items would be clear if that of the basic elements were). This question seems to me to be of crucial importance: once it is seen what these elements are and how they are related (by «physical» as well as logical operations) then the question of whether the distribution law holds can be resolved. There seem to be three main possibilities. (i) The elements or atomic proposition are of the simple sort mentioned above (i.e., like p^1 or $\neg x^1$); Heelan's «simple empirical propositions» such as «The observables X, Y ... fall within the range $\Delta xy \in B(Rx \otimes Ry)$» can also be treated in this way. (ii) They are «semi-theoretical» or more complex empirical propositions, e.g., an intuitive interpretation of Birkhoff and von Newmann's elements φ and ψ which preserves the central logical idea is that they are of the form $\alpha \vee \beta$ where α and β are *incompatible* propositions, i.e., $x_1 \vee x_2$ or $x_1 \vee p_1$. (3) (There are other possibilities as well. I am not using φ and ψ here as Fine does. It is impossible to use a simple notation consistent with the usage of all authors or even with that of a single author.) (iii) The elements are, e.g., of the form $E_1 \subset C(p_1 \cup p_2)$: an experiment of sort E_1 will give a value for the momentum.

We have already considered sort (i) propositions to some extent. With regard to statements of sort (ii) consider the proposition $(A_1)\ \Phi . (\psi \vee \neg\psi)$; this could correspond (for example) to the statement $(x_1 \vee p_1) . ((x_2 \vee p_2) \vee \neg(x_2 \vee p_2))$. By the distribution law this in turn would be equivalent to (A_2): $((x_1 \vee p_1) . (x_2 \vee p_2)) \vee ((x_1 \vee p_1) . \neg(x_2 \vee p_2))$. Remembering that the negation here is non-standard (one species of what van Fraassen (4) calls «choice-negation» and corresponding to the idea of an orthocomplement), we can see that $A_1 \neq A_2$, i.e., that distribution *appears* to fail, for the following reason. Suppose $\varphi = 1$ (φ is true) because $x_1 = 1$; $\psi \vee \neg\psi = 1$ even where ψ and $\neg\psi$ are both 0 because $\psi \vee \neg\psi$ is a lowest upper bound (See Putnam [15] and Fine [6]); $x_2 \vee p_2$ is not a lowest upper bound – x_2 and p_2 are NOT elements of the lattice on this interpretation ($x_2 \vee p_2$ is the simplest example of an element) – and so can take the value 0 where x_2 and p_2 are each 0. Lastly (oddly enough) $\neg(x_2 \vee p_2)$ is also 0 since this represents the orthocomplement of ψ which also fails to hold

(3) HOOKER ([12], footnote 125, pp. 273-274) unlike many writers in this area discusses explicitly the fact that the terms «compatible» and «incompatible» propositions are used in two importantly different senses by commentators on quantum logic; (i) in the sense in which p (an object has a momentum) and x (an object has a position) are incompatible and (ii) the sense in which p_n (an object has a specific momentum n) and $Pn^{\perp}$ (an object has a momentum other than n) are incompatible. Here we are normally talking about incompatibility in the first respect.

where $\varphi = 1$ since φ and ψ are incompatible. Given this and the fact that where $x_1 = 1$ all the other individual propositions (not elements) are false, it then follows that $A_1 \neq A_2$ (it is easy to confirm that the Φs and ψs will serve generally as elements of, e.g. Heelan's example of a non-Boolean lattice. See Appendix to [11]). Type (iii) propositions will be discussed at the end of this section.

Given all this my argument that a quantum logic is not really non-distributive (though it is *related to* non-Boolean lattices in the way set out above) can be stated, by simply answering the question, «Why are the elements related in this apparent non-distributive way?» The answer, of course, is that it is impossible to simultaneously assign them values on experimental grounds («operationally» distribution fails to hold as Putnam claims). It has often been noted that quantum logic is not truth-functional (e.g. by Quine; see [10] p. 156 and Gardiner [9]). It is also true that *if* it is truth-functional it *cannot* be bivalent. (5) But I do not think the full significance of these facts has been clearly recognized (except by Hooker and van Fraassen) for the sense in which it is not truth-functional, is, in effect, that the relevant statements *tacitly employ an operator* analogous to a *modal* one; *once this operator is made explicit it is seen that distribution does not fail.* To take the simplest example consider the reason for saying that

$$(B_1)\ p\,.\,(x \vee \neg x)$$

is not equivalent to $(B_2)\ (p\,.\,x) \vee (p\,.\,\neg x)$ – the reason of course is that if p is experimentally determined neither x nor $\neg x$ is. Let $\Box$ stand for experimentally determined (or measured or assigned a value on the basis of a yes-no experiment etc.) then B_1 is *really* the statement

$$(B_1{*})\ \Box p\,.\,\Box(x \vee \neg x)$$

'$x \vee \neg x$' might be assigned $\Box$ on the «trivial» grounds that it is always determined to be true (this, of course, is arguable – see below).

Then $(B_2{*})$ is

$$\Box(p\,.\,x) \vee \Box(p\,.\,\neg x)$$

(4) See VAN FRAASSEN, [20], esp. pp. 581-582.

(5) See VAN FRAASSEN [19] p. 177 and STRAUSS, [16], pp. 41, 42.

And, of course, $(B_1*) \neq (B_2*)$, but this does *not* involve any failure of the distribution law: *the failure of equivalence is just due to the presence of the modal operator* [6]: it is no more astonishing than the fact that from the ordinary modal statement $Lp \,.\, L(q v \sim q)$ we cannot necessarily infer $L(p \,.\, q)\ vL(p \,.\, \sim q)$ for $L(p \,.\, q)$ implies Lp and Lq (normally), but obviously $L(qv \sim q)$ may by true where neither q nor ~ q is necessary. And, of course, this is just the situation in quantum logic for we have that $\Box p \rightarrow \sim \Box x \wedge \sim \Box \sim x$ and $\Box x \rightarrow \sim \Box p \wedge \sim \Box \sim p$. The same point obviously holds with elements of the second sort mentioned above. (Heelan argues that his simple empirical propositions form *Boolean* sublattices within a non-Boolean lattice whose statements are formulated in *languages* which are to some extent exclusive. To relate this to the above I think his distinct languages would correspond to distinct experimental situations, i.e., ones in which $\Box (p \cup p^{\perp})$ held as opposed to $\Box (x \cup x^{\perp})$; his point about simple propositions could then be put: Distribution holds where no modal operators are even tacit, e.g., amongst propositions which are compatible in *one* sense).

The case is even clearer where the elements are conditional propositions involving reference to possible experiments for then $p \,.\, (x \vee \neg x)$ becomes either

$$(C_1)\ (E_1 \rightarrow p) \,.\, (E_2 \rightarrow (x \vee \neg x))$$

or $(C_2)\ (E_1 \rightarrow p) \,.\, ((E_2 \rightarrow x) \vee (E_2 \rightarrow \neg x))$

and we also have as an axiom, $\sim M(E_1 \wedge E_2)$. The M here is not as strong as logical possibility but means, rather, «possible relative to the theory in question.» There is no way of expanding C_1 using the distribution law, and C_2 *validly* becomes,

$$(C_3)\ ((E_1 \rightarrow p) \,.\, (E_2 \rightarrow x)) \vee ((E_1 \rightarrow p) \,.\, (E_2 \rightarrow \neg x))$$

(This involves tacit use of temporal operators since E_1 and E_2 can't be

(6) If, as I think is the case, (B_1*) should be, rather, $\Box\, p. (x \vee \neg x)$ then the case against a rejection of distribution is even stronger. Following a suggestion of Jauch's van Fraassen develops a highly technical modal interpretation of quantum logic, but his idea is essentially different from the (non-technical) one of this paper for he treats *each* elementary proposition as modally qualified and in a way different from that suggested here.

simultaneously performed nor non-simultaneously performed on one and the same system. This distinction is discussed in more detail on p. 108. The reason that E_1 and E_2 cannot be non-simultaneously performed on one and the same system is that, *for example* an E_1 experiment designed to determine the momentum of a particle appears to change the system in such a way as to make it impossible at that time or at a later time to determine the position the particle had at the time the momentum was measured.)

Bohr holds (Hooker [12] pp. 173-147) that $p.(x \vee \neg x)$ is not wellformed or meaningless (it would have to be expanded as in C_1 above), but that quantum theory employs standard logic. This accords with the «modal» interpretation, that $p.(x \vee \neg x)$ needs to be spelled out as above, having just as it stands, no clear interpretation. (7)

5. *The Metaphysical Issue*

Consider the following analogy (after [4] p. 53). Suppose we have a box containing bits of wire and four «measuring instruments», two magnetic circles of radius r_1 and r_2 and two squares of side length l_1 and l_2. For some reason we only have access to the box via the measuring instruments; suppose further that the wires are such that if we make a measurement (E_1) using r_1 and r_2 we will get the result r_1 or the result r_2 and similarly with the squares. Thus if we have $r_1 \vee r_2$ and $l_1 \vee l_2$ (which is arguable), then there will be a «failure of the distribution law» which obviously needs to be spelled out as in the previous section. My purpose here is only to provide a simple intuitive analogy

(7) A modal interpretation seems preferable to that in terms of non-standard negation, or rather, if the latter is to make sense at all it seems that it must tacitly employ a modal operator. To say that both $\varphi = 0$ and $\neg\varphi = 0$ makes sense where $\neg$ represents choice negation (e.g., where φ says that *a* lies on a diameter orthogonal to the first) but then to *combine* this with a definition of 'v' that makes $\varphi \vee \neg\varphi$ a «logical truth» (see, e.g., FINE [5] pp. 17, 18) seems anomalous to say the least. At any rate an interpretation which preserves excluded middle (and the apparent needs of quantum mechanics) would say that the geometrical account represents this «operational» meaning: to say φ and $\varphi^{\perp}$ can both be false really means that its possible that $\sim\Box\varphi$ and $\sim\Box\varphi^{\perp}$(but $\varphi \vee \sim\varphi = 1$). Otherwise this non-standard negation just looks like an attempt to retain two truth-values and something like truth-functionality where this is in fact impossible.

that shows the consistency of Eisenbud's view (and that of others) that in microphysics, the fundamental classical assumption of objective physical properties of systems fails ([4] p. 59), e.g., «although an electron with precise momentum cannot be said to have a position, nevertheless a suitable measurement will find the electron at some definite position» ([4] p. 52).

(The way in which this analogy is an over-simplification is, of course, that the manner in which the method of measurement disturbs or «creates» the phenomena is perfectly clear and of no great interest whereas this is not the case in quantum theory.)

Putnam thinks that a position of this sort is wildly paradoxical and that it is acceptance of the distribution law rather than «quantum logic» that produces «the idea that momentum measurement brings into being the value found...» ([15] p. 229). His argument is simply this. Suppose the momentum has been given a precise value (within an allowable range) then according to Putnam both $p \,.\, x_1$ and $p \,.\, x_2$ are false, so $(p \,.\, x_1) \vee (p \,.\, x_2)$ must be false and since p is true, $x_1 \vee x_2$ (i.e., on the assumption of only two possibilities, $x \vee \neg x$) must be false but this amounts to the view that where the momentum is experimentally determined a particle just has no position (that measuring position somehow brings it into existence etc). Putnam's view is that to avoid this «paradox» it is preferable to drop the distribution law. It might of course be argued that this paradox about measurement though metaphysically very odd is not a logical antinomy (at least not necessarily an antinomy in any obvious formal sense – it might, of course, be artificially transformed into one) whereas a distribution law represents a logically necessary truth; hence it is preferable to accept the paradoxical metaphysical account and retain distribution. This argument however begs the question at issue from Putnam's point of view since he argues that the logical connectives should be operationally defined and that whether a logical law holds should be decided on empirical grounds. Hence he in effect claims that the objective existence of physical properties is «more of a necessary truth» than the distributive law. I will not try to resolve this very difficult metaphysical issue but will argue instead (as against Putnam) that once the point made above about modal operators is recognized one can accept that the logic of quanta is Boolean without being committed to «subjectivism» with respect to observable properties or, indeed, to any metaphysical position about quanta. For what everyone agrees on (with some qualifications) is that certain variables are not simultaneously decidable in quantum theory (at least with regard to the future) *whether or not* they *really* have values independent of observation. Hence once this is spelled out in the way described above, e.g., that $\Box p \rightarrow (\sim \Box x . \sim \Box \sim x)$ then, because of the modal character of this statement the question of whether the distribution law is obeyed or not does not really arise, for what looks like, but is *not* the distributive expansion is clearly *not* equivalent by *standard* modal logic. The important point then with regard to Putnam's purely interpretative or metaphysical claim is that once the modal character of the relevant statements is recognized one can hold that the distribution law holds without being committed to «subjectivism», i.e., to the view that $x \vee \neg x$ must be false where $\Box p$ is true, for the relevant disjunction is not $x \vee \neg x$ but $\Box x \vee \Box \neg x$ and everyone would agree that the latter is false where $\Box p$ is true no matter what interpretation one puts on quantum theory, i.e., since $x \vee \neg x$ can be true where $\Box x \vee \Box \neg x$ is false, one can accept the distribution law without being committed to erroneous statements like $\Box (p_1 . x_1)$.

In Putnam's quantum logic (see also Fine, [6]) $\varphi \vee \psi$ can be true where φ and ψ are each false (e.g., if the value of momentum is determined and φ and ψ describe position); this does have the result that «distribution» is violated in a sense (I would agree with Fine that the sense, however, is innocuous) since $p . (x \vee \neg x)$ can be true where x and $\neg x$ are each false. To say this is counterintuitive seems an extreme understatement until one realizes that what is actually *meant* is that where $\Box p = 1$ although $x \vee \neg x = 1$ neither $\Box x$ nor $\Box \neg x$ is true (and, when this is made explicit distribution is seen not to fail). The non-modal way of expressing the point has the added disadvantage that disjunctive syllogism must be rejected (as well as $p \supset q = \sim p \vee q$; see Birkhoff and von Newmann [1], pp. 16, 17); otherwise it is easy to derive a contradiction from, for example, the basic propositions of Finkelstein's non-Boolean lattice.

6. *Conclusions*

Jauch is, of course, correct that quantum «logic» is not a logic in the sense that some of its properties depend upon «empirical» relations, on theoretical considerations which are not part of formal logic in the traditional sense (such as $\Box p \rightarrow \sim\Box x \wedge \sim\Box\sim x$). I assume that Putnam would agree that a statement like $\Box p \rightarrow \sim\Box x \wedge \sim\Box\sim x$ is not itself a logical truth but that the formal structure of quantum mechanics that results from such statements being true is a «logic» and thus that logic can be revised in accordance with scientific findings. The point I hope to have established in this paper is that *whether or not* one calls this «formal structure» a «logic» this much is clear: if one spells out the relations among the elements involved by means of a modal-like operator then one can see that there is no violation of the standard distribution law. ([8]) If this operator is left out of account it is true that the resulting «elements» form a non-Boolean lattice, but it is difficult to see why this should be thought to have any general logical importance since it *depends* on the fact that the elements do *not* correspond precisely to the relevant propositions. (It is like, though not exactly analogous to, the claim that since one cannot simultaneously determine both that a person can run the four minute mile and sit still for five minutes, propositions describing people do not necessarily form a Boolean lattice. The way in which this is not exactly analogous brings out an ambiguity in the use of the expression «simultaneously decidable» in these discussions. Sometimes it is used in this way: exact values for, e.g., energy and time cannot be determined at the same time. But this sort of situation holds for all sorts of cases outside quantum mechanics and would not seem to be of any particular interest. The point must be, rather, that it is impossible to determine through experiments at *different* times that, e.g., an electron has precise momentum and position at a particular time. There is also a dispute about whether this holds generally or just with respect to the future, but fortunately the resolution of this is not relevant to the problem dealt with here.) It might be claimed that the whole thesis of this paper overlooks the crucial point, that «true» has a slightly unusual sense in quantum logic, viz., either (i) 'it is

([8]) The fact that this relatively simple point is overlooked seems to be a desire to treat what is essentially an intensional formal structure extensionally. It is interesting for example that the only interpretation of modal logics that Birkhoff mentions in his treatise *Lattice Theory* (American Mathematical Society, 1973, p. 284) is that they are many-valued (i.e., truth-functional) logics.

experimentally confirmed that,' or even (ii) 'it *can* be experimentally determined that' and that this obviates the need for a modal-like operator. Let this sort of truth be represented by Td. Then xv ~ x cannot have this «truth-value» where p does since assigning this value to (x v ~ x) could only mean that either (x) or (~x) has it; but in that case distribution again does not fail. (9)

Unfortunately the question of *which* modal logic quantum logic is cannot be answered since no standard modal logic contains propositions which depend for their logical relations on their empirical content; in this respect Jauch's position is correct.

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(9) This might appear to be inconsistent with the assigning of □ to x v ¬x in (B_1*) but I do not believe that it is since □ is a non-truth-functional operator whereas Td is supposed to be a truth-value. In any case if (B_1*) is re-written □ p . (x v ¬x) the point still holds that □ (p . x) v □ (p . ¬x) does not follow from it by *ordinary* modal logic. (Of course (p . x) v (p . ¬x) *will* follow but that is allright since it is the modal statement that is supposed to be false. What I mean by this is that, if the argument of this paper is correct, quantum logic is not extraordinary in one way: the distribution law holds, it is only a modal version of it that fails and the latter fails in *ordinary* modal logic as well.

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