ANALYTICAL DEONTIC LOGIC: AUTHORITIES AND ADDRESSEES

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1. Introduction

It is now a well established fact that the study of so-called deontic logic must take into account the individuals who play a rôle in the norms. To avoid undertaking a historical study of the question I shall mention only three of the authors on this subject: J.L. Gardies [3], B. Hansson [5] and Z. Ziemba [7].

Whatever the techniques used, the two categories of individuals most commonly considered are on the one hand the *addressee* of the norm (Z. Ziemba), i.e. the individual stated as being obliged or not (the one affected by the obligation or the permission), and on the other hand the subject or the *authority* of this same norm, i.e. the individual obliging or giving permission. The authority, sometimes identified with the juridical *creditor*, may also be hidden by the impersonal entity of a code (as understood by Z. Ziemba). He can even be totally obliterated as was the case in many of the early systems of deontic logic; then, one claims to speak only about a single universal morality or a single natural law.

On the contrary, the analysis of the norm can be made more subtle, eventually leading to the distinction of four individuals. Indeed, the institution of the *security* necessitates the separation of the addressee of the norm from the addressee of the prestation concerned by the norm: thus, in the statement «x guarantees y that z will fulfill that prestation», «x» is the addressee of the norm and «z» is the addressee of the prestation. But it may also be that the creditor of an obligation and the *beneficiary* of the corresponding prestation are two different people, although «in most cases the person establishing a credit intends to do so for his own benefit» (1).

I am not going to pursue this analysis here, more especially as it does not seem at all certain to me that the creditor should be taken as a

(1) Cf. [3], p. 152.

truly normative authority, the subject responsible in the norm. But I did think it worthwhile drawing attention to this point as an example of a borderline case in order to show how far the analysis can be developed.

This paper will restrict itself to an axiomatic and semantic study of normative structures taking into consideration, firstly a plurality of authorities, secondly a plurality of addressees, and finally both of these categories together. Following one of B. Hansson's ideas, I shall use different quantifications to show the level of generality that can be reached and the harmony with ordinary intuition that results from this.

2. *A deontic system with several authorities*

Let Oxp mean that «x makes it obligatory that p». The search for a coherent deontic system with several authorities involves an extrapolation of the structures well-known in the case of the unity into a multiplicity. Thus, when just a single authority «x» is under consideration, it is obvious that the laws and the rules sought are formally identical with those of classical deontic logic. Therefore, starting with the standard system D, axiomatically defined by:

Def P	$Pp = \sim O \sim p$
A1	$Op \supset \sim O \sim p$ (or $Op \supset Pp$)
A2	$O(p \supset q) \supset (Op \supset Oq)$
RO	$\vdash \alpha \rightarrow \vdash O\alpha$

(where Op means «it is obligatory that p» and Pp «it is permitted that p»), we come to the definition:

Def Px $Pxp = \sim Ox \sim p$

to the theorems:

Tx1	$Oxp \supset \sim Ox \sim p$ (or $Oxp \supset Pxp$)
Ax2	$Ox(p \supset q) \supset (Oxp \supset Oxq)$

and to the rule:

ROx $\vdash \alpha \rightarrow \vdash Ox\alpha$

to which must be added all the axioms and rules of the classical propositional calculus (PC), as in the case of the system D.

Let us now consider which other formulae may be characteristic not of a single authority but of several together. To obtain a coherent system, the only condition to be satisfied is that there should not be any conflict between the obligations emanating from the different authorities. In cases where there are only two authorities, it is necessary and sufficient for each of them not to forbid – in other words to permit – what the other makes obligatory. (similarly, when there is just a single authority, the axiom A1 states his own coherence). The dual-authority generalization of the standard axiom A1 will be therefore:

(i) $Oxp \supset \sim Oy \sim p$ (or $Oxp \supset Pyp$)

However the case of a set of more than two authorities is more complex. On first inspection, one would think that *formula (i) simply generalized for all pairs x, y of authorities* would be suitable, but in fact agreements among mere pairs of authorities do not imply agreements among *n-tuples* (where $n > 2$) of authorities.

This last point is clearly illustrated by the example of a set of three authorities x, y, z. Thus, if x makes it obligatory that p and y makes it obligatory that q, then for the norms issued by x, y and z to be coherent, z must permit that p & q. Indeed, if any authority z did not permit it, i.e. made obligatory that $\sim(p \,\&\, q)$, this obligation would be in contradiction with the one that p and q, which can be logically derived from the norms issued by x and y. It is therefore necessary to lay down as a thesis:

(ii) $(Oxp \,\&\, Oyq) \supset Pz(p \,\&\, q)$

Now, the instances of formula (i):

$Oxp \supset Pzp$ and $Oyq \supset Pzq$

allow us to deduce the formula:

(iii) $(Oxp \,\&\, Oyq) \supset (Pzp \,\&\, Pzq)$

which can easily be derived from (ii), whereas (ii) cannot be derived from (iii). Indeed, it is known that standard deontic logic contains the thesis:

$$P(p \& q) \supset (Pp \& pq)$$

(which $Pz(p \& q) \supset (Pzp \& Pzq)$ corresponds to), but not the converse. (ii) is therefore stronger than (iii), i.e. than (i).

Then by analogy with formula (ii) the following axiom schema must be laid down for a domain $D = \{x, y, ...\}$ or any entire number of authorities:

Ax1 $(Oxp \& Oyq \& ... \& Ots) \supset Pu(h \& q \& ... \& s)$

Let Dx be the deontic system with a plurality of authorities defined by the following: the definition Def Px, the axioms Ax1 and Ax2, the rule ROx and the axioms and rules of PC. By reference to widely used semantic methods and more particularly by following G.E. Hughes and M.J. Cresswell's formulation (applied to modal logic), I am going to study the different properties of Dx.

Consistency – This can easily be proved by means of the «PC-transformation» [2] (for Dx, the PC-transform of a wff α will be formed by rewriting α (if necessary) in primitive notation, and then deleting every occurrence of Ox, Oy, etc.).

Validity – A Dx-model is an ordered quadruple $<W, D, S, V>$ where W is a set of «worlds» w_i; D is a domain of individuals x, y, ... ; S is a triadic relation whose first argument is an element of D and the other two are elements of W, a relation subject to the condition that, for any $w_i \in W$, there is some $w_j \in W$ such that, for any $x \in D$, $Sxw_i w_j$; V is a value assignment (1 or 0) satisfying the usual conditions for PC and the following for the operator Ox:
[VOx]. For any wff α, for any $w_i \in W$ and for any $x \in D$, $V(Ox\alpha, w_i) = 1$ if for every $w_j \in W$ such that $Sxw_i w_j$, $V(\alpha, w_j) = 1$; otherwise $V(Ox\alpha, w_i) = 0$.

A formula α is Dx-valid if for every Dx-model and for every w_i, $V(\alpha, w_i) = 1$.

The definition of the relation S calls for explanations and comments. As for its significance, $Sxw_i w_j$ should be interpreted as meaning that *the world w_j is permissible for the world w_i with respect to x* (or w_j is a deontic alternative world to w_i with respect to x, or

(2) [6], pp. 41-42.

more simply, but less appropriatly, w_j is «good» or «positive» relatively to w_i and to x).

Finally, the normative coherence between the different authorities is given by the last condition imposed on S. Whatever the world w_i and several authorities are, there will be a world w_j permissible for w_i with respect to all these authorities: consequently, the norms issued by different authorities will be consistent. As in the axiomatic presentation, I want to point out once again that the strength of this property is greater than the strength of the property corresponding to agreements only between couples of authorities. For, let w_i, w_j, w_k, w_1 be four worlds such that $Syw_i w_j$ and $Szw_i w_j$, $Szw_i w_k$ and $Sxw_i w_k$, $Sxw_i w_1$ and $Syw_i w_1$; these four worlds realize a normative agreement between the couples (y, z), (z, x), (x, y). However, their existence in no way implies the existence of a single world w_m such that $Sxw_i w_m$, $Syw_i w_m$ and $Szw_i w_m$, and it is only this world which would settle a proper normative agreement between the three authorities under consideration.

Decision procedure – I present a decision procedure in the style of Hughes and Cresswell [6]. The semantic diagrams are as for the system T, except that I shall assume all occurrences of the operator Px (or Py, etc.) are eliminated by applications of Def Px, and that the «rules for a new world» are as follows:

A. If in a world w_i there occurs a formula $Ox\alpha$ with an asterisk above the O ($Ox\alpha$ is assigned 1), then in every world w_j such that $Sxw_i w_j$ α must be assigned 1.

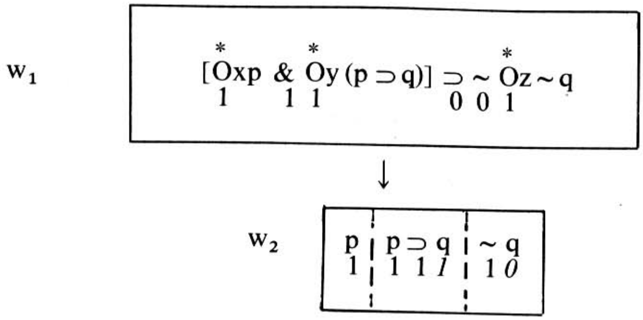
B. If in a world w_i there occurs a formula $Ox\alpha$ with an asterisk below the O ($Ox\alpha$ is assigned 0), then there must be a world w_j such that $Sxw_i w_j$ in which α is assigned 0.

B′. If in a world w_i there occurs no formula $Ox\alpha$ with an asterisk below the O, but at least one with the asterisk above the O, then there must be a world w_j such that, for all $t \in D$, $Stw_i w_j$.

For example, let the formula to be tested be:

$$[Oxp \ \& \ Oy(p \supset q)] \supset Pzq$$

meaning that «if x makes it obligatory that p and y makes it obligatory that if p then q, then it is permitted by z that q». I write its consequent $\sim Oz \sim q$ in primitive notation and I build the following diagram.



The world w_2 (required by rule B′) is permissible for w_1 with respect to x, y and z. The inconsistency in w_2 is immediate and therefore the formula put forward is Dx-valid.

With this sort of procedure, it is easy to verify that the axioms of Dx are valid. Moreover, given that the rules of Dx (substitution generalized to Dx, Modus ponens, ROx) preserve validity, one must conclude that every thesis of the system is valid. Then, it remains to be proved that, inversely, every Dx-valid formula is also a Dx-thesis, i.e. that the system is (weakly) complete.

Completeness – As previously, there is a great deal of similarity between the present *exposé* and the study carried out by G.E. Hughes and M.J. Cresswell. The simplest approach is to take-up and modify the proof of completeness for modal system T given by these authors.

The four lemmas used for T are to be retained and the only important modification affects the second one. There are no change at all in lemmas 1 and 4; lemma 3 ([6], p. 98), whilst its statement remains unchanged, now needs only a part of the original proof (for $Oxp \supset p$ is not a axiom [3]). Concerning lemma 2, the statement is left unmodified, but the proof is partly different.

Indeed, according to the rules for a new world of Dx, there are two cases in which an arrow can have been drawn from a rectangle w_i to a rectangle w_j: those that correspond to the rules B and B′. The first of these cases leads to a situation quite similar to that studied in [6] (pp. 99-100). On the contrary, the second case is an original one.

Case 2 (rule B′). The initial formulae of w_j (permissible world for w_i

(3) Cf. [1], p. 301.

with respect to the authorities x, y, ..., t, u) are a set of formulae, $\alpha_1, \ldots, \alpha_k, \beta_1, \ldots, \beta_1, \ldots, \delta_1, \ldots, \delta_m, \varepsilon_1, \ldots, \varepsilon_n$, each assigned 1 in w_j and being all the formulae such that $Ox\alpha_1$, ..., $Ox\alpha_k$, $Oy\beta_1$, ..., $Oy\beta_1$, ..., $Ot\delta_1$, ..., $Ot\delta_m, Ou\varepsilon_1$, ..., $Ou\varepsilon_n$ are assigned 1 in $w_i\,(k + 1 + \ldots + m + n > 0)$.

Thus w_j' will be the formula:

$$\sim\alpha_1 \vee \ldots \vee \sim\alpha_k \vee \sim\beta_1 \vee \ldots \vee \sim\beta_1 \vee \ldots \vee \sim\delta_1 \vee \ldots \vee \sim\delta_m \vee \sim\varepsilon_1 \vee \ldots \vee \sim\varepsilon_n,$$

and so (by the hypothesis of the lemma) we have:

$$\vdash \sim\alpha_1 \vee \ldots \vee \sim\alpha_k \vee \ldots \ldots \vee \sim\delta_1 \vee \ldots \vee \sim\delta_m \vee \sim\varepsilon_1 \vee \ldots \vee \sim\varepsilon_n$$

Hence by the rule ROx (in this case more exactly by the instance ROu of this rule):

$$\vdash Ou\,(\sim\alpha_1 \vee \ldots \vee \sim\alpha_k \vee \ldots \vee \sim\delta_1 \vee \ldots \vee \sim\delta_m \vee \sim\varepsilon_1 \vee \ldots \vee \sim\varepsilon_n)$$

i.e., by Def $\supset$:

$$\vdash Ou\,(\varepsilon_1 \supset (\ldots \supset (\varepsilon_n \supset (\sim\alpha_1 \vee \ldots \vee \sim\alpha_k \vee \ldots \vee \sim\delta_1 \vee \ldots \vee \sim\delta_m))\ldots))$$

whence by repeated applications of the instance with «u» of Ax2:

$$\vdash Ou\varepsilon_1 \supset (\ldots \supset (Ou\,\varepsilon_n \supset Ou\,(\sim\alpha_1 \vee \ldots \vee \sim\alpha_k \vee \ldots \vee \sim\delta_1 \vee \ldots \vee \sim\delta_m))\ldots)$$

Now, Ax1, by contraposition and several applications of Def Px, gives the thesis:

(iv) $Ou\,(p \vee q \vee \ldots \vee s) \supset (Pxp \vee Pyq \vee \ldots \vee Pts)$

Then, by (iv) with the same authority x for different propositional variables equal to $\sim\alpha_1, \ldots, \sim\alpha_k$, with the same authority y for other propositional variables equal to $\sim\beta_1, \ldots, \sim\beta_1$, and so on until the authority t, and besides taking into account Def Px and the extensionality permitting the replacement of $\sim\sim p$ by p in the scope of every normative operator, it becomes:

$$\vdash Ou\,(\sim\alpha_1 \vee \ldots \vee \sim\alpha_k \vee \ldots \vee \sim\delta_1 \vee \ldots \vee \sim\delta_m) \supset$$
$$(\sim Ox\alpha_1 \vee \ldots \vee \sim Ox\alpha_k \vee \ldots \vee \sim Ot\delta_1 \vee \ldots \vee \sim Ot\delta_m)$$

and therefore by syllogism:

$$\vdash Ou\varepsilon_1 \supset (\ldots \supset (Ou\varepsilon_n \supset (\sim Ox\alpha_1 \vee \ldots \vee \sim Ox\alpha_k \vee \ldots \vee \sim Ot\delta_1 \vee \ldots \vee \sim Ot\delta_m))$$

i.e., by Def $\supset$:

$\vdash \sim Ou\varepsilon_1 \vee \ldots \vee \sim Ou\varepsilon_n \vee \sim Ox\alpha_1 \vee \ldots \vee \sim Ox\alpha_k \vee \ldots \vee \sim Ot\delta_1 \vee \ldots \vee \sim Ot\delta_m$

which is:

$\vdash \sim Ox\alpha_1 \vee \ldots \vee \sim Ox\alpha_k \vee \ldots \vee \sim Ot\delta_1 \vee \ldots \vee \sim Ot\delta_m \vee \sim Ou\varepsilon_1 \vee \ldots \vee \sim Ou\varepsilon_n$

Now in w_i each of $Ox\alpha_1, \ldots, Ou\varepsilon_n$ is assigned 1. Hence (by Lemma 3) we have:

$\vdash \sim Ox\alpha_1 \supset w'_i, \ldots, \vdash \sim Ou\varepsilon_n \supset w'_i$

and hence (by PC):

$\vdash (\sim Ox\alpha_1 \vee \ldots \vee \sim Ou\varepsilon_n) \supset w'_i$

which, together with the formula previously derived, gives us by Modus Ponens:

$\vdash w'_i$

Q.E.D.

3. A deontic system with several sets of authorities

Starting with the system Dx, when one wants an increase of one degree in the level of generality and to express an «anonymous» norm, the existential quantification over the authorities seems appropriate. Indeed, as noted by B. Hansson, there is an obligation if at least one authority obliges. Thus it seems apposite to lay down the definition ([4]):

Def O_1 $\quad O_1 p = \exists x\, Oxp$

from which it follows $P_1 p = \forall x\, Pxp$ (by Def Px and Def P_1: $P_1 p = \sim O_1 \sim p$). I can then establish without difficulty the validity of the formula:

$O_1 p \supset P_1 p$

and of the rule:

$\vdash \alpha \rightarrow \vdash O_1 \alpha$

([4]) Cf. [5], pp. 243-244. Because of quantification the system is no longer purely propositional. However, we do not obtain a general quantification theory, since Def O_1 is the sole way for a quantor to appear.

but it is noteworthy that I cannot do so for:

$$O_1(p \supset q) \supset (O_1 p \supset O_1 q)$$

However, the validity of this latter formula is indispensable for the operator O_1 to be sure of possessing the «standard» character of the general usual obligation.

The invalidity can be explained intuitively in the following way. If the authority x makes it obligatory that $p \supset q$ and the authority y ($\neq$ x) makes it obligatory that p, then every addressee must *ipso facto* be obliged that q. And yet if no authority (identical or different to x and/or y) makes explicit this obligation, with the definition of O_1 it cannot be said that the addressee is obliged that q. This shows therefore that the definition of O_1 is badly chosen and that it should be modified (5). Indeed in the last example, even though the obligation of q emanates neither from x nor from y, it does emanate from their set, and this leads to an enlarging of the system Dx into a system DX where the norms emanate from sets of authorities.

Let D = {x, y, ...} be a set of authorities and let X, Y, ... be various subsets of D (therefore sets of authorities). «OXp» signifies that «(some member of) the set of authorities X makes it obligatory that p». I lay down:

Def Px	$PXp = \sim OX \sim p$
AX1	$(OXp \ \& \ \dots \ \& \ OZr) \supset PT(p \ \& \dots \& \ r)$
AX2	$OX(p \supset q) \supset (OXp \supset OXq)$
ROX1	$\vdash \alpha \rightarrow \vdash OX\alpha$
ROX2	$X \subseteq Y \rightarrow \vdash OXp \supset OYp$

According to Def PX, «PXp» means that «(every member of) X permits that p». The system DX is determined so that if X = {x}, the operator OX is formally identical with Ox. But the essential innovation is to be found in rule ROX2, which states this intuitive truth that if one set of authorities is included in another set, every obligation

(5) Cf. [5], pp. 243-244. B. Hansson reaches a similar conclusion. But the definition he proposes as a replacement, where there occurs «a *primary* obligation (...) or a finite conjunction of primary obligations (...)» is formally inadequate. It is not Oxp & Oyp that have to be considered for example, but O {x, y} p. There is not a conjunction of obligations, but obligation by a conjunction of authorities, or more precisely by their *set*.

emanating from the former is also an obligation emanating from the latter. Thanks to this rule we could have simply laid down:

AX1′ $OXp \supset PXp$

instead of AX1, for the rule makes possible to derive AX1 from AX1′ (6).

Let us now consider the thesis (useful for what follows):

TX1 $OX(p \supset q) \supset (OYp \supset OX \cup Y\, q)$

whose deduction can be laid out thus:

$X \subseteq X \cup Y$ therefore $\vdash OX(p \supset q) \supset OX \cup Y(p \supset q)$
$Y \subseteq X \cup Y$ therefore $\vdash OYp \supset OX \cup Y\, p$

From there, by PC:

$$[OX(p \supset q) \& OYp] \supset [OX \cup Y(p \supset q) \& OX \cup Y\, p]$$

By AX2 (with $X \cup Y$ instead of X) and by PC:

$$[OX \cup Y(p \supset q) \& OX \cup Y\, p] \supset OX \cup Y\, q$$

from which, by syllogism:

$$[OX\,(p \supset q) \& OYp] \supset OX \cup Y\, q$$

and finally TX1 by PC.

Validity – Here it will suffice to say that the validity in DX can be based on the same triadic relation $Sxw_i w_j$ as for Dx. With $X = \{x, \ldots, z\}$, the notation $SXw_i w_j$ can be introduced as a simple ab-

(6) From ROX2 we obtain the derived rule:

$X \subseteq Y \rightarrow \vdash PYp \supset PXp$

Therefore, $OXp \supset OX \cup Yp$ and $PX \cup Yp \supset PYp$ hold, from which by AX1′ $(OX \cup Yp \supset PX \cup Yp)$ and syllogism: $OXp \supset PYp$. Now, we have also:

$(OXp \& \ldots \& OZr) \supset (OX \cup \ldots \cup Zp \& \ldots \& OX \cup \ldots \cup Zr)$
i.e. $(OXp \& \ldots \& OZr) \supset OX \cup \ldots \cup Z(p \& \ldots \& r)$

and by $OXp \supset PYp$:

$OX \cup \ldots \cup Z(p \& \ldots \& r) \supset PT(p \& \ldots \& r)$

and therefore AX1 by syllogism.

breviation for Sxw_iw_j and ... and Szw_iw_j. SXw_iw_j satisfies the following condition, which amounts to the one on the triadic relation of Dx: for any $w_i \in W$, there is some $w_j \in W$ such that SDw_iw_j.

Decision procedure – Once again there are three rules for a new world. The last two are quite similar to those of the system Dx, but all the authorities x, y, etc. are simply replaced by sets X, Y, etc. The first however must be reformulated:

A. If in a world w_i there occurs a formula $OX\alpha$ with an asterisk above the O, then in every world w_j and for every Y such that $X \subseteq Y$ and SYw_iw_j α must be assigned 1.

Thus, for example, the validity of the thesis TX1 from earlier is proved by the following simple diagram, where $S\,X \cup Y\,w_1w_2$

w_1

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*           *
OX(p ⊃ q)⊃(OYp⊃OX ∪ Y q)
1         0 1   0 0
                *
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↓

w_2

p⊃q	p	q
1 1 *1*	1	*0*

With this procedure, the proofs of the validity of the axioms and rules of DX present no difficulty. Completeness can be proved in the same manner as previously (the situation of case 1 (rule B) being different, the proof must be modified on this point).

4. *A deontic system with several addressees*

Let «O-y p» mean that «it is obligatory for y that p», y being the addressee of the norm but not necessarily the subject of proposition p. Certainly, in most cases this subject is identical with the addressee of the norm; but, as I have said, the juridical institution of the security demands that they should be distinguished, and – what is more – there are sound structural reasons for not confusing them. Indeed, it is only in this situation that variable p in the expression O-y p retains its true status of a propositional variable(7) and besides the system D-y to be built will then be made up of axioms and rules similar in form to those of Dx. In this way Def P-y, A-y1, A-y2, RO-y, analogous to Def Px, Ax1, Ax2, ROx will be found.

One of the instances of A-y1 is:

$$\text{O-y p} \supset \text{P-z p}$$

It is advisable to note that this formula appears paradoxical only if the normative addressee is unduly identified with the subject of proposition p. The formula does not say for example that «if y is obliged to go into that house, then z is also permitted to go in». The formula only corresponds to the following: as soon as an addressee is obliged that such and such a thing is accomplished, normative coherence makes it necessary that all other individuals are not obliged that this thing is not accomplished (in other words, that they are permitted that it is accomplished).

Semantically, D-y can be made to depend on a triadic relation S-y w_i w_j satisfying conditions analogous to those of Sxw_i w_j for Dx. Consistency, validity, decision procedure and completeness are established and are proved using methods similar to those in the preceding cases.

5. A deontic system with several sets of authorities and several addressees

Nothing prevents the grouping of the results of the last two sections in a single basic system that I shall call DXy. We have:

Def PXy	PXyp = ~ OXy ~ p
AXy1	(OXyp & ... & OStr) ⊃ PTu (p & ... & r)
AXy2	OXy (p ⊃ q) ⊃ (OXyp ⊃ OXyq)

(7) As B. Hansson notes, the opposite choice necessitates a modification of this status ([5], p. 246): then «propositions describing acts (...) with an empty space for the agent» are dealt with; in fact it would be better to talk in terms of *predicates*.

ROXy1 $\vdash \alpha \rightarrow \vdash OXy\alpha$

ROXy2 $X \subseteq Z \rightarrow \vdash OXyp \supset OZyp$

The semantics depends on a tetradic relation $SXyw_i w_j$ which satisfies the condition that for any $w_i \in W$, there is some $w_j \in W$ such that for any $y \in D$, $SDyw_i w_j$. Once again, consistency, validity, etc. could be studied as previously. I will not do so here for the sake of brevity.

6. *Generalised deontic operators*

To begin with I shall take the system DX. From what we have seen in Section 3, we can say that there is a mere obligation if at least one set of authorities obliges. Therefore I lay down:

Def O_2 $\quad O_2 p = \exists X\, OXp$

from which it follows by Def P_2 ($P_2 p = {\sim} O_2 {\sim} p$) and Def PX:

$$P_2 p = \forall X\, PXp$$

Indeed it is reasonable to consider that something is (merely) permitted when it is universally permitted by all the sets of authorities. Then, starting with AX1 (more precisely starting with the instance $OXp \supset PYp$) and with ROX1, we immediately obtain:

$$O_2 p \supset P_2 p$$

$$\vdash \alpha \rightarrow \vdash O_2 \alpha$$

as was the case of the generalisation starting with Dx.
Besides, by TX1 universally closed, we have:

$$\forall X\, \forall Y\, [OX(p \supset q) \supset (OYp \supset O\, X \cup Y\, q)]$$

i.e., by the passage rules of the lower predicate calculus (LPC):

$$\forall X\, [OX(p \supset q) \supset \forall Y\, (OYp \supset O\, X \cup Y\, q)]$$

and also by the thesis $\forall x\, (fx \supset gx) \supset (\exists x\, fx \supset \exists x\, gx)$ of LPC:

$$\exists X\, OX(p \supset q) \supset \exists X\, \forall Y\, (OYp \supset O\, X \cup Y\, q)$$

Hence by the thesis $\exists x\, \forall y\, fxy \supset \forall y\, \exists x\, fxy$ and by syllogism:

$$\exists X\, OX(p \supset q) \supset \forall Y\, \exists X\, (OYp \supset O\, X \cup Y\, q)$$

i.e., by the passage rules of LPC applied to the consequent:

$$\exists X\, OX(p \supset q) \supset \forall Y\, (OYp \supset \exists X\, O\, X \cup Y\, q)$$

and whence by the former thesis of LPC already used and by syllogism:

$$\exists X\, OX(p \supset q) \supset (\exists Y\, OYp \supset \exists Y\, \exists X\, O\, X \cup Y\, q)$$

Now, $X \cup Y$ being a set of authorities, $\exists Y\, \exists X\, O\, X \cup Y\, q$ implies $\exists Z\, OZq$; from which it follows finally:

$$\exists X\, OX(p \supset q) \supset (\exists Y\, OYp \supset \exists Z\, OZq)$$

which is none other than, by Def O_2:

$$O_2(p \supset q) \supset (O_2 p \supset O_2 q)$$

Then, starting with D-y, the mere obligation is obtained this time by a universal quantification over the addressees. Thus I lay down:

Def O_3 $\quad O_3 p = \forall y\, O\text{-}y\, p$

from which it follows by Def P_3 ($P_3 p = \sim O_3 \sim p$) and Def P-y:

$$P_3 p = \exists y\, P\text{-}y\, p$$

But what does one usually mean when, without specifying for whom, one asserts that it is permitted that p? It certainly does not mean that *there is an individual* for whom it is permitted that p, but rather that such a permission is applicable to *every individual* [8]. It is therefore advisable to introduce at this point a *strong permission*, defined by:

Def P'_3 $\quad P'_3 p = \forall y\, P\text{-}y\, p$

and to qualify the previous permission as a *weak* one.

Then, using the ordinary theses and rules of LPC as above, we obtain:

$$P'_3 p \supset P_3 p$$

$$O_3 p \supset P'_3 p$$

$$O_3(p \supset q) \supset (O_3 p \supset O_3 q)$$

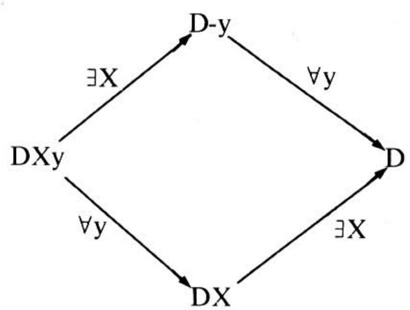
[8] Cf. [5], p. 246.

$O_3(p \supset q) \supset (P'_3 p \supset P'_3 q)$

$\vdash \alpha \rightarrow \vdash O_3 \alpha$

These four formulae and this rule are identical with the four axioms A1, ..., A4 and with the rule RO that I have given in a previous study in which the norms O_3 and P'_3 (then simply noted as O and P′) were defined as the standard norms of obligation and permission preceded by the modal operator of *necessity* ([9]). Of course, the identity of the results comes from the well-known analogy between the quantification and the modality.

Both obligations O_2 and O_3 satisfy the axioms and the rules of the standard system D. Thus, they appear to be structurally equivalent. Broadly speaking, from the basic system DXy, by including concepts of strong permissions in DX and D, different quantifications can be effected leading to D via stages D-y and DX. For instance, existential quantification over the sets of authorities and the definition Op = ∃X OXp enable us to obtain the system D on the basis of the system DX. We can draw the following schema:



These results are important for the structure of moral and juridical thought. They show that, when the logic of norms takes into account a multiplicity of individuals, the standard system, which could be imagined to be a simple and obvious point of departure, appears on

([9]) Cf. [2]. The identity in question is only formal; it does not concern the meaning of the different concepts of weak and strong norms. Besides, it is important to note that there is not any identity, even formal, between the present strong permission and the one of [1].

the contrary as a final result. Then, some more complex underlying structures (DXy, D-y, DX) seem to be logically anterior. More generally, the consideration not only of the individuals but also of dimensions such as alethic, temporal, epistemic modalities is the sole efficient means allowing an analysis of the deonticity: then the standard system, with all the well-known paradoxes affecting it, only appears as a summary of our normative intuitions.

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REFERENCES

[1] BAILHACHE, P. – Sémantiques pour des systèmes déontiques intégrant permission faible et permission forte, *Logique et analyse*, 79, 1977, pp. 286-316.
[2] BAILHACHE, P. – Several possible systems of deontic weak and strong norms, *Notre Dame Journal of Formal Logic*, 21, 1980, pp. 89-100.
[3] GARDIES, J.L. - Logique déontique et théorie générale des fonctions complétives, *Logique et analyse*, 61-62, 1973, pp. 143-220.
[4] HANSON, W.H. – Semantics for deontic logic, *Logique et analyse*, 31, 1965, pp. 177-190.
[5] HANSSON, B. – Deontic logic and different levels of generality, *Theoria*, 36, 1970, pp. 241-248.
[6] HUGHES, G.E. & CRESSWELL, M.J. –*An introduction to modal logic,* London, 1968.
[7] ZIEMBA, Z. – «Deontic logic», an appendix in Z. ZIEMBIŃSKI, *Practical logic,* Warsaw and Boston, 1976, pp. 360-430.