PREPOSITIONAL LOGIC

James W. GARSON

0. Introduction

An adequate prepositional logic would give a formal account of the validity status of arguments which contain prepositional phrases, such as the following:

(1)	Randy ran at midnight / Randy ran	(valid)
(2)	Sam stares at Sally / Sam stares	(valid)
(3)	Randy runs at midnight / Randy runs	(invalid)
(4)	Logic is easy for Kripke / Logic is easy	(invalid)

The purpose of this paper is to make some new suggestions about the foundations of such a logic.

There have been numerous attempts to show that prepositions can be handled within standard theories of first, or second order logic. (1) Schwartz [22] makes the most recent proposal of this kind. In sections 1-3 we will argue against his position, and for the need for non-standard logics. The main non-standard approach involves adding functors to the syntax of first order logic which take predicates into more complex predicates, (2) on analogy with numerical functors like minus (−) which take a term (such as '3') into a new term ('−3'). For example, if R stands for the predicate «runs» and M for the phrase «at midnight», then «runs at midnight» is represented as the (complex) predicate [R]M which results from applying the predicate functor M to

(1) See CLARKE [3] and SCHWARTZ [22] for descriptions of Reichenbach's and Davidson's attempts along these lines.

(2) Main papers in this line of research include [3], [4], [16], [18], [19], [25], and [27].

the predicate R. Although we will agree with the main outlines of this approach, we will point to the need for a more general syntax, and a more detailed semantics than is found in the literature.

Of course, prepositional phrases are only one of a wide variety of modifiers in English. Though our main purpose will be to give an adequate account of prepositions, a theory of other modifiers will be a natural by-product.

1. The classical treatment of indefiniteness

One of the most interesting features of natural languages is the pervasiveness of indefinite expressions, expressions whose meaning depends on the context of their evaluation. This accounts for the flexibility of natural language, and at the same time, for its ambiguity when considered out of context. Standard examples of indefinite expressions include indexicals ('I', 'this', 'here'), and tensed verbs, but indefiniteness can be found in just about any use of English expressions, for example, in adjectives: «large», descriptions: «the president», quantifiers: «everybody», and even prepositional phrases: «until 1975».

Prepositions are typically used in English as tools for eliminating indefiniteness. So, we might say «large for a man», «the president of the Coca Cola Company in 1952», «everybody on the committee», and «from 1977 until 1985» to make explicit what is otherwise left implicit in the context of use.

To a traditional logician, these phenomena, and the arguments (1)-(4) are not really proper subject matter for logic. According to him, one symbolizes the proposition expressed, not the utterance, and so one must filter out all indefinite expressions to prepare a sentence for consideration by the logical machinery. On this view, one should not attempt to characterise the validity status of (say) (3) as it stands, because, at the very least, the conclusion «Randy runs» does not make clear the time of Randy's running. Instead, we need to introduce extra term places in the predicates to remove indefiniteness, so as to get, for example, «Randy runs at t in place p in manner m at speed v...»

There are two fundamental reasons why the traditional view is inadequate as a proper analysis of prepositions. First, it is doubtful that we can ever analyse away all indefiniteness in language. Perhaps we should have said «large for a man of Caucasian ancestry inhabiting North America in the twentieth century», and even then we forgot that «the twentieth century» is indefinite since it presupposes the dating system which is being used. One of the difficulties with this translation process is that it is very difficult to tell when one has completed it successfully. To put it another way, we don't have a theory of all the ways in which an expression might be indefinite, and until we do, we will not be clear on exactly how the translation process ought to go.

The second difficulty is that even if we had a correct theory of indefiniteness, translation from utterance to symbolic formulation would be immensely complicated, requiring great sensitivity to all the ways indefiniteness can «infect» language. A logical theory would do best to minimize translation from ordinary language, for if the gap is too wide, then the formal theory becomes too cumbersome to be of practical value to a philosopher or layman who wants to use it to evaluate his reasoning.

One of the main flexibilities in English and other natural languages is that we can generate predicates with as many places as we like from a finite stock of transitive and intransitive verbs using prepositions. Since complex predicates cannot be formed in traditional theories of logic, one must begin with an infinite stock of predicate letters since we can put no finite bound on the number of places in a predicate. If the logical form of English were correctly represented in this kind of formal theory, it would be a mystery how a speaker could master the language. (3) Clearly, the learnability of English depends on the fact that the meaning of complex predicates is a function of the meaning of their parts. The traditional approach makes it impossible to investigate these meaning relationships within the theory.

2. *A first order logic of modifiers?*

Schwartz [22] argues that the difficulty of an infinite number of core

(3) See DAVIDSON [5].

predicates can be overcome because we actually need only finitely many predicates to represent all sentences of English. (4) This means that there will be an upper bound on the number of term places we could conceivably need for a predicate, and so requires that the dimensions of indefiniteness are finite. He remarks that there are only finitely many prepositions in English, so that if none is ever repeated in a predicate, then the upper bound will be two more than the number of prepositions in the language. The trouble is, of course, that the same preposition *is* repeated in many predicates, for example in «Lance lunged at Lana at 11 m.p.h. at lunchtime at the lighthouse». But Schwartz claims that while «at» appears here four times, it has a different sense or use in each occurrence. Since the number of senses is finite, and since no preposition with the same sense needs to be repeated in a predicate, we may still locate an upper bound for the number of places in a predicate.

But his claim that the number of senses is finite and that no preposition with the same sense needs to be repeated should be supported with a *semantical* theory which allows us to individuate senses of prepositions, and which shows why repetitions are never needed. This underscores the need for the development of a semantics for prepositions. If Schwartz is right, we will still need the semantics to *establish* that he is right.

Even if an upper bound on the number of places in an English predicate could be established, Schwartz' theory suffers from other difficulties which are characteristic of attempts to handle prepositions within a standard logic. His tactic for ruling (1) valid is straightforward: One translates the conclusion «Randy ran» by «There is a time at which Randy ran» so that it follows by existential generalization from the premise «Randy ran at midnight». He realises, however, that the same strategy will not do for (2), for the conclusion: «Sam stares» cannot be represented using logical notation for «there is something at which Sam stares», since Sam might stare at nothing. Schwartz suggests that for such cases we add a new predicate letter S′ which is to be read '____ stares-but-not-at-anything-in particular'. I use hyphens to stress that S′ is atomic, despite its complex form in English. Then 'Sam stares' is symbolized by ∃xSsx v S′s, which

(4) SCHWARTZ [22] p. 367 ff.

follows from Sss′, the formulation of the premise 'Sam stares at Sally.'

The plausibility of this tactic evaporates quickly when a language contains more than one preposition. If we want to handle a portion of English which contains 'stares', 'at' and 'through', we will need to account for the validity of such arguments as

(6) Sam stares at Sally through spectacles
———————————————————
Sam stares at Sally,

and this will force us into the introduction of predicate letters for '____ stares through ____ but-not-at-anything', '____ stares-at ____ but-not-through-anything', etc. In this simple minded language the quaint expression 'Sam stares' is rendered by $\exists x \exists y Ssxy \vee \exists x S'sx \vee \exists x S''sx \vee S'''s$, and the complication increases exponentially as we go to larger languages.

As if this weren't enough, the technique still does not work for arguments (3) and (4). According to Schwartz (3) is ruled valid, since its conclusion translates to 'Randy runs at some time'. But the conclusion means that Randy runs now, and the argument is invalid. In (4) 'Logic is easy' ought not to follow from 'Logic is easy for Kripke', and clearly the conclusion is not parsible as 'there is someone for whom logic is easy or logic is easy but not for anyone in particular.' With a little practice the reader will be able to generate hosts of other examples like these.

The fact that (1) and (3) differ only in the tense of the verb should help us appreciate that characterization of validity in prepositional logic is a subtle matter, and not liable to yield to any one simple translation technique. These difficulties with Schwartz' theory point out the need for adjusting both the syntax and semantics of standard theories if we want a natural prepositional logic. The syntactic change is to start with a finite list of primitive predicate letters and to introduce prepositional phrases as functors which form new predicates from old ones. ([5]) The semantical change is to introduce the concept of the linguistic situation into the description of the truth

([5]) We could think of prepositions as functors that take n-ary predicates into n + 1-ary predicates, but there is little difference in the end result, and our treatment is a bit simpler.

conditions of sentences. In a semantics (or pragmatics) (6) where truth values are assigned to sentences *in a given situation* one can examine the truth conditions of indefinite sentences, for although they do not have truth values apart from a context, they do once the situation is provided. We will be able to analyse sentences of varying degrees of indefiniteness in such a theory, and so translation to remove indefiniteness is no longer a precondition for the use of logic, but part of its subject matter. Providing a correct theory of this kind is not an easy project, but the only alternative I know is anarchy disguised in the form of translation rules.

3. *The role of syntax*

Since I will be adopting the predicate functor strategy in its main outlines, I should defend myself against Schwartz' main objection to it. He points out that certain combinations of modifiers do not make sense, for example, those in 'Randy rapidly lives next door.' So he claims that any predicate functor language must have either an immensely complicated syntax to avoid counting such monstrosities as well-formed, or a semantics «which is strained to interpret» (7) them.

But the same «problem» arises in first order logic. The sentence

(7) All coronations think green bacon

is also senseless, and if Schwartz is right about modifiers, then by parity of reasoning, we ought to adjust the syntax of first order logic to avoid it. What is odd is that (7) is a sentence of English, not a formula of logic, so no difficulties with its syntax or interpretation could arise there. Of course there are deep and difficult problems about the syntactical and semantical analysis of (7) in English, but first order logic does not pretend to answer them. We might be able to address these problems in *some* formal theory, but first order logic does not attempt to, and so has no advantage over any alternative theories in handling senselessness in English.

(6) We do not make a distinction between pragmatics and semantics in this paper though we agree with STALNAKER [23] that it makes sense to do so, for certain purposes.

(7) SCHWARTZ [22] p. 366.

Schwartz may believe that the purpose of alternative theories is to offer a final solution to the problem of characterizing the syntax and semantics of English, but that is not so. The predicate functor strategy abstracts from the surface structure of natural languages just as does first order logic. Its advantage is that it provides a more natural foundation for displaying the structure of language.

If we want, the embarrassment of senselessness can be partly overcome for both first order logic and predicate functor languages. There is no obstacle to developing a semantics which lets certain combinations of expressions have extension gaps. In this way, the formal correlates of senseless expressions of a natural language are well-formed, but we are not strained to give them an interpretation because we record their senselessness with an extension gap. (8) Of course it would be very difficult to provide an adequate theory of how and why extension gaps arise in a natural language such as English. But a properly general semantics would not attempt to do that since it would want to accomodate all languages.

In any case, Schwartz is mistaken if he suggests that the recognition of senselessness is best handled by adjusting the syntax, for it is only once we have given the primitives of a formal theory an interpretation that meaningfulness becomes an issue. When $\forall x (Cx \supset \exists y (Gy \& By \& xTy))$ is interpreted to mean that all communists think great thoughts, it makes sense, but if it is interpreted to mean that all coronations think green bacon, it is not. In a way, the same point holds for English as well, for 'the bank gives away free pencils' is meaningless when 'the bank' is interpreted to mean the side of a river.

4. *Syntax for prepositional logic*

The syntax we will develop for prepositional logic is more general than the predicate functor strategy, so it is important to explain why and how our approach differs from suggestions in the literature. The main idea in our syntax will be to let prepositional phrases and other

(8) Both LEWIS [16], p. 131 and CRESSWELL [4], p. 78 (see also pp. 224-225) make this sort of suggestion for semantical theories of natural language, though neither endorses it for bearing the full weight of imposing selection restrictions.

modifiers bind predicate letters, sentences, terms, and even other modifiers. In our system, modifiers are syntactic chameleons. The same modifier may be applied to expressions in several distinct semantical categories. Part of our motivation for this is formal. Completeness results for semantics we will present are much easier to come by when modifiers can bind all expressions. But there are strong non-formal reasons as well.

First, natural languages display this sort of syntactic structure. We know that some prepositions play in English the role of inflections in inflected languages. (For example consider the possessive use of 'of'.) But some inflections modify predicates, adjectives, nouns, and even pronouns. If our theory is to be strong enough to handle a full variety of inflections, modifiers must be applicable to expressions in the appropriate syntactical categories. Furthermore, modification of terms by prepositions occurs in English as witnessed by these examples:

(8) 10 to the base 2 is 2 to the base 10,

(9) Muhamid Ali in his prime could have defeated Jack Dempsey in his prime.

One might argue that (8) and (9) contain terms which really ought to be symbolized as definite descriptions so that the prepositional phrases modify the sentential portion of the description. But this asks us to resort to complicated translation therapy in order to preserve an antiquated syntax. If there *are* equivalences between (8) and (9) with corresponding sentences containing descriptions, then these should appear as theorems of a formal theory with a general syntax which accomodates both modification of terms and sentences. We should not prejudge the issue by forcing correspondence *via* the use of translation rules, especially since we can simply reduce the gap between natural and formal languages by simply using the more liberal syntax.

Modification of other modifiers is also possible in English, in a way that is irreducible. In one reading of

(10) John ran quickly intentionally

it is the quickness that is intentional, not the running. Modification of modifiers is commonly cited as a major obstacle for theories of modification within first, or second order logic. Our syntax accomodates it directly, and so does not force us to analyse it in other terms.

The fact that the same modifier can be applied to sentences as well as predicates provides a straightforward device for distinguishing the *de re* from the *de dicto* applications of the modifiers. If we use square brackets to indicate the scope of a modifier, then *de dicto* application of M to the sentence aF is written [aF]M, while the *de re* application is written a[F]M. (Notice that we have preserved the structure of English by placing our term a before our predicate letter.)

Those who believe in certain theories of modality may feel that there is no need to introduce the distinction between *de re* and *de dicto* applications of a modifier. Whatever the merits of this claim for necessity, it is dubious when we consider modifiers in general.

(11) The president was popular in 1972

On one interpretation, (11) claims that Carter was popular in 1972, while on another, the assertion is about Nixon. The difference in reading involves the scope of the modifier 'in 1972', so that on the first interpretation it modifies the predicate:

(12) The president [was popular] in 1972,

and on the second, the whole sentence:

(13) [The president was popular] in 1972.

So to disambiguate (11), we need to distinguish *de re* and *de dicto* applications of 'in 1972'. (9)

(9) THOMASON and STALNAKER [25] argue for a syntactical distinction between predicate and sentence adverbs, even when abstraction is present. They argue in note 27 that abstraction is inadequate for expressing certain formulas, given that predicate modifiers are translated away by $M[\lambda x \Phi x](t) = \lambda x M[\Phi x](t)$. But their *reductio* fails because this is not the correct translation. The correct one is $M[\lambda x \Phi x](t) = M[\Phi t]$. They also claim that predicate and sentence modifiers have different deductive behavior, in that a) sentence modifiers generate opaque contexts while predicate modifiers do not, and b) for predicate modifiers we want $aM[P] \supset aP$. As for a), the proper behavior is a simple result on our approach of differences in scope, and as for b) they give some counterexamples to their claim, to which we would add (3) and (4) of this paper. Another argument for the syntactic distinction is that the surface structure of English makes it. However, as we explained in section 3, we do not want to be bound by the surface structure of any one language.

Those familiar with the calculus of abstraction (or λ-conversion, or variable binding) might wonder why we don't let modifiers bind sentences only, and then use abstraction to produce modified predicates. For example, where we would write a[F]M, for the *de re* application of M, one could get the same effect with λx[xF]M(a). One problem with this is well known. The usual formulation of the principles of abstraction includes

$$\text{(Ab)}\ \lambda x\, \Phi x\, (a) \equiv \Phi a,$$

which has as a special case λx[xF]M(a) ≡ [aF]M, and this is fatal for our purposes because it rules that *de re* and *de dicto* applications of a modifier are equivalent.

One way out of this is to develop an intensional theory of abstraction that restricts (Ab) appropriately. Stalnaker and Thomason [24] present such a system, but it has its limitations. They propose the following modification of (Ab):

$$\text{(TS-Ab)}\ x = a \supset (\lambda y\, \Phi y\, (a) \equiv \Phi x).$$

In their language, variables are rigid designators, and under those conditions (TS-Ab) is valid. It is not valid, however, in logics that do not make that assumption. Because the variables are rigid, the domain of quantification in this system consists of those individual concepts which are constant functions. Thomason [25] argues that this is a good thing, since the constant functions mirror perfectly the substances. But this is a serious limitation for any language that handles tense, for there, substances are functions from times to time slices, and unless substances never change, they could not be represented by constant functions. (10)

Bressan [2] proposes another intensional theory of abstraction. He counts the *extension* of a predicate as a set of individual concepts, rather than a set of individuals with the result that (Ab) is valid. He draws the distinction between *de re* and *de dicto*, nevertheless, by adding machinery to distinguish different sorts of descriptions. To distinguish the two senses of (11) he might write:

$$\text{(12B)}\ M\ I_r x P x C$$

and

(10) See GARSON [9] for further details on this point.

(13B) M IxPxC.

Here M binds a sentence and the description I_rxPx has the president in the real world as its extension, while the extension of IxPx depends on the world in which it is evaluated.

Bressan does not treat the ambiguity of (11) as a matter of scope of the modifier, but as different kinds of occurrences of 'the president'. As a result, he requires us to resort to more complicated translation techniques from natural language. Furthermore, his device is not available for distinguishing *de re* from *de ricto* when a sentence contains no descriptions, but only primitive terms.([11]) This may be acceptable in a modal logic, where one might argue that all names are rigid designators. But we cannot assume this in a general theory of modifiers. To see why consider

(14) [Noon is one o'clock] on Eastern Standard Time

which is clearly false, and

(15) Noon [is one o'clock] on Eastern Standard Time,

which is true. 'Noon' clearly cannot be treated as a rigid designator. Of course one might claim that all this shows is that 'noon' hides an implicit description, but that would require translation theory which we have argued against.

Though we don't accept Bressan's theory, we are able to mirror his approach in our own. In our theory of modifiers, 'in 1972' obeys the principle: [aF] M ≡ [a] M [F] M, and so (13) is equivalent to

(16) [The president] in 1972 [was popular] in 1972.

So the *de dicto* formulation is equivalent to (16) which differs from the *de re* formultion (12) only in that 'the president' is modified in one and not in the other. We *can* claim that the difference depends on how we read the description if we like. The advantage of our theory, however, is that this emerges as a theorem within our theory, whereas for

([11]) It is true that one might *define* an r-subscript notation for terms in Bressan by something like $t_r \cong I_r x\, x = t$, where ≅ stands for identity of intension. But this involves us again in translation therapy. It is so much easier to introduce modification of terms directly.

Bressan the relationship is presupposed in translating from the natural language to his theory.

Even if a satisfactory intensional theory of abstraction were available, we would still argue against the widespread use of abstraction to display the syntactic form of natural languages as is done by Stalnaker and Thomason [24], Cresswell [4], and Montague [18]. Abstraction involves a cumbersome notation, and it is rarely used overtly in normal English. There is no excuse for using a device that is little used in natural languages as the main technique for analysing deep structure. It is true that English is capable of expressing abstraction, but if we want to keep as many parallels as possible between the syntax of English and the syntax of our formal theory, we will introduce abstraction only were it is absolutely necessary. It is interesting to note that any sentence containing abstraction can be adequately expressed in our syntax without it. (12)

We won't present the syntax for our system here because we will want to make adjustments to it in light of coming remarks. A final version appears in the Appendix.

5. *A starting point: semantics for predicate functors*

The semantics we will give for prepositions has a lot in common with a semantics which is often given for predicate functor languages. We will outline it to help us display the ways in which our approach differs. Such a semantics is built on the standard account (due to Carnap) of the intensions of terms predicates and sentences. We begin with a set of situations S, which we may imagine contains places, possible worlds, cases, coordinates, and the like, or sequences formed from these. Puzzlement about what situations are may be relieved in section 10. Given that the appropriate extension for a term is an object, for a predicate, a set of sequences of objects, and for a sentence, a truth value, the *intension* of a term, predicate or sentence is a function which takes each $s \in S$ into an appropriate extension. If δ

(12) For example $\lambda y[\lambda x[x F y] M(a)] M'(b)$ is expressed in our syntax by $[a[F]M]M' b$, while $\lambda x[xFy\ Ox\ Ow]M(a)$ is just a $[FyO]a[Ow]M$. In these and similar cases our notation has the advantage of simplicity, and straightforward display of which terms fall into the scopes of which modifiers.

is an expression, (i.e. a term, predicate, sentence, or modifier) then its intension on an interpretation u is written u (δ), and its extension at situation s is just the result of applying the function u (δ) to s, i.e. u (δ) (s). For convenience, we will often write this extension as u_s (δ).

So far we are clear about what the intension of a term, predicate or sentence ought to look like, but what about modifiers? In predicate functor semantics the role of a modifier, i.e. predicate functor, is to convert the intension u (P) of a predicate P into a new intension u ([P] M). This means that the intension of a predicate functor ought to be a function that takes predicate intensions into predicate intensions. To calculate the intension of a modified predicate [P] M, we just apply the function u (M) to the intension u (P). More formally:

(GM) u ([P] M) = u (M) (u(P)).

It would not be too difficult to modify (GM) to build a semantics for the syntax we have adopted. All we need to do is let u (M) be a function from the intension of any expression to a new intension for that sort of expression. So if δ is any expression, the modified truth clause would read:

(δM) u([δ] M) = u (M) (u (δ)).

Now a full-fledged semantics can be developed around (δ M), but the result is very weak. It is a generalization of the weakest neighborhood semantics,[13] and its only formal principles merely guarantee the substitution of provably equivalent or provably identical expressions.[14] Though we could define stronger systems by laying down

[13] The reader may not see the relationship between the semantics we have given and Scott-Montague, or neighborhood semantics. When the language contains a single modifier □ which modifies sentences only then (δ M) reads u (□ A) = u (□) (u (A)), where the modal operator □ is given an intension which is a function that takes sentence intensions into new sentence intensions. This semantics validates the same formulas as one where we introduce a relation R on S X $\mathscr{P}$(S) and define the extension of □ A by u_s(□ A) is 1 iff Rs |A|, where $|A| = \{s' : u_{s'}(A) = 1\}$, and this is a standard way of formulating neighborhood semantics.

[14] It may not be clear what the equivalence of two expressions comes to when they are predicates or modifiers. We say predicates P, Q are provably equivalent when $\vdash$ Ps ≡ Qs, for every string s of terms (of appropriate length), and modifiers M, M′ are provably equivalent when $\vdash$ [A] M ≡ [A] M′ for each wff A. Later, we will allow ≡ to bind predicates directly, in which case P and Q are provably equivalent when $\vdash$ P ≡ Q.

conditions on the intensions of our modifiers, it is difficult to do so in a way that makes strong contact with our intuitions about our use of prepositions.

So we will be adopting a different semantical clause for the modifiers, though within the same Carnapian framework. Our strategy will be to investigate those occurrences of prepositional phrases where our intuitions seem to be fairly straightforward, namely when they are used to indicate place and time.

6. *Topological semantics*

The semantics we will describe in this section is designed primarily for locative prepositions, i.e. those cases where the preposition indicates spatio-temporal location. I believe that non-locative uses of prepositions in English are «metaphorical» and that their semantics can be given in a way which mirrors the semantics for the locative uses. Prepositions which are most often used in non-locative ways, such as 'to' and 'by' have important locative uses as well, and even 'for', which if I am right, has lost a former locative use, reflects its «heritage» in the prefix of such words as 'forearm' and 'foresight', and has several uses, such as expression of purpose or goal which bear obvious analogies to a locative use. ([15]) Of course the success of our system for handling non-locative uses depends on linguistic and logical, not etymological evidence. Nevertheless, it is a good guess that if the locative uses were primary, the non-locative ones could be analysed using similar methods.

So let us begin by thinking of our set S of situations as a set of locations in space-time. The main idea is that each prepositional phrase picks out a location in S which the phrase describes. So for example, 'on Main Street' and 'under the chair' refer to locations in space. If, for simplicity, we assume that the extension of a term is also a location, so that objects are identified with their positions in space-time, we discover that a preposition ought to refer to a function from S into S. If the identification of objects with their locations is not

([15]) It turns out that 'off' and 'of' are derived from the same Anglo-Saxon preposition, which meant (roughly) away from. This meaning is preserved in such phrases as 'east of Eden', and '5 miles off the coast'.

acceptable, we can always add a location function that maps each object into its position. However, that does not make any formal difference, and so we will omit it for simplicity.

Given this decision about the referent of a prepositional phrase, the extension of a modified expression δ can be given by

$$(\mathrm{TM})\ u_s([\delta]M) = u_{u_s(M)}(\delta)$$

which says, for example, that '[walks] on Main Street' has as its extension (at point s) the set of things which satisfy the predicate 'walks' at the point refered to by 'on Main Street'. We have assumed that the extension of M depends on a point in S. One obvious reason for this is that a prepositional phrase can contain a term n whose extension depends on s, for example 'at noon', and so the extension of the whole phrase must depend on s as well. But there is another reason. Some prepositions have extensions that depend on the point at which they are evaluated. We have already mentioned 'since'. The temporal location picked out by 'since 1942' depends on the time at which it is evaluated, and the same goes for phrases containing 'until'. This means that the extension $u_s(O)$ of a preposition O ought to be a function from S into S, and so the intension for a proposition ought to be a function that takes each $s \in S$ into a function from S into S. The extension of a prepositional phrase On would then be calculated by:

$$(\mathrm{On})\ u_s(On) = u_s(O)(u_s(n)).$$

A complete definition for a semantics of this kind is given in the appendix. Apart from the fact that we will want to allow extension gaps, the truth clauses for expressions of the language other than modifiers are fairly standard.

This semantics can be axiomatized by adopting and generalizing the rules and axioms of topological logic. ([16]) In this sort of system, the modifier Tn is introduced to modify sentences, so that Tn A is read 'it is the case at (or as of) n that A'. The only difference between topological logic and prepositional logic is that the latter has many modifiers, not just one for 'at n', and we allow them to bind all expressions. A system which is adequate for the semantics of this section is given in the appendix. Its salient feature is that modifiers

([16]) See GARSON [10], [12] and [13] for more details on these systems.

distribute through all connectives, the quantifer, and even into the terms and predicates of sentences. We have already noted the last property in section 4, namely that [nF] M is equivalent to [n] M [F] M.

7. Cases

In the previous section we concentrated on locative uses of prepositions. But prepositions like 'for' and 'of' rarely, if ever, take locative uses, and even phrases which appear to have locative uses out of context, such as 'at the corner', do not have locative uses in some situations. For example, this phrase indicates direction, not location, in one reading of:

(17) He stared at the corner.

Similarly 'by the canyon' in

(18) He was trapped by the canyon

indicates agent on one reading. Almost all prepositions, including those with clearly defined locative uses, may be used to indicate one or more of the cases, such as direction, goal, agent, object, manner, instrument, possession, etc., which are indicated by inflexions in other languages. Some prepositions have taken on the job of indicating one or two cases, for example 'of' commonly indicates possession, and 'with' commonly indicates instrument or accompaniment. Since prepositions in English play the same role as inflexions in other languages we should hope that an analysis of prepositions would include a semantical analysis of case inflexions.

The topological semantics we presented in section 6 will handle cases with only minor revision. So far, we have suggested that our situations are positions in space-time, so that on a finer analysis, we might take them to be pairs $<p, t>$ composed of a spatial position p and a time t. Where at first it seemed that we were handling the case of «scientific» position, we may also take it that topological semantics is a simultaneous analysis of two cases: (spatial) location, and time. But if we make such a division, won't we have to modify the semantical clause for modification (TM)? If 'at noon' expresses time, then we would want its extension to be a *coordinate* of a situation, rather than the entire situation, and we will want the semantical clause (TM) rewritten so that:

(19) $u_{<p,t>}([P]\text{ 'at noon'}) = u_{<p,\, u_{<p,t>}(\text{'at noon'})>}(P) = u_{<p,\,\text{noon}>}(P)$

that is, 'It is raining at noon' should be true at situation $<p, t>$ just in case 'It is raining' is true at position p at noon.

But we can get the same effect without altering (TM). We simply let $u_{<p,t>}$('at noon') be the *situation* $<p, \text{noon}>$, in which case the standard clause will do:

(20) $u_{<p,t>}([P]\text{ 'at noon'}) = u_{u_{<p,t>}(\text{'at noon'})}(P) = u_{<p,\,\text{noon}>}(P)$

Although we count the intension u ('at noon') as a function from situations to *situations*, the fact that it modifies along the time coordinate will be reflected by the fact that u (at noon) is the identity function with respect to the coordinate p, i.e. when we apply u ('at noon') to $<p, t>$, the position coordinate of the result is still p. So the fact that a preposition modifies only along a given coordinate can be insured by placing the appropriate condition on its extension. The condition may force an axiom in the axiomatic system which is adequate. For example when two such modifiers have constant intensions along their respective coordinates, the modifiers may be permuted. Nevertheless such systems will be extensions of topological logic, and so it provides a good foundation for case modifiers.

To handle cases other than location and time, all we need to do is to expand our situations to include new coordinates. For example, we might include a coordinate for the instrumental case so that

$u_{<c,i>}$('John hit Mary with the brick') is T iff
$u_{u_{<c,i>}(\text{'with the brick'})}$('John hit Mary') is T iff
$u_{<c,\,\text{the brick}>}$('John hit Mary') is T,

and so on for other cases.

In cases like manner, for example, it may not be clear what sort of entity goes into the coordinate. That may be because the case (say manner) is a wastebasket category including many dimensions (such as rate, 'at 5 m.p.h., mental state, 'in pain', etc.) which are at least somewhat more concrete. We needn't be bound by any traditional notion of case in introducing dimensions into our situations, and so we needn't hesitate to use this analysis on uses of prepositional phrases that don't seem to belong to any case commonly recognised by grammarians, for example, 'to the music' in 'dance to the music' or 'with age' in 'grow wise with age'.

8. *Terms as modifiers*

Agent and object are indicated by word order around verbs in English, so we might expect to be able to treat the application of terms to predicates as the application of modifiers. Fillmore [6] argues that noun phrases in English are best thought of as prepositional phrases in deep structure, and that the prepositions (usually for agent and object) are deleted during passage to the surface structure, and indicated instead by word order around verbs. Given that we have coordinates for the other cases, it seems natural to add dimensions for agent and object in our situations. The result is that we can unify the account of the extension of a predicate, for when agent and object are supplied by the situation, we can let the extension of a predicate be a truth value. As a result, we will need to make no distinctions between predicates of differing numbers of places in the syntax, and we may allow the logical connectives to bind predicates as well as sentences. (17) An atomic sentence nF can be considered to be the result of modifying the predicate F with a (prefix) modifier for the agent coordinate. (18)

Passive constructions can be understood in this scheme quite simply. 'John was hit with a brick' can be seen as the result of modifying 'hit' by phrases in the object and instrumental cases, leaving the agent to be supplied by the situation. The passive form in the surface structure of this sentence can then be derived by transformation rules characteristic of English.

This approach may also throw light on pronouns. Nothing in our semantics prohibits a modifier from modifying along more than one

(17) In effect our logic is anadic. (See [14].) Our technique here is similar to the definition of satisfaction of open sentences which employs sequences of objects. We are using virtually the same method for all sentences.

(18) Notice that since a term n, and a prepositional phrase On have exactly the same kind of intension, (namely a function from S into S) there is no need to symbolise the preposition explicitly when we treat nF as a predicate bound by a modifier. Terms can come with their cases built in exactly as they do in inflected languages.

dimension. By adjusting the preposition, we can get a modifier that modifies in more than one dimension. One way, then of treating the pronoun in

(21) John wants Sally to go with him

is to think of the phrase 'with him' as determining the preposition implicit in 'John' so that this term modifies in both subject and comitative cases. The same maneuver can be used to cross reference variables. Though we do not present a theory which handles pronouns and passive constructions in the appendix, the approach shows considerable promise.

The theory that terms are really just modifiers has interesting consequences when we consider general terms. It is clear that

(22) Bill saw «Jaws» somewhere

follows from

(23) Bill saw «Jaws» in New York

by some form of existential generalization. If we treat 'somewhere' as short for in 'in some place', the structure of the argument is made clear. So 'somewhere' in (22) is a general term which modifies in the locative case. The 'where' in 'somewhere' not only indicates the sort, or domain of discourse of the quantifier, but it determines the case as well, and so partly plays the role of a preposition.

We can already treat sentences like (22) in our theory if we are willing to engage in a little translation, for (22) would become

(24) Bill saw «Jaws» in some place,

which has the form

(25) $\exists x\,(xP\,\&\,[bSj]\,Ox)$.

It would be more in the spirit of this paper, however, if we had a way to represent the syntactic form of (24) so that «in some place» is formalized as a unit.

One simple way of doing that is to introduce the notation $O\,\exists\,P$ so that $[Q]\,O\,\exists\,P$ abbreviates $\exists x\,(xP\,\&\,[Q]\,Ox)$. We may then investigate the logical behavior of the «modifier» $O\,\exists\,P$.

Despite the outward similarity of 'in some place' and 'in New

York', these phrases behave differently. The second distributes through all the connectives, while the former behaves like the weak modal operator (◇) in systems as strong as K. The kinship with ◇ is immediately apparent when we give ◇ its standard translation into possible worlds talk: 'in some possible world'. □ and ◇ then, can be treated in our system as prepositional phrases $O\,\forall\,P$ and $O\,\exists\,P$ containing embedded general terms. In fact, our theory of modality will automatically deal with the *de re* applications since our modifiers bind predicates.

The recognition that modifiers with embedded general terms behave as strong and weak, rather than as «topological» modifiers will help us resolve a potential problem. Consider this pair of sentences:

(26) John whistled throughout the concert

(27) John whistled during the concert.

Our intuitions rule that the interval picked out by 'throughout the concert' is the same as the interval picked out by 'during the concert', namely the period between its beginning and end. So (26) and (27) should have the same truth conditions, and they obviously do not. But if we assume that 'throughout the concert' is a strong, and 'during the concert' the corresponding weak modifier, then the semantical behavior of (26) and (27) is captured correctly, right down to the way the phrases distribute through conjunction and disjunction. ([19])

Many prepositional phrases, primarily those used to indicate position and time, turn out to exhibit the behavior of strong and weak modifiers. Part of the reason may be that we use singular terms which

([19]) Although the evidence is fairly good on the behaviour of T ('throughout the concert') and D ('during the concert') when we look at behavior with respect to & and v, it is ambiguous with respect to ~. For example, if T is a strong and D the corresponding weak operator, then we should have $T\sim A \equiv \sim DA$, and $D\sim A \equiv \sim TA$. It is hard to test for these equivalences because English is poor at indicating scope of negation. It is very difficult, for example to find any English sentence corresponding to $D\sim A$, 'During the concert, John had a bout of non-whistling' is about as close as I can get. Even 'During the concert John failed to whistle', has the truth conditions of $\sim DA$. It seems then, that omission the surface structure of English has rejected the forms $D\sim A$ (and to some degree $T\sim A$), so that the choice between phrases D and T helps disambiguate between 'John failed to whistle at every concert-time,' and 'John failed to whistle at some concert-time.'

ordinarily refer to events or things in an oblique way to pick out sets of times or locations. For example, 'Pittsburgh' in 'Pittsburgh is where my grandfather was born' refers to a region rather than a city. So while a term may be singular in its ordinary use, it has the effect of picking out the sort and case in a prepositional phrase that contains a general term. When this happens we can always rephrase the modifier to make the general term appear. For example, for 'throughout the concert' we might read 'at all concert-times'. (20)

The trouble with this is that we are forced to resort to the kind of translation therapy we have criticised in this paper. A fully adequate theory would introduce the strong and weak modifiers directly, and characterise their intensions, and those of general terms separately in the semantics. This can be done within the very general semantics used by Cresswell [4] and Lewis [16], although intensions for the modifiers have to be carefully chosen so that they behave as they ought to. However, the simple system we have given provides a framework in terms of which it is easy to determine what that semantical behavior should look like.

9. *Degrees of Indefiniteness*

We still have not given an account of the meaning relations between predicates P, [P]M of different degrees of definiteness, for example between 'runs' and 'runs at midnight', or 'stares' and 'stares at Sally'. In a way, this problem is partly solved at the formal level in Garson [13], at least for sentence modifiers. For example, it follows from what was shown there that

(28) $\exists x (P \equiv [P] Ox)$

corresponds to the following condition on the intension of O

(20) The fact that our language handles strong and weak modifiers may help us over a problem for Filmore's theory of case. As MELLEMA [17] points out, there are semantic differences between 'John smeared paint on the wall' and 'John smeared the wall with paint' in that the latter implies the wall was fairly well covered while the first does not. We may claim that 'on the wall' in the first, and 'the wall' in the second are «in the same case», but differ in interpretation because of an implicit quantifier, which is selected (in part) by features in the surface structure.

(29) $u_s(On) = s$ for some $n \in N$ for all $s \in S$.

Condition (29) has an odd ring because we have adopted the substitution interpretation of the quantifier. But it seems perfectly acceptable, at least for prepositions that modify along a given coordinate. If O «picks out» (say) the object coordinate and situations are triples *aoi* of agent *a*, object *o*, and instrument *i*, then for a given situation *aoi*, we should be able to find a term n such that On picks out *o*, as long as we have terms for each of the objects. Then u_{aoi} (On) = *aoi*, and (28) will be satisfied. It seems, then, that (29) should be accepted merely on grounds that the domain of quantification (in our case the set of all objects) should match the set of all things named. ([21])

But there is a problem with accepting (29), for (28) entails $P \supset \exists x\,[P]\,Ox$ and that means we must rule all arguments of the form:

$$(30)\quad \frac{P}{\exists x\,[P]\,Ox}$$

valid. Yet we know of invalid arguments that have this form

(31) $\dfrac{\text{John hides}}{\text{John hides something}}$

(32) $\dfrac{\text{Superman flew}}{\text{Superman flew in something}}$

(33) $\dfrac{\text{Sam stares}}{\text{Sam stares at something}}$

Perhaps we can handle (31) by claiming that it is indeed valid, for if John hides, he must hide himself, and so he hides something. Then we might say that we fail to see the validity of (31) for the same reason we fail to see the inconsistency of 'John is bigger than everyone', namely that we mean to say 'everyone else' but leave 'else' understood. I'm not sure that is the best way to deal with (31), but if it works, it raises a new issue: how do we insure that 'John hides' is equivalent to 'John hides himself'. More generally, how do we handle other verbs such as 'washes', 'bathes', and 'shaves' which behave in the same way? One

([21]) See [13] for more information on such clauses as (28).

answer is that the term 'John' in 'John shaves' is a modifier along both agent and object coordinates, and that this is masked in the surface structure because the preposition has been deleted.([22]) If this answer is right, we must be careful to employ it only where appropriate, for it is clear that 'everyone' in 'Everyone shaves someone' does not modify in both coordinates when this sentence is read 'Everyone shaves someone or other.' In that case, it has the form 'Everyone [shaves someone]', and if 'everyone' modified in both agent and object cases, this would be equivalent to 'Everyone shaves himself.'

The difficulties with (32) and (33) however, resist any such maneuvers. One tactic for dealing with (32) is to claim that we simply have two different verbs here, the transitive and intransitive forms of 'flies'. Then we could claim that (32) involves equivocation of 'flew'. We would then use two separate letters F_I and F_T for these forms, and give their intensions separately. This raises an issue that may have been bothering the reader since section 7. What are we to do when we are asked to give the extension of a verb (say 'sleeps') at a situation which includes an irrelevant coordinate (say the instrumental coordinate)? We really have but one choice. We simply ignore that coordinate, and consider only those which are relevent. For if the extension of 'sleeps' depends on the value of the instrument coordinate, then this coordinate wouldn't be irrelevant. So when it comes to giving the intension for F_I (the intransitive form of 'flies') we will assign an extension ignoring the object coordinate. That means we will have to validate

(34) $nF_I \equiv [nF_I]On'$

for no matter what the value of the term n', it shouldn't affect the extension of F_I, at least when the prepositional phrase On' modifies along the object coordinate. But from (34) it follows that $nF_I \supset \exists x[nF_I]Ox$, and so we must rule (32) valid, even if 'flew' is understood in its intransitive form. The tactic of distinguishing different versions of 'flies' won't work. Even if it did, we would undercut one of the advantages of our whole approach, namely to allow the intension of a modified predicate to depend on the intension

([22]) The mandatory pronoun in the French 'Jean se lave' might help support this sort of analysis.

of the predicate and the intension of the modifier, instead of inventing a brand new intension for the modified predicate. For instance, if we tried to avoid the validity of (33) by distinguishing different senses of 'stares', we would still have to avoid the validity of

$$(35)\ \frac{\text{Sam stares}}{\text{Sam stares through something}}$$

by making a further distinction, and so on with other, prepositional phrases such as 'into something', 'next to something', etc... We would end up with exactly the implausible features that plagued Schwartz' theory.

Luckily, there is an interesting alternative. It is to allow our coordinates to take null values, and to allow the extensions of predicates to remain undefined at certain coordinates. Let us suppose that Superman flies, that he also flies an X-15, but not a DC-10 and he does not sleep. Then in the situation s-, where Superman is the agent and nothing is the object, the extension of 'flies' is 1. In the situation sx, where x is an X-15, its extension is also 1, while it is 0 at the situation sd, where d is a DC-10. The extension of 'sleeps' at s- is 0, but we leave its extension at sx and sd undefined, since it is senseless to give an extension for 'sleeps' in a situation where an object is present. By allowing the null value, –,in coordinates we may record the intensions of the transitive and intransitive forms of 'flies' in one intension; by leaving the extension of 'sleeps' undefined for sx and sd, we will avoid the problem we had with F_I.

Since we have allowed the extensions of predicates to be undefined, we have to adjust the definitions of 'true on an intrepretation' and 'validity' in the appropriate way. To make a long story short, a sentence is true on u just in case every full interpretation that agrees with u for whatever extensions are defined on u gives the sentence the value 1. If we adjust the extensions of each prepositional phrases On so that it never picks out the null coordinate –, we will get plausible results on the interpretation we outlined in the previous paragraph. Most importantly, 'Superman sleeps ≡ Superman [sleeps] an X-15' comes out false. Furthermore, when we adjust the interpretation so that Superman does not fly an X-15, or anything else, it turns out that $sF \supset \exists xs\,[F]\,x$ is false, and so we are not forced to claim the validity of (32).

Not only that, this tactic handles arguments of the forms (30) which we would rule valid, for example:

(36) $\dfrac{\text{John runs}}{\text{John runs at some time}}$

and (37) $\dfrac{\text{John throws}}{\text{John throws something}}$

In (36), it turns out that when we evaluate 'runs' at a situation with a null time coordinate, we leave its extension undefined, and similarly for 'throws'. If this is written into the semantics as conditions on the possible interpretations of 'runs' and 'throws', it turns out that both arguments are ruled valid.

This is only a beginning of a thorough account of the relationships between predicates of different degrees of definiteness. Much more has to be done, particularly when it comes to the interaction between modifiers. I hope this paper provides the foundation to make this task worth trying.

10. What are situations?

We have been purposely vague about what a situation is in this paper. In effect, we have left indefinite the number and kinds of indefiniteness to be handled by the semantics. For example, even in our locative semantics, we did not say whether situations were points or intervals or volumes of space-time. Some may think this undercuts our entire approach. If we cannot give a complete account of what the dimensions of a situation are, then the central concept of the semantics is vague and unsatisfying. It may seem that all I have done is to spirit away the problem of determining the «real» form of the predicates (i.e. their number of places in an «exact» language) into the semantics, hiding it in the notion of a situation.

But I argued, or at least strongly hinted, in section 2 that we cannot give a definitive account of the dimensions of indefiniteness, and I don't believe there is any reason to give one even if it were possible. That would be necessary if the metalanguage used to present the semantics had to be freed of indefiniteness. But there is no reason to take a position with respect to the metalanguage that differs from the account given for the object language; in fact loyalty to our account demands otherwise. ([23])

If we allow indefiniteness in the metalanguage, then 'situation' can itself be one of those terms whose interpretation depends on the situation in which it is used. Semantics offers us a framework with which to understand the meaning relationships in a language. When we apply it to a given problem concerning those meaning relationships, we may pick out those dimensions of indefiniteness that are relevant. In a discussion of time and determinism, for example, we may get away with having to consider only dimensions for possible worlds and times, because other dimensions are not fixed by the portion of the language we are considering, and so become «parameters» of our discussion, i.e. elements which would never change anyway, and so are not worth making explicit. We do not need, or want, to pack all imaginable dimensions into the situation, nor do we need, or want, an exhaustive list of them in order to do so. If two dimensions will suffice for a given purpose, then by all means let the situations be pairs. If semantics is a practical tool which helps us make sense of the meaning relations in a given argument, or even philosophical position, then we can be expected to come to the situation armed with ideas about what kinds of indefiniteness have to be dealt with. That will help us fix the meaning of 'situation' during that episode of using the semantics.

Some may want to look at semantics in a more abstract way, as an account of the «connection» between language and the world. They may feel prompted to find a definitive account of all the possible styles of indefiniteness in order to give a thorough account of all «slippages» between them. Yet it very well may be that the correct account abstracts properly by eschewing any attempt at giving a detail account

([23]) The same issue arises in the semantics for many-valued logic. Rescher [21] points out that one who heartily endorses many-valued logic, does not need to give it a semantics in a two valued language.

([24]) We might want to add another set for two place prepositions, such as 'between'. It may be better to analyse 'between John and Mary' as a single place preposition filled with a complex term 'John and Mary'. Phrases such as 'between the sheets' provide support for that analysis. We leave a full fledged theory of complex terms to another paper.

of the nature of a situation, just as a properly abstract theory of geometry does not attempt to establish any one axiom for parallels. Ordinary language has a habit of leaving a lot «understood». I don't think there is any reason to believe that there is any complete theory of what could be implicit in what is explicit. If there is one, it probably tells us 'Just about anything'. So it is only proper that this is the answer we give to the question 'What is a situation?'.

Appendix

We will give here the syntax, semantics and proof theory for our basic system of prepositional logic.

Syntax

We will assume that atomic sentences are built up out of predicates and modifiers as we explained in section 8. This means that predicates and sentences fall into a single syntactic category, and are anadic in the sense that we do not distinguish them according to the number of their term places. This has the advantage that we may use a single symbol for transitive and intransitive forms of the same verb. The result is that logical operators bind predicates as well as sentences.

A syntax for prepositional logic is built from a set of logical operators: $\forall$, $\supset$, $\sim$, $=$ for first order logic with identity, sets C and X of constants and variables, a set *Pred* of (anadic) predicate letters, a set *Ad* of primitive modifiers (which may include symbols for «topological» adjectives and adverbs), and a set *Prep* of prepositions.

Sets of modifiers **M**, terms **N**, and predicates **P** are defined by simultaneous recursion as the smallest sets satisfying the following conditions: $Ad \subset \mathbf{M}$; $\mathrm{On} \in \mathbf{M}$; $[\mathrm{M}]\mathrm{M}' \in \mathbf{M}$; $Pred \subset \mathrm{P}$; $\mathrm{n} = \mathrm{n}'$, $\sim\mathrm{P}$, $(\mathrm{P} \supset \mathrm{Q})$, $\forall\mathrm{xP}$, and $[\mathrm{P}]\mathrm{M} \in \mathbf{P}$; $X \cup C \subset \mathbf{N}$; $[\mathrm{n}]\mathrm{M} \in \mathbf{N}$; where $\mathrm{O} \in Prep$; $\mathrm{n}, \mathrm{n}' \in \mathbf{N}$; $\mathrm{M}, \mathrm{M}' \in \mathbf{M}$; $\mathrm{P}, \mathrm{Q} \in \mathbf{P}$ and $\mathrm{x} \in X$.

We could have introduced both prefix and suffix modifiers into this syntax to keep closer contact with the structure of natural languages. This complicates the statement of the semantics and proof theory in an annoying way, so we will deal only with suffix modifiers for simplicity.

Semantics

In this semantics we will allow the extensions of expressions to be undefined at certain situations. We explained the reasons for wanting to do this in sections 3 and 9. It is worth noting, however, that it would not change the final definition of satisfaction if we had insisted that every expression has an extension at every situation. We will use the substitution interpretation of the quantifier in order to insure that our system is axiomatizable. The standard interpretation may be used, but only when proper care is taken. For more on what is required see Garson[14].

A model U is a pair <S, u> consisting of a non-empty set of situations such that u is a function that assigns intensions to members of sets $C, X, Pred, Ad$, and $Prep$ in the following way. Let MI be the set of partial functions from S into S. Let OI be the set of all partial functions from MI into MI. Let PI be the set of all partial functions from S into $\{1, 0\}$. Then if $\delta \in C \cup X \cup Ad$, then $u(\delta) \in MI$, and if $\delta \in Pred$ then $u(\delta) \in PI$, and if $\delta \in Prep$, then $u(\delta) \in OI$.

A partial interpretation v for a model <S, u> is a function which agrees with the intensions that are given to the primitive expressions by u, and extends the definition of the intension to expressions in sets **P**, **M**, and **N** according to the following definition:

1. $u(\delta) \in PI$ for $\delta \in \mathbf{P}$
2. If either $u(\delta)$ or $u(\gamma)$ is not defined at s, then neither are $u((\delta \supset \gamma))$, $u(\forall x \delta)$, $u([\delta]\gamma)$, or $u(\sim\delta)$.
3. If $u(n)$ and $u(n')$ are not defined at s, then neither is $u(n = n')$.
4. If one of $u(n)$, $u(n')$ is defined at s and the other not, then $u(n = n')$ is defined at s so that $u_s(n = n')$ is 0.
5. When $u(n)$, $u(n')$, $u(P)$, $u(Q)$, $u(O)$, $u(M)$ and $u(\delta)$ are all defined at s then the following hold:
   a. $u_s(n = n')$ is 1 iff $u_s(n)$ is $u_s(n')$
   b. $u_s(\sim P)$ is 1 iff $u_s(P)$ is 0.
   c. $u_s((P \supset Q))$ is 1 iff $u_s(P)$ is 0 or $u_s(Q)$ is 1
   d. $u_s(\forall x P)$ is 1 iff $u_s(P^n / x)$ is 1 for all $n \in N$
   e. $u_s([\delta]M)$ is $u_{u_s(M)}(\delta)$ or $u([\delta]M)$ is undefined at s
   f. $u(On)$ is $u(O)(u(n))$ or $u(On)$ is undefined at s

A full interpretation for a partial interpretation V of a model <S, u> is a function w that gives each expression δ an appropriate intension such that $w(\delta)$ is defined at each $s \in S$ for each expression δ, and $w(\delta)$ agrees with $v(\delta)$ wherever $v(\delta)$ is defined.

A set G of predicates is satisfiable just in case for some syntax that includes the expressions in G, there is a model <S, u> and an $s \in S$ such that for every full interpretation w for the partial interpretation v of u, each $P \in G$ is such that $w_s(P)$ is 1.

Semantical concepts such as validity and semantical entailment are defined from satisfaction in the usual way.

Proof Theory

The system which is adequate with respect to the preceding semantics results from adding the following principles to first order logic with identity

(~) $[{\sim} P]M \equiv [P]M$

($\supset$) $[(P \supset Q)]M \equiv ([P]M \supset [Q]M)$

($\forall$) $[\forall x P]M \equiv \forall x [P]M$, where M does not contain x

([P]) $[[P]M]M' \equiv [P][M]M'$

([,]) $[[n]M]M' = [n][M]M'$

(=) $(n = n' \supset (A \supset \bar{A}^{n'}/n))$, where $A^{n'}/n$ is the result of replacing n′ properly for one or more occurrences of n in A which do not lie in the scope of any modifier.

(R[]) If P is a theorem, then so is $[P]M$

(R =) If $(P \supset ([Q]On \equiv [Q]On'))$ is a theorem, and Q does not appear in P, then $(P \supset n = n')$ is a theorem.

This is by no means the most efficient way to axiomtize the system, but it displays its salient features. The completeness proof is given in [10].

University of Illinois
at Chicago Circle

James W. GARSON

REFERENCES

[1] BELNAP, N, and DUNN, M., «The substitution Interpretation of the Quantifiers», *Nous* 2 (1968), pp. 177-185.

[2] BRESSAN, A., *A General Interpreted Modal Calculus*, Yale U. Press, 1972, Section N55.

[3] CLARKE, R., «Concerning the Logic of Predicate Modifiers», *Nous* 4 (1970), pp. 309-335.

[4] CRESSWELL, M.J., *Logics and Languages*, Methuen and Company, London, 1973.

[5] DAVIDSON, D., «Theories of Meaning and Learnable Languages» in *Proceedings of the 1964 International Congress for Logic, Methodology and The Philosophy of Science*, Bar-Hillel, Y. (ed.), North Holland, Amsterdam, pp. 383-394.

[6] FILLMORE, C. «Toward a Modern Theory of Case» in *Modern Studies in English*, REIBEL, D. and SCHANE, S. (eds.), Prentice Hall, Englewood Cliffs, 1968, pp. 361-375.

[7] GABBAY, D., *Investigations in Modal and Tense Logics*, D. Reidel, Dordrecht, Holland, 1976, p. 175.

[8] GARSON, J. «Completeness of Generalized Scott-Montague Systems» (ditto).

[9] —— «Completeness of Some Quantified Modal Logics», forthcoming in *Logique et Analyse*.

[10] —— «De Re Topological Logic», forthcoming.

[11] —— «Free Topological Logic», (ditto).

[12] —— «Indefinite Topological Logic», *Journal of Philosophical Logic* 2 (1973), pp. 102-118.

[13] —— «The Substitution Interpretation in Topological Logic», *Journal of Philosophical Logic* 3 (1974), pp. 109-132.

[14] —— «The Substitution Interpretation and the Expressive Power of Intensional Logics», forthcoming in the *Notre Dame Journal of Formal Logic*.

[15] GRANDY, R., «Anadic Logic» *Synthese*, (1976), pp. 395-402.

[16] LEWIS, D., «General Semantics», in *Semantics of Natural Languages*, DAVIDSON, D., and HARMAN, H., (eds.), D. Reidel, Dordrecht, Holland, 1972, pp. 169-218.

[17] MELLEMA, P., «A Brief Against Case Grammar», *Foundations of Language* 11 (1974), p. 50.

[18] MONTAGUE, R., *Formal Philosophy*, THOMASON, R. (ed.), Yale University Press, 1974, Chapters 6 and 8.

[19] PARSONS, T., «Some Problems Concerning the Logic of Grammatical Modifiers», *Synthese* 21 (1970), pp. 320-334.

[20] REICHENBACH, H., *Elements of Symbolic Logic*, McMillan and Company, New York, 1947, pp. 284-286.

[21] RESCHER, N., *Many Valued Logic*, McGraw Hill, New York, 1969, Ch. 3, section 7.

[22] SCHWARTZ, T., «The Logic of Modifiers», *Journal of Philosophical Logic* 4 (1975), pp. 361-380.

[23] STALNAKER, R., «Pragmatics» in *Semantics of Natural Language*, D. DAVIDSON & G. HARMAN (eds) Reidel, 1972 pp. 380-397.

[24] STALNAKER, R. and THOMASON, T., «Abstraction in First-Order Modal Logic», *Theoria* 3 (1968), pp. 203-207.

[25] THOMASON, R., «Modal Logic and Metaphysics» in *The Logical Way of Doing Things*, Yale University Press, 1969, pp. 137-138.

[26] THOMASON, R., and STALNAKER, R., «A Semantic Theory of Adverbs», *Linguistic Inquiry* 4 (1973), pp. 195-220.

[27] WHEELER, S., «Attributives and their Modifiers», *Nous* 6 (1972), pp. 310-334.