A SEMANTICS FOR A DIALECTICAL LOGIC

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During the last forty years, some authors have been interested in the (possible) relations between logic and dialectics, either trying some formalization of dialectics or, at least, discussing some relevant topics about this problem.

I will quote only some of these authors: Apostel [1,2], Allinson [3], Asenjo [4,5], Beth [6], Butler [7], Colletti [8], Dalla Chiara [9], Dubarle [10], Findlay [11], Gauthier [12], Gorren [13], Guccione [14], Günter [15], Havemann [16], Jaskowski [17], Kosok [18], Mc Gill [19], Parry [20], Narski [21], Patzig [22], Popper [23], Rogowski [24, 25], Thomason [26], Von Wright [27], Zinov'ev [28].

First of all, I think that if you face the problem of building some dialectical logic, you can't avoid the question regarding the aims of such an enterprise.

If you don't merely want to refuse the so-called classical logic and replace it with a dialectic one, basing yourself on some metaphysical a-priori, you will immediately face the so-called problem of *the plurality of logics* [29] and the task of overcoming the traditional equivalence: rational reasoning = reasoning in terms of classical logic[14].

It is my opinion that the most correct procedure starts from the acceptance of a standpoint coherent with the plurality of logics.

Moreover this point of view is very close to the other one, namely that logic is empirical [30] and different kinds of experience suggest different logics [31,32].

Then, which kind of experience could suggest a dialectical logic?

You would answer – following Marx – «the experience of social history», or – following, in a sense, Popper – «the experience of the activity of scientific research». In any case, you can undoubtedly assert that the central point of any – possible – *construction* of a dialectical logic are three.

a) – A dialectical logic should be able to express formally notions such as those of *process, movement, change*. (Classical logic

doesn't seem to express these notions([1])).

b) – The contradiction should be a central element of a dialectical logic.

c) – Relations between negation and contradiction.

Moreover, it is clear that the three points stressed so far are closely intertwined.

Now, as regards contradiction, two seem to be the ways to avoid its well-known, awful consequences.

The first way consists in changing the notion of contradiction used by dialectical philosophers, or, more properly, in trying an explication of a precise notion of contradiction – different from that one used in classical logic – which is able to grasp and maintain the most part of what dialectical philosophers want to express (for instance, some kind of relation between contradiction and change).

A second way might consist in an attempt to express the notion of contradiction in a logical structure different from classical logic.

These two different approaches are not to be seen – of course – as mutually exclusive. On the contrary it might be likely to suppose that a modified notion of contradiction still maintaining the fundamental connotations given by dialectical philosophers, suggests a proper logical structure – different from classical logic – philosophically meaningful and rich of possible inferences. Only then we might introduce – in relation to other logics, among which the classical logic – problems of *inclusion* and *translation*. Hereafter, I will try to supply some sufficient conditions to set up the core of a formal dialectical semantics, giving an account for both the quoted approaches. A philosophical interpretation will be supplied later on.

At first, I give an informal justification of such conditions. Let p be any proposition: you write 'Ap' for 'p is in the state A' and 'Pp' for 'p is in the state P'.

We want a semantics S_D making valid the following statement:

'If Ap holds then P (p and not-p) holds'

which says that some weak form of contradiction (contradiction in the

([1]) *Classical logic* is meant the traditional mathematical logic and not every kind of formal logic: in fact nowadays you see many attempts to express the above mentioned notions by the so-called *tense logics*.

state P) is seen as a *necessary* condition for a change (transition of p from the state P into the state A). First of all we must *inject* the dialectical aspects closely bound up with contradiction.

We will carry out it, stating that:

I) in some state of affairs M_i, Pp holds if and only if in the same state of affairs P (not-p) holds.

II) in some state of affairs M_i, P (p and q) holds if and only if in the same state of affairs both Pp and Pq hold.

Furthermore, we must reflect the change, that is, the transition from a state P into a state A. We will carry out it stating that:

III) if in some state of affairs M_i, Ap holds, then Pp held beginning from some former state of affairs M_j.

Let be:

- $\{A_1, A_2, A_3, \ldots\ldots\}$ a denumerable set of sub-propositional letters
- « $\wedge$», « $\vee$», « $\supset$», « $\sim$» symbols for logical connectives
- '('the symbol of leftward bracket and')' the symbol of rightward bracket.

Let us give now the definition of sub-formula.

0^1) If A_i is a sub-propositional letter then A_i is a sub-formula;

0^2) If p is a sub-formula then $\sim p$ is a sub-formula;

0^3) If p and q are sub-formulas then $(p \wedge q)$, $(p \vee q)$ and $(p \supset q)$ are sub-formulas.

Now let us introduce two more symbols A and P, and let us give our definition of well formed formula (wff):

I^1) If p is a sub-formula then Ap and Pp are well formed formulas.

I^2) If α is a well formed formula then $\sim\alpha$ is a well formed formula.

I^3) If α and β are well formed formulas then $(\alpha \wedge \beta)$, $(\alpha \vee \beta)$ and $(\alpha \supset \beta)$ are well formed formulas.

Now the proposed formal semantics will be a 3-uple $<\{M_i\}_{i\in I}$, R, $\models>$ in which $I \neq \emptyset$, R is a transitive relation on $\{M_i\}$ and $\models$ is a relation between elements belonging to $\{M_i\}_{i\in I}$ and wffs, which satisfies the following conditions for each $M_i \in \{M_i\}_{i\in I}$:

0) If $M_i \models Pp$ then for any k such that $M_i R M_k$ we have $M_k \models Pp$
1) $M_i \models Pp$ iff $M_i \models P \sim p$
2) $M_i \models P(p \wedge q)$ iff $M_i \models Pp$ and $M_i \models Pq$
3) If $M_i \models Ap$ then $\exists j \neq i$ such that $M_j R M_i$ and $M_j \models Pp$
4) If $M_i \models A \sim p$ then $M_i \not\models Ap$
5) $M_i \models Ap \supset Pq$ iff $M_i \not\models Ap$ or $M_i \models Pq$

It is the case to remark that for any semantics S_D carried out starting from conditions 0) – 5) one has to claim that for any wff α and for any state of affairs M_i you cannot have $M_i \models \alpha \wedge \sim\alpha$. Moreover, I would like to stress that while condition 0) will turn out particularly meaningful in connection with the epistemological interpretation provided at the end of this paper, condition 4) shows how the interpretation 'is true that' is not admissible for the operator A. Indeed, in the case you make a substitution in 4) replacing 'if ..... then' with 'iff', you would obtain the disagreable consequence that, for any p and any M_i, you have $M_i \models Pp$. As concerns condition 3), I would like to stress the following two points:

i) you cannot find something analogous to Putnam's objection [33] (that is, things that *have been*, of which it has never been true that they *are*) which here takes the form: «a situation is in *Actu* in some M_i and it has never been in *Potentia*».
ii) It is perhaps stimulating to consider a transitive relation ϱ on the set of wffs such that: $\alpha \varrho \beta$ iff $\exists M_i$, $\exists M_j$, with $M_j R M_i$, such that $M_j \models \alpha$ and $M_i \models \beta$.
Moreover we claim that ϱ is *objective* or *physically intrinsic* (according to Stein [3]).
Then ϱ will be admitted in the special theory of Relativity just with the interpretation: «$\alpha \varrho \beta$» is «α is in the past of β».
This is particularly meaningful if α is Pp and β is Ap, for any p.

Of course the dialectical core expressed by conditions 0) – 5), could be completed in several ways by means of conditions consistent with the given ones, supplying a complete set of suitable rules stating whether any wff holds in a given state of affairs M_i. It is not the case, in this paper, to choose a complete system of conditions rather than another one: this choice being possibly performed on the basis of different philosophical background.

It is on the contrary worth remarking now that for any semantics S_D constructed starting with our dialectical core, if you state valid a wff α of our language in the case that $M_i \models \alpha$ for any M_i of any model $<\{M_i\}_{i\in I}, R, \models>$ satisfying our conditions, then the wff 'Ap $\supset$ P(p $\wedge \sim$p)' is valid for any p.

Indeed we have:

A) If $M_i \models Ap$ then $M_i \models Ap \supset P(p \wedge \sim p)$ [5]

B) If $M_i \models Ap$ then:

I) $M_j \models Pp$ [3]

II) $M_j \models P \sim p$ [I and 1]

III) $M_j \models P(p \wedge \sim p)$ [I, II and 2]

IV) $M_i \models P(p \wedge \sim p)$ [III and 0]

V) $M_i \models Ap \supset P(p \wedge \sim p)$ [IV and 5]

A stimulating interpretation of the two symbols A and P, which premised to sub-formulas supply the wffs in our calculus, could be to see them as expressing Aristotle's notions of *Actus* and *Potentia.*

In 'Metaphysica', book IX, Aristotle points out – in order to explain the process of the becoming – the necessity to admit *Potentia* besides *Actus*. He emphasizes, against the *Megaricos* it is clear that *Potentia* and *Actus* are distinct.

Besides, in our semantics we try to show the non-deterministic nature of the whole process.

In fact, if p is *in Potentia,* also $\sim$p results *in Potentia* and *vice versa* ($M_i \models Pp$ iff $M_i \models P \sim p$), and it is a matter of fact whether p or $\sim$p will become actual. (At any rate, in our semantics it is not allowed that both p and $\sim$p, result *in Actu* !).

Thus, while on the one hand the process *Potentia - Actus* can be seen as an adequate descriptive modality of the change, on the other hand contradiction seems to be admissible within the modality of *Potentia* and nothing seems to prevent us from using the peculiar negation of classical logic.

Therefore, we have a sort of *weak contradiction,* a contradiction *in Potentia.*

Hence, the proposition which asserts the truth of wff Ap $\supset$ P(p $\wedge \sim$p), might be interpreted saying that within our semantics a weak contradiction (or contradiction in Potentia) can be seen as a necessary condition of the transition to *Actui* and, consequently, it

may express in a way the change as a function of the contradiction.

It is easy to observe that we give priority to change in respect to time, according to Aristotle and Augustine rather than to Kant.

Indeed, if one follows Von Wright's sharp distinction [27], it is possible to stress that, while change presupposes time logically, time presupposes change epistemologically. One experiences change rather than directly time.

In other words my schema intends to be a very general one which could be specified according to choice among different possible notions of time.

Such a choice must be performed, obviously, on an empirical ground.

Moreover, my schema allows that for some sub-formulas p, we have that either p or, of course, $\sim p$ never appear in *Potentia* (and, accordingly, never *in Actu*). Also the choice of this class of sub-formulas, too must be performed on an empirical ground. In my opinion this choice is one of the most interesting open philosophical problems.

Let me now remark that the notion of *being in Potentia,* might contrast the intuitive (classical) notion of *being possible.*

In detail, if you read P as M that is to say «being (classically) possible», condition 2) trivially fails in one of the two directions: in fact, you cannot admit if $M_i \models Mp \wedge Mq$ then $M_i \models M(p \wedge q)$.

I think that the difference between my notion of *being in Potentia* and any (classical) notion of *being possible* can be found exactly in that: while it is acceptable that p and q are both possible even if contradictory, it is not accepted that a contradiction is possible.

On the contrary, a contradiction *in Potentia* is accepted in the case that a sentence p *in Potentia* is accepted and a sentence q *in Potentia* is accepted, being p and q contradictory.

That is to say, the (classically) possible would be what is *in Potentia* but free from contradiction. This is not surprising if one minds that the classical notion of being possible seems to be suggested by the experience of everyday life.

In other words, let Γ be the set of sub-formulas *in Potentia* in M_i and not containing binary logical connectives. All these sub-formulas should also be *possible* in an explication wide enough(²) of the intuitive notion of *being possible.*

(²) I would like to remark that in the so-called Quantum Logic it is not the case that

If we take into consideration sub-formulas of the kind $p \wedge q$, with p, $q \varepsilon \Gamma$, then, while they result to be *in Potentia* in any case, on the contrary they result to be *possible* only in the case that p and q are not contradictory.

It is evident, however, that the problem regarding the relations between the notion of being *in Potentia* and the different formalized notions of being *possible* remains open.

At any rate I think this problem should be possibly faced after that a certain epistemological relevance has been tested – above all by dialectic philosophers – for the notion we tried to make explicit in this paper.

I would add a final observation.

It is my opinion that the semantics exposed in this paper might be adopted to express some ideas about the development of scientific theories, following a way close to Grzegorczyk's views |36|.

Following Popper's work 'The Logic of Scientific Discovery' Grzegorczyk introduces some *weak assertions* of the form: «In my investigation E I cannot refute the sentence Φ in the moment t».

Then, limiting himself to atomic (namely, without quantifiers) theoretical sentences, he states a set of postulates, which he considers relevant.

Among the others:

i) «An atomic sentence if once refuted can not be admitted later»
ii) «We are never obliged to refute Φ and $\sim\Phi$»
iii) «But there are many moments in which we can admit Φ as well as $\sim\Phi$, especially at the beginning of the research when we have no information».

Then, following our scheme, a Grzegorczyk's weak assertion can be broken in – so to say – two dialectic assertions:

i) «In my investigation E I consider the sentence p a potential hypothesis of my theory in the configuration M_i of the world and I write '$M_i \models Pp$'. But I must consider also the sentence $\sim p$ a

the negation of a tautology is necessarily false.
For instance, in Quantum Logic

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

does not hold [30].

potential hypothesis of my theory because I have no information and therefore I must write '$M_i \models P \sim p$' too».

ii) «In my investigation E I consider the sentence p an acceptable one (on the ground of the empirical available experience and of inductive methods) and now falsifiable hypothesis of my theory in the configuration M_j of the world and therefore write '$M_j \models Ap$'.
I can write '$M_j \models Pp$' and '$M_j \models P \sim p$' also, but I can't write '$M_j \models A \sim p$' anymore».

In this way, we can split the two steps, which are, in my opinion, central.

The first one consists in setting a potential hypothesis. (This step seems to be not sufficiently analyzed yet by the epistemological enquiry).

The second one consists in setting an acceptable (possible) hypothesis – which has to be submitted to the judgement of falsification.

It is evident that this schema is still very rough and that the proposed semantics should be strengthened by other conditions, first among the others one able to show how to erase a falsified hypothesis p.

Moreover, let me point out that all the argumentations above exposed are included in the range of the methodology that Lakatos [37] defines naïve methodological falsificationism.

However this methodology is modified by the introduction of a step of inductive nature.

In any case the semantics proposed in this paper is not yet able to describe how to pass from assertions «$M_i \models Pp$» and «$M_i \models P \sim p$» to the assertion «$M_i \models Ap$» (or $M_i = A \sim p$).

ACKNOWLEDGEMENTS

I wish to thank prof. Leo Apostel for many interesting conversations on this topic.

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