FREE SEMANTICS FOR DEFINITE DESCRIPTIONS

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Applications of free logic to languages with definite descriptions are at least as old as free logic itself. The reason is quite obvious: when singular terms are not analyzed, the assumption that they all be denoting does not entail any dramatic consequence, but when they are analyzed, we discover that assigning a denotation to some of them might contradict some of the theories or even the very logic we accept (simply because the theories, or the logic, allow us to *prove* that no such denotation exists). Hence the analysis of terms is probably the most powerful motivation for the construction of a logic without existential assumptions, and definite descriptions are the most typical instances of analyzed (singular) terms.

We can't say, however, that free logic has been particularly successful in dealing with these matters. The main problem, as I see it, is of a semantical nature: it is the problem of what conception of truth justifies our evaluation of sentences like

(1) t is red

or

(2) t is t

when «t» is non-denoting. And to this problem, free logicians have usually had one of the following two reactions:

(a) to assign to «t» some non-existing denotation, and then apply the usual (Aristotle-Tarski) conception of truth to (1) and (2), or

(b) to establish that the truth-value of such sentences (if any) is only a matter of convention, and then perhaps try to show that the set of logically valid sentences is independent of which convention you adopt.

Now it is easy to see that (a) is not so much a way of solving as a way of avoiding our problem, and that it creates additional problems connected with the status of the non-existing denotations. As to (b), it certainly looks more promising, especially if the independence result we mentioned can be proved in general. Unfortunately, however, the semantics constructed along these lines (most notably, the semantics of supervaluations proposed by van Fraassen and extended to languages with definite descriptions by van Fraassen and Lambert) (1) do not allow us to prove such a general result: in particular, they have to rule out all conventions that assign the truth-value False to any self-identity like (2), and then of course the nature of this «ruling out» is going to be questioned. Is it again the result of accepting a specific convention (in this case, a convention about conventions, or a *meta*convention, as we might also say)? And why should we accept such a metaconvention?

In a number of recent papers, (2) I have been proposing a new semantics for free logic that gives a different solution of our problem, and one that is not involved with the difficulties mentioned above. The principles on which the semantics is grounded are the following two.

(i) When all the singular terms occurring (free) in a sentence are denoting, the sentence has the same truth-value as in standard semantics (and this truth-value is called a *factual* one).

(ii) When some of the singular terms occurring in a sentence are non-denoting, the sentence in general has no factual truth-value. Also, in general some of its parts have factual truth-values and some have not. Keep constant the factual truth-values of the parts which do have some, and combine them with the factual truth-values the other parts have in an extension of the present world in which the singular terms occurring in them are all denoting. Repeat this operation for every such extension. If for every such repetition the whole sentence is true (false), then the sentence is *formally* true (false); if this is not the case, then the sentence is formally (as well as factually) truth-valueless.

Principles (i) and (ii) allow for a natural extension to languages with descriptions, and indeed, in a different context I have already extended them to a language with *indefinite* descriptions. (3) In the

(1) See «On Free Description Theory», *Zeitschrift für mathematische Logik und Grundlagen der Mathematik*, 1967, pp. 225-40.

(2) See for example «Free Semantics», *Boston Studies in the Philosophy of Science*, 1980, and «Truth, Correspondence, and Non-Denoting Singular Terms», *Philosophia* 1980, pp. 219-29.

(3) See «Free Semantics for Indefinite Descriptions», *Journal of Philosophical Logic* 1978, pp. 389-405 (from now on, *FID*).

present paper, I want to extend them to a language with definite descriptions, and also prove for this language a non-axiomatizability result that already holds for my language with indefinite descriptions.

The primitive symbols of my language *LDD* are denumerably many individual variables, denumerably many individual constants, denumerably many n-ary predicates (for every n) and the logical symbols $\neg$, $\&$, $\forall$, E!, $=$, ℩. Terms and wffs are defined as usual, and so are the symbols $\vee$, $\supset$, $\equiv$ and $\exists$ and the notions of a free and a bound occurrence of a term and of a closed and an open wff. Closed wffs are also called sentences. I will use x, y, z as metavariables for variables, a, b, c as metavariables for constants, P, Q as metavariables for predicates, t as a metavariable for terms and A, B, C as metavariables for wffs, all possibly with superscripts or subscripts. $A^{t'}/t$ will be the result of substituting occurrences of t' for all free occurrences of t in A (possibly after relettering some bound variables to avoid scope problems). Finally, due to my substitutional treatment of the operators binding variables (and to my decision to use it only as a simplifying device, not as a significant departure from more standard treatments), I will also consider all parametric extensions of *LDD*, that is, all languages obtained from *LDD* by adding some new individual constants to it, and indeed, I will give the key definitions of my semantics for the whole family $\mathscr{L}$ of such extensions (which of course includes *LDD*, as a vacuous extension of itself). (4)

And now for the semantics.

(D1) A *model-structure for* $\mathscr{L}$ is an ordered pair $M = \langle D, f\rangle$, where D is a set, possibly empty, to be called the *domain,* and f is a unary function, to be called the *function of interpretation,* assigning to every n-ary predicate of $\mathscr{L}$ a set of ordered n-tuples of members of D and to every member of a (possibly empty) set of individual constants of $\mathscr{L}$ a member of D.

(D2) A model-structure $M = \langle D, f\rangle$ (for $\mathscr{L}$) (5) is *full* if and only if, (6) for every member d of D, there is an individual constant a (of L) such that $f(a) = d$.

(4) To simplify things, I will not consider any uncountable extensions of *LDD* as members of $\mathscr{L}$.

(5) I will often omit such qualifications.

(6) I will sometimes use «iff» as an abbreviation of «if and only if».

(D3) The *primary auxiliary valuation* V^*_M relative to a full([7]) model-structure $M = \langle D, f\rangle$ is the partial unary function W from the set of all terms and sentences (of L) into $D \cup \{T, F\}$ such that

(a) if t is an individual constant and $f(t)$ is defined, then $W(t) = f(t)$;

(b) if A is of the form $Pt_1 \dots t_n$ and $W(t_i)$ is defined for every i such that $1 \leqslant i \leqslant n$, then (i) $W(A) = T$ if $\langle W(t_1), \dots, W(t_n)\rangle \in f(P)$, and (ii) $W(A) = F$ otherwise;

(c) (1) if A is of the form $t = t'$ and both $W(t)$ and $W(t')$ are defined, then (i) $W(A) = T$ if $W(t) = W(t')$, and (ii) $W(A) = F$ otherwise;

(c) (2) if A is of the form $t = t'$ and exactly one of $W(t)$ and $W(t')$ is defined, then $W(A) = F$;

(d) if A is of the form $E!t$, then (i) $W(A) = T$ if $W(t)$ is defined, and (ii) $W(A) = F$ otherwise;

(e) if A is of the form $\neg B$ and $W(B)$ is defined, then (i) $W(A) = T$ if $W(B) = F$, and (ii) $W(A) = F$ otherwise;

(f) if A is of the form $B \,\&\, C$ and both $W(B)$ and $W(C)$ are defined, then (i) $W(A) = T$ if $W(B) = W(C) = T$, and (ii) $W(A) = F$ otherwise;

(g) (1) if A is of the form $\forall xB$ and $W(B^a/x) = T$ for every individual constant a such that $W(E!a) = T$, then $W(A) = T$;

(g) (2) if A is of the form $\forall xB$ and $W(B^a/x) = F$ for at least one individual constant a such that $W(E!a) = T$, then $W(A) = F$;

(h) if t is of the form ιxA and there is an individual constant a such that (i) $W(A^a/x) = W(E!a) = T$ and (ii) for every individual constant b, $W(A^b/x) = W(E!b) = T$ only if $W(b) = W(a)$, then $W(t) = W(a)$, where a is the first individual constant c in the alphabetical order such that $W(A^c/x) = W(E!c) = T$;

([7]) Unless explicitly noted, all the model-structures I will consider will be full. For this reason, I will often avoid making it explicit.

(i) $W(t)$ and $W(A)$ are not defined if not by virtue of (a) – (b).

(D4) The *secondary auxiliary valuation* $V^{**}_{M'(M)}$ relative to two full model-structures M and M' (in this order) is the partial unary function W from the set of sentences into $\{T, F\}$ such that

(a) (1) if A is an atomic sentence and $V^{*}_{M}(A)$ is defined, then $W(A) = V^{*}_{M}(A)$;

(a) (2) if A is an atomic sentence, $V^{*}_{M}(A)$ is not defined and $V^{*}_{M'}(A)$ is defined, then $W(A) = V^{*}_{M'}(A)$;

(b) if A is of the form $\neg B$ and $W(B)$ is defined, then (i) $W(A) = T$ if $W(B) = F$, and (ii) $W(A) = F$ otherwise;

(c) (1) if A is of the form $B \,\&\, C$ and $W(B) = W(C) = T$ then $W(A) = T$;

(c) (2) if A is of the form $B \,\&\, C$ and either $W(B) = F$ or $W(C) = F$, then $W(A) = F$;

(d) (1) if A is of the form $\forall xB$ and $W(B^a/x) = T$ for every individual constant a such that $W(E!a) = T$, then $W(A) = T$;

(d) (2) if A is of the form $\forall xB$ and $W(B^a/x) = F$ for at least one individual constant a such that $W(E!a) = T$, then $W(A) = F$;

(e) $W(A)$ is not defined if not by virtue of (a) – (d).

(D5) A model-structure $M' = \langle D', f' \rangle$ is an *extension* of a model-structure $M = \langle D, f \rangle$ if and only if

(a) $D \subseteq D'$;

(b) for every predicate $P, f(P) \subseteq f'(P)$;

(c) for every individual constant a such that $f(a)$ is defined, $f'(a) = f(a)$.

If $D = D'$ and f and f' differ at most on constants not in LDD, then M' *is a parametric* extension of M.

(D6) A model-structure M' is a *restriction* of a model-structure M if and only if M is an extension of M'.

(D7) A full model-structure M' is an *A-world for* a full model-structure M if and only if

(a) there is at least one model-structure M'' such that M'' is an extension of M and M' is a restriction of M'';

(b) $V^{**}_{M'(M)}(A)$ is defined.

(D8) The *valuation* V_M relative to a full model-structure M is the partial unary function W from the set of sentences into {T, F} such that

(a) $W(A) = T$ if (i) there is at least one A-world for M and (ii) for every A-world M' for M, $V^{**}_{M'(M)}(A) = T$;

(b) $W(A) = F$ if (i) there is at least one A-world for M and (ii) for every A-world M' for M, $V^{**}_{M'(M)}(A) = F$;

(c) $W(A)$ is not defined if not by virtue of (a) – (b).

(D9) Let M be any non-full model-structure, and M' any full parametric extension of it. Then the valuation V_M relative to M is the partial unary function W from the set of sentences of *LDD* into {T, F} such that

(a) $W(A)$ is defined if and only if $V_{M'}(A)$ is defined;

(b) if $W(A)$ is defined, then $W(A) = V_{M'}(A)$. [8]

(D10) A sentence A is *valid* if and only if $V_M(A) = T$ for every model-structure M.

(D11) A sentence A is *essentially truth-valueless* if and only if $V_M(A)$ is not defined for any model-structure M. [9]

[8] It is easy to prove that this definition does not depend on the choice of M.

[9] Some discussion is in order here of a number of points on which I have changed my mind since writing my previous papers on this subject. As the reader who compared the above semantics with the one given in *FID* will have noticed, there are substantially three such points.

First, as a result of discussions with Hugues Leblanc and Bas van Fraassen, I have changed my attitude towards individual constants. In *FID* and other papers, I dispensed with them, primarily for economical reasons. Now I think that, whatever value these reasons might have, they are overwhelmed by the greater simplicity of a language with two categories of atomic terms, and so I decided to have individual constants in the language. As a result of this and of my substitutional treatment of the operators binding variables, I can limit myself to defining valuations for sentences.

Second, I have partly modified my view on the evaluation of wffs (or sentences, in the present case) containing descriptions. One of the main sources of trouble with descriptions, as was pointed out in *FID*, is that some of them do not denote anything in any possible world, and as a result some of the wffs containing them never get a truth-value. Hence before looking for the formal truth-value of a wff containing descriptions, which is to be established in terms of a universal (metatheoretical)

To give an idea of how the semantics works (and to facilitate comparison with other systems) I will list here a number of valid and invalid sentences (or better, sentence-schemata). Proofs of validity and invalidity are omitted.

quantifier, we had better find out whether or not there are values over which this quantifier ranges. In *FID* I established this by asking the question

(i) Does the given wff receive somewhere a truth-value?

and in a footnote I pointed out that (i) was to be preferred to

(ii) Do all the terms occurring in the given wff receive simultaneously denotations somewhere?

since «it is the truth-value of a wff (or better, the truth-values that the wff receives in the various circumstances) that is (are) essential to decide on its validity, and the denotations of the terms are important only in so far as they are relevant to the determination of such truth-value(s).» Afterwards however I realized that, just for the same reason for which (i) was to be preferred to (ii), (i) could be regarded as inadequate. For consider the sentence

(iii) $\forall x \neg Px \,\&\, P\,\alpha x Px$,

and suppose that we are trying to evaluate it in a world *M in which there are no P's* (and in which, as a consequence, (iii) has no factual truth-value. Of course there will be a lot of alternative worlds in which (iii) will have a truth-value, and in all of them such a truth-value will be F, but none of these truth-values will be relevant to our present concerns. What *will* be relevant will be the truth-values of (iii) in the various secondary auxiliary valuations relative to *M*, and it is easily seen that all of these truth-values will be T's. Hence why should we make sure of the existence of truth-values that might not be relevant at all? It would be much more reasonable, and much more consistent with out general semantical approach, to ask instead the question

(iv) Does the given wff receive somewhere a truth-value *from the point of view of the actual world* (that is, the world in which we are trying to evaluate the wff in question)?

And this indeed is the course I took in the present context, by slightly modifying the definition of the key notion of an *A*-world.

Thirdly (and most importantly) I have also changed my mind relative to the general notion of a factual (or primary auxiliary) valuation. In *FID* and other papers primary auxiliary valuations were so defined that they assigned the truth-value T to all the tautological consequences of a factually true wff. This practice allowed me to obtain some results which I found desirable, as for example the truth of

(v) $Pa \vee Q\,\alpha x\,(Px \,\&\, \neg Px)$

in every world in which *a* (exists and) is a *P* and the validity of the tautology

(vi) $(Pa \vee \neg Pa) \vee Q\,\alpha x\,(Px \,\&\, \neg Px)$,

but at the same time it was hardly consistent with the characterization of the valuations in question as *factual*. Further reflection on the subject convinced me that I should stick

The following sentence-schemata are valid:

(3) $\mathrm{E}!\iota xA \equiv \exists y\, \forall x\, (A \equiv x = y)$

(4) $(\mathrm{E}!\iota xA \,\&\, B^{\iota xA}/y) \equiv \exists y\, (\forall x\, (A \equiv x = y) \,\&\, B)$

(5) $\mathrm{E}!\iota xA \supset A^{\iota xA}/x$

(6) $\forall y\, (y = \iota xA \equiv (\forall x\, (A \supset x = y) \,\&\, A^{y}/x))$

(7) $\iota x\, (x = a) = a$

(8) $P(\iota xPx)$.

All of these schemata have an interesting history. (3) is the biconditional corresponding to the first of the two Russellian definitions given in chapter 14 of *Principia*, and is accepted by most free logicians. (4) is a revised version of the second of the above definitions, and was originally proposed by Jaakko Hintikka in 1959. ([10]) In the theory presented by Hintikka in that occasion, something much stronger that (5), that is

(9) $A^{\iota xA}/x$,

was provable, but Lambert showed later on ([11]) that (9) has contradictory instances, and Leonard had already shown ([12]) that (9) has other dubious instances, as for example

(10) $\mathrm{E}!\iota x\mathrm{E}!x$.

Lambert also suggested (5) as a convenient compromise between (9) and the Russellian

(11) $\mathrm{E}!\iota xA \equiv A^{\iota xA}/x$,

and was able to prove (5) by assuming (6) as an axiom-schema. The theory resulting from the addition of (6) to a free logic with identity is usually referred to as *FD*, and often considered the minimal free

more consistently to the factual nature of primary auxiliary valuations, and that this does not create any problems as to (v) or (vi) in so far as the definition of a *secondary* auxiliary valuation is not modified. This will explain the new form of (D3) (f).

([10]) In «Towards a Theory of Definite Descriptions», *Analysis*, pp. 79-85.

([11]) In «Notes on E ! III: a Theory of Descriptions», *Philosophical Studies* 1962, pp. 51-9.

([12]) In «The Logic of Existence», *Philosophical Studies* 1956, pp. 49-64.

description theory.[13] Some people however (including Lambert) regarded it as too weak, and thought of adding further axiom-schemata to it. One of the candidates Lambert considered was

(12) $\imath x\,(x = t) = t$,

of which (7) is a special case, but (12) (as on the other hand both (9) and (11)) is not valid in our semantics. One other candidate that is also not valid for us is

(13) $(\neg E!t\ \&\ \neg E!t') \supset t = t'$,

whereas of course other schemata that are not provable in *FD* (including (7) and the special case (8) of (9)) are valid for us.

At this point, then, it might seem natural to characterize our «theory» as just another extension of the minimal theory *FD*, perhaps one not considered before in the literature. Things however are more complicated than that, for as we will see it is the very existence of a theory that is in question here.

As I pointed out elsewhere, there is an assumption of possibility of existence hidden in our first-order logic, whether free or otherwise, and such an assumption is going to be contradicted by our analysis of terms. Not only are some (analyzed) singular terms non-denoting: they can't possibly denote. But this means that a reasonable analysis of terms is going to take a revisionist attitude with respect to first-order logic, whether free or otherwise, exactly as free logic takes a revisionist attitude with respect to standard logic when it drops the assumption that all singular terms are denoting. Very simply stated, this revisionist attitude will imply that some schemata that were valid *as a result of our holding the above assumption* might turn out not to be valid any more.

And sure enough, this is exactly what happens in our present case. Schemata like

(14) $t = t$

or even

(15) $Pt \vee \neg Pt$

are not valid in our semantics; indeed, they are essentially truth-va-

([13]) See for example «On Free Description Theory» quoted in footnote 1.

lueless when the analysis of terms shows the structure of «t» to be contradictory.

At this point, a very typical move for a free logician would be to extend the language in such a way as to become able to make the assumption explicit. A new symbol like, say, «P!», to be read «... possibly exists», might do the job, and there are very natural ways of interpreting «P!» within our semantical framework that would make such schemata as

(16) $\mathrm{P}!t \supset t = t$

and

(17) $\mathrm{P}!t \supset (Pt \vee \neg Pt)$

valid. I will try to explore this possibility in another context; as to the present paper, I will conclude it by proving an unaxiomatizability result for the set of valid sentences of my semantics. It is important to notice that such a result would *not* be modified by just extending the language; the only way to modify it that I (or, as far as I know, anybody) can think of is by adopting one or the other of the two «solutions» presented at p. 393. I already pointed out that I consider these two solutions (however popular they might be) very unsatisfactory from a philosophical point of view; for this reason, I am inclined to regard the negative result I am about to prove as expressing a deep theoretical limitation of our formal activity.

A few auxiliary steps are required to establish the result in question: to be precise, three more definitions and two lemmata.

(D12) L' is the language obtained from *LDD* by dropping all the individual constants and the logical symbols E!, =, ℩.

(D13) A model-structure $M = \langle D, f \rangle$ is *standard* if and only if $f(a)$ is defined for every individual constant a of *LDD*.

(D14) A sentence A of *LDD* is *standardly valid* if and only if $V_M(A) = \mathrm{T}$ for every standard model-structure M.

Lemma 1: Let B be a sentence of *LDD* of the form

(18) $\iota x(A \,\&\, x = a) = \iota x(A \,\&\, x = a)$,

where A is a sentence of L'. If there is at least one model-structure M such that $V^*_M(B)$ is defined then for every model-structure M there is at least one B-world.

Proof. Let $M = \langle D, f \rangle$ be a model-structure such that $V^*_M(B)$ is defined, and let $M' = \langle D', f \rangle$ be any model-structure. If $V^*_{M'}(B)$ is defined then obviously M' is a B-world for M'; hence suppose that $V^*_{M'}(B)$ is not defined. Let $L*$ be an extension of LDD that contains all the individual constants that receive an interpretation either in M or in M', and let $L**$ be an extension of $L*$ obtained by adding to it denumerably many new individual constants. Define a one-one correspondence g between the individual constants of $L*$ and the new individual constants of $L**$; then consider a set $D*$ which has the same cardinal number as D and whose members are neither members of D nor of D', and define a one-one correspondence h between D and $D*$. Let $D** = D*$ if $f'(a)$ is not defined, and $D** = D* \cup \{f'(a)\}$ otherwise. Let some arbitrary member $d*$ of $D*$ be associated with a. Let $M'' = \langle D'', f'' \rangle$ be the extension of M' such that

(i) $D'' = D' \cup D*$;

(ii) for every individual constant c, $f''(c)$ is defined iff either $f'(c)$ is defined or c is a or, for some individual constant b of $L*$, $c = g(b)$ and $f(b)$ is defined;

(iii) if $f'(a)$ is not defined, $f''(a) = d*$;

(iv) for every individual constant c such that (a) c is not a, (b) $f''(c)$ is defined and (c) $f'(c)$ is not defined, $f''(c) = h(d)$, where $d = f(b)$ and $c = g(b)$;

(v) for every n-ary predicate P and every ordered n-tuple $\langle d_1,\ldots,d_n \rangle$ of members of D'', $\langle d_0,\ldots,d_n \rangle \in F''(P)$ iff $\langle d_1,\ldots,d_n \rangle \in f'(P)$ or, for some ordered n-tuple $\langle d'_1,\ldots,d'_n \rangle$ of members of D, $d_1 = h(d'_1),\ldots,d_n = h(d'_n)$ and $\langle d'_1,\ldots,d'_n \rangle \in f(P)$, or $f'(a)$ (is defined and) $= d_i$ for some $i\,(1 \le i \le n)$ and $\langle d*_1, \ldots, d*_n \rangle \in f''(P)$, where for every $i\,(1 \le i \le n)$ $d*_i$ is $d*$ if d_i is $f'(a)$ and is d_i otherwise.

Now let $M** = \langle D**, f** \rangle$ be the restriction of M'' such that

(vi) for every individual constant c, $f**(c)$ is defined iff either c is a or c belongs to $L**$ but not to $L*$;

(vii) for every n-ary predicate P and every ordered n-tuple $\langle d_1, \ldots, d_n \rangle$ of members of $D**$, if no d_i is $f'(a)$ then

$<d_1, ..., d_n> \in f_{**}(P)$ iff $<d_1, ..., d_n> \in f''(P)$, and if some d_i is $f'(a)$ (14) then $<d_1, ..., d_n> \in f_{**}(P)$ iff $<d^*_1, ..., d^*_n> \in f_{**}(P)$, where for every i $(1 \leq i \leq n)$ d^*_i is as specified above.

It is easy to prove, by induction on the number of connectives and quantifiers occurring in a sentence C of $L*$ not containing any occurrences of E!, = or ℩, that if $a_1, ..., a_n$ are all the individual constants occurring in C then $V^*_M(C)$ is defined iff $V^*_{M**}(C^{g(a_1)}/a_1 ... {}^{g(a_n)}/a_n)$ is defined, and if $V^*_M(C)$ is defined then $V^*_M(C) = V^*_{M**}(C^{g(a_1)}/a_1 ... {}^{g(a_n)}/a_n)$ (15). Since A contains no individual constants, this implies that $V^*_{M**}(A) = V^*_M(A) = \text{T}$. On the other hand, since $f_{**}(a)$ is defined, $V^*_{M**}(a = a) = V^*_{M**}(\text{E}!a) = \text{T}$; hence $V^*_{M**}(A \ \& \ a = a)$ (or, which is the same, $V^*_{M**}((A \ \& \ x = a)^a/x)) = V^*_{M**}(\text{E}!a) = \text{T}$. Suppose now that, for some individual constant b, $V^*_{M**}((A \ \& \ x = a)^b/x)$ (that is, $V^*_{M**}(A \ \& \ b = a)) = V^*_{M**}(\text{E}!b) = \text{T}$. Then $V^*_M(b = a) = \text{T}$, and $V^*_{M**}(b) = V^*_{M**}(a)$. In conclusion, $V^*_{M**}(\imath x(A \ \& \ x = a))$ is defined; hence $V^*_{M**}(B)$ and $V^{**}_{M**(M')}(B)$ are defined, and $M**$ is a B-world for M'. Q.E.D.

Lemma 2: Let B be a sentence of LDD of the form (18). B is valid iff $\neg A$ is not standardly valid.

Proof. It is easy to see that B is valid iff for every model-structure M there is a B-world for M; hence (by Lemma 1) iff there is at least one model-structure M such that $V^*_M(B)$ is defined.(è[e]) Then we will obtain the desired result if we can prove that there is at least one model-structure M such that $V^*_M(B)$ is defined iff $\neg A$ is not standardly valid.

First, suppose that there is at least one model-structure $M = <D, f>$ such that $V^*_M(B)$ is defined. Then $V^*_M(\imath x(A \ \& x = a))$ is defined; hence there is an individual constant b such that $V^*_M((A \ \& x = a)^b/x) = V^*_M(\text{E}!b) = \text{T}$; hence (since A is a sentence) $V^*_M(A) = \text{T}$ and $V^*_M(\neg A) = \text{F}$.

Take any individual constant c such that $f(c)$ is defined, and extend M to a standard model-structure $M' = <D', f'>$ such that

(14) Which of course implies that $f'(a)$ is defined and $D** = D* \cup \{f'(a)\}$.

(15) To prove this in the particular case in which $f'(a)$ is defined, it is crucial that $f'(a)$ has been made indiscernible from $d*$, and that the language does not contain the identity symbol.

(16) To be precise, Lemma 1 supplies only one direction of this biconditional, but the other direction is trivial.

(i) $D' = D$;

(ii) for every individual constant c' of LDD such that $f(c')$ is not defined, $f'(c') = f(c)$.

It is easy to prove, by induction of the number of connectives occurring in a sentence C of LDD not containing occurrences of E!, = or ℩, that if $V^*_M(C)$ is defined then (a) $V^*_{M'}(C)$ is defined and (b) $V^*_{M'}(C) = V^*_M(C)$. But then $V^*_{M'}(\neg A) = V^*_M(\neg A) = \mathrm{F}$; hence $V_M(\neg A) = \mathrm{F}$, and $\neg A$ is not standardly valid.

On the other hand, suppose that $\neg A$ is not standardly valid. Then there is a standard model-structure M such that $V_M(\neg A) \neq \mathrm{T}$. But since A contains no descriptions or individual constants, $V^*_M(\neg A)$ is defined, and it cannot of course be T; hence $V^*_M(\neg A) = \mathrm{F}$ and $V^*_M(A) = \mathrm{T}$. Since M is standard, $V^*_M(a)$ is defined; hence $V^*_M(a = a) = V^*_M(\mathrm{E}!a) = \mathrm{T}$; hence $V^*_M(A \,\&\, a = a)$ (or, which is the same, $V^*_M((A \,\&\, x = a)^a/x)) = V^*_M(\mathrm{E}!a) = \mathrm{T}$. Suppose now that, for some individual constant b, $V^*_M((A \,\&\, x = a)^b/x)$ (that is, $V^*_M(A \,\&\, b = a)) = V^*_M(\mathrm{E}!b) = \mathrm{T}$. Then $V^*_M(b = a) = \mathrm{T}$; hence $V^*_M(b) = V^*_M(a)$. In conclusion, $V^*_M(\text{℩}x\,(A \,\&\, x = a))$ is defined, and $V^*_M(B)$ is also defined. Q.E.D.

Theorem 3: The set of valid sentences of our semantics is not recursively enumerable.

Proof. Suppose that the set of valid sentences of this semantics be recursively enumerable. Then the set of valid sentences of LDD of the form

(19) $\text{℩}x(\neg C \,\&\, x = a) = \text{℩}x(\neg C \,\&\, x = a)$,

where C is a sentence of L', is also recursively enumerable. Then by Lemma 2 the set of sentences of L' which are (i) of them form $\neg\neg C$ and (ii) not standardly valid is recursively enumerable; hence (since $\neg\neg C$ is standardly valid iff C is), the set of sentences of L' which are not standardly valid is recursively enumerable. But, as is easily seen, this implies that the set of non valid sentences of a standard quantification theory (without identity and individual constants) be recursively enumerable, and we know that this is impossible.

Q.E.D.

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