SLIGHTLY NON-STANDARD LOGIC

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I shall here outline a slighly non-standard account of validity and argue for its superiority over the standard account. The theory I shall recommend is a species of «free logic», but the arguments I'll offer for it are (I think) somewhat different from and (I hope) somewhat better than the usual ones. Essentially, I shall urge that certain features of the standard account are not in harmony with its basic notions and that their removal would therefore produce a more satisfying theory. Let me begin, then, with a review of these basic notions.

Arguments are taken to consist of sentences, and validity is characterized by the following semantic condition: if the premisses were true, then the conclusion would be true. How is this subjunctive conditional to be explicated? The fact that truth depends both on the way the world is and on the meaning of words suggests two ways of dealing with the hypotheticality of the antecedent. On what might be termed the 'possible world' approach (in which meanings remain the same but the world may change so that the premisses become true) the condition becomes: there's no possible world in which the premisses are true and the conclusion false, words being accorded their usual meanings. On what might be termed the 'possible meaning' approach (in which the world remains as it is while meanings may change so that the premisses become true) the condition becomes: there are no possible meanings of words according to which the premisses are in fact true and the conclusion in fact false.

It is not clear just where the possible world approach leads, because little is clear about possible worlds and meanings. Suppose we think of possible worlds on a model of the actual one, i.e., as consisting of individuals possessing certain properties and standing in certain relations. Then the crucial question is: which putative combinations of individuals, properties, and relations really constitute a possible world? Suppose, for example, that p = 'Abraham Lincoln's parents were Dwight and Mamie Eisenhower' appears as a premiss in an argument A. If we ask about the validity of A, we shall on the possible world account be asking about the possible worlds in which p is true.

But are there any? Not on some currently popular views. Hence, on such views A is valid no matter what its conclusion and other premisses are. Those who find this bizarre might object that the notion of possibility operative in such views is not *logical* possibility but *metaphysical* possibility. However, the possible world approach is here being pursued precisely to get at the former notion, and it is not clear what objection there can be if it turns out that one's parents are invariant across possible worlds. But just how do such matters turn out? I really have no idea.

If we take possible worlds and meanings seriously, i.e., think of possible worlds as above and regard words as having meanings (or as backed by concepts) which enable names to pick out the very same individual and predicates the very same property or relation in each possible world, then a workable characterization of validity appears to await solution of difficult metaphysical problems. This seems a situation best avoided, if possible. The difficulties to which taking possible worlds and meanings seriously leads can perhaps be avoided by not taking them seriously, i.e., by *stipulating* their natures. But how is this to be done? It seems to me that what will guide us here is simply a theory that arises from the possible meaning approach (roughly, we take possible worlds and meanings to be given by interpretations). Let me therefore turn to an elaboration of this more promising approach.

If words can mean anything at all, then there aren't going to be any valid arguments on the possible meaning approach. Hence, if the notion of validity is to be useful, the possible meanings of words must be somehow constrained. In contrast to the possible world approach, there seems a natural way of delimiting possibility here. Consider a sentence like

(1) 'Smith's bank is open'.

(1) is the sort of sentence that makes English ill-constructed from Frege's point of view, since it could mean (among other things) either that the institution where Adam Smith kept his money is doing business at the moment or that Jedediah Smith's land along the river is unforested. If we were concerned with writing a truth definition for English, the fact that different meanings can attach to the same word

might seem a nuisance to be overcome by some device such as indexing. Thus we might construe (1) as schematic of a number of sentences 'Smith_i's bank_i is open_i' (where 'Smith_i', ''s bank_i', and 'open_i' are different expressions corresponding to the triples of senses that may attach to 'Smith', ''s bank', and 'open' to give meaningful English sentences) and aim for a truth definition from which ''Smith_i's bank_i is open_i' is true iff Smith_i's bank_i is open_i' followed for each i.

If, however, we think about how such a truth definition is going to have to be constructed, noting in particular the constraints that it be «finitely presented» but applicable to a potential infinity of sentences, we shall see that indexing is unnecessary. Implicit in whatever truth definition we construct will be a truth definition schema based on a recursive syntax, which applied to (1) yields

(2) "Smith's bank is open' is true iff the value of the 's bank' mapping at the thing named by 'Smith' belongs to the extension of 'open'.

Thus we might as well begin with such a schema and obtain a truth definition for English by specifying, in terms of the categories mentioned in the right-hand portion of sentences like (2), the particular combinations of meanings which produce meaningful English sentences. Thus in the case of (2) we would specify the triples of things, mappings from things to things, and sets of things (resp.) which, when regarded as the (extensional) senses of 'Smith', ''s bank' and 'open' (resp.) make (1) a meaningful English sentence.

This suggests the following approach to characterizing validity. (a) Undertake a syntactic/semantic analysis of English, obtaining a truth definition schema based on a recursive syntax. (b) Instead of restricting the possible meanings of the expressions which are on this analysis primitive to those appropriate to English, let them range as widely as possible, i.e., let them be restricted only by the categories of the analysis. Thus in the case of (1), instead of troubling ourselves with the question of what senses of 'Smith', ''s bank' and 'open' give us a meaningful English sentence (as if there were a non-conventional answer here), we let the appropriate referents of 'Smith' be things, the appropriate designata of ''s bank' be mappings from things to things, and the appropriate extensions of 'open' be sets of things.

First-order logic may be - and in my opinion is best - considered an

elaboration of this approach. Here the truth definition schema is based on a rather abstract recursive syntax, generally expressed not in terms of English but in terms of a formal language. Sentences of this formal language can be taken to be the *logical forms* of corresponding English sentences in the sense of revealing the features upon which their logical relations depend. This abstract approach enables us to obtain a theory of logical relations without having to do a detailed semantic analysis of English, but of course the applicability of the theory is then problematic. While we can mechanically describe translations from the formal language into (quasi-) English, obtaining logical forms of English sentences – their first-order analysis – is definitely not mechanical. In essence, the theory provides us only with an array of semantic roles that we must construe English words and phrases assuming; in general, it does not tell us which words and phrases in which contexts actually assume which roles.

Readers who wish to refresh their acquaintance with such formal languages can consult a good text (e.g., [1]). Suffice to say that the full first-order theory conceives English sentences as built up from certain syntactic elements (variables, function-names, and predicates) via syntactic rules which explain how terms arise from variables and function-names, how subject-predicate sentence(-form)s arise from terms and predicates, how compound sentence(-form)s arise from sentence(-form)s and sentence connectives, and how quantified sentence(-form)s arise from sentence(-form)s, quantifiers and variables. Corresponding to these syntactic rules are semantic rules – the truth definition schema - which together reduce the truth or falsity of sentences to the meanings assigned their syntactic elements. Such assignments are called 'interpretations'. On the standard first-order theory, an interpretation specifies a non-empty set U (the universe) as the range of the variables, total functions $U \rightarrow U$ as designata for the function-names, and (set-theoretic) relations in U as extensions for the predicates. Validity is then defined by the condition that there is no interpretation under which the premisses are true and the conclusion false.

The features of the standard theory that seem to me anomalous are the requirements that universes be non-empty and that functions be total. These requirements are anomalous because they represent a qualification of the idea that the possible meanings of variables and function-names are to range as widely as possible, a qualification for which I can discover no good reason. As such they function as extra hidden premisses which constrain meaning.

In particular, the requirement that universes be non-empty enables us to draw existential conclusions from non-existential premises. Consider, e.g., the following argument:

- (3) 'Something is good iff it loves everything'
- (4) 'There's something that loves all good things'.

On the standard theory the argument is valid. For either there's something good or there isn't. If there is, it loves everything, hence all good things. If there isn't, pick anything; since there aren't any good things, there aren't any good things it doesn't love, hence it loves all good things. But why should there be anything to pick in the latter case? The 'something' in (3) isn't existential.

The requirement that functions be total also involves existential commitment. Thus if we construe the logical form of

(5) 'God exists'

in the simplist and most natural way as ' $\exists x (g = x)$ ', where 'g' is a 0-ary function-name, we find that (5) is logically true because the function designated by 'g' must have a value (taken as a name, 'g' must have a referent). (¹) But if (5) is logically true, matter it not be actually true? And if we construe the logical form of

(6) 'Mary's lover is bald'

in the simplist and most natural way as 'B (l(m))', we find ourselves committed also to the truth of 'Everyone has a lover' because 'l' must designate a total function.

These requirements are easily eliminated. We simply relax the notion of an interpretation so that universes can be empty and function-names can designate partial functions. The only problem is

⁽¹⁾ We speak of there being just one 0-tuple z of objects taken from a universe U. Thus a total 0-ary function $f: U \rightarrow U$ is defined by specifying the value of f at z. 0-ary function-names thus correspond on the standard view to names with referents. The extension of a 0-ary predicate in U will be $\{z\}$ or Φ . Hence 0-ary predicates correspond to ananalyzed declarative sentences.

how the semantic rules should be modified to accommodate reference-less terms. It seems to me that the most natural approach is to withhold truth values from subject-predicate sentence(-form)s containing such terms and let compound and quantified sentence(-form)s inherit this lack – not indeterminacy – of truth values as necessary. (In particular, taking negation, disjunction, and existential quantification as primary: if A lacks a truth value, so does the negation of A; if A and B lack truth values, so does the disjunction of A and B; but the existential quantification of a sentence-form A will be true if A is true of at least one object in the universe and false otherwise.) When defining logical concepts, we can simply ignore such valueless cases; e.g., an argument will be valid iff there's no interpretation under which the premisses are true and the conclusion false.

Why should these features of the standard theory be eliminated? The strongest consideration seems to me to be theoretical. The approach to validity that leads to the standard theory involves letting the possible meanings of (certain) words be unrestricted except by a broad construal of what sorts of meanings are appropriate to what sorts of words (e.g., functions to function-names). But the standard requirements that universes be non-empty and functions be total represent a pulling-back from the idea that meanings may range as widely as possible. (Indeed, the standard theory is not even consistent about it, for predicates can fail to be true of anything but names cannot fail to refer.) Thus the standard theory's restrictions on possible meaning need special pleading; unless there is some good reason to incorporate them, they should be banished. But such reasons seem to me to be lacking.

How might such restrictions be justified? I'll first consider «observational» arguments, then «theoretical» ones. By an «observational» argument I mean one that displays a clearly valid or invalid argument about which the non-standard theory gives the wrong answer. However, I very much doubt that such arguments can be produced. As anyone who has tried to teach introductory logic can testify, validity is not a concept possessed by the untrained mind (it may be there, but only in the sense that statues are there in the uncut stone). A characterization like 'the conclusion would be true if the premisses were' may give one some grasp of the notion – perhaps even the ability to decide certain gross cases. But a working knowledge of

validity requires a theoretical elaboration of the definition; and when one is asked to judge the validity of an argument, one will generally have to fall back on this theory. In particular, if one were to urge that a particular argument required the standard treatment, one would (I think) be merely construing validity in the standard manner.

Because of the way I've suggested defining validity in the non-standard approach, plausible candidates are hard to find. But if we shift from validity to truth, perhaps

(7) 'Either Pegasus has wings or he (she?) doesn't'

will do. On the non-standard theory, (7) isn't true on the natural construal of its logical form as ' $(W(p) \lor - W(p))$ '. What is upsetting about this? Presumably its violation of the law of the excluded middle. But what is the justification of this law? It presupposes the truth or falsity of every sentence, and what justification can be given for that? At this point, it seems to me, one must invoke a theory of logical form in which names have referents, universes are non-empty, etc. — in short, a standard theory. But this is patently circular.

My view is essentially that the notion of validity is theory-laden, at least to the extent that there are no reliable, theory-independent intuitions about validity that would decide between the standard and non-standard approaches. Judgments about the validity of English arguments either involve construal and assessment of the logical form of premisses and conclusion (the «argument form») or they don't. In the former case, assessment will employ methods that reflect a particular theory; different theories will interpret the same argument form differently. In the latter case, we are to imagine that we can just «see» the validity or invalidity of certain unformalized arguments. But if a standard analysis confirms our intuition here, so will a non-standard analysis. For if there is no argument form to constrain us, we can simply add the restrictions of the standard theory as extra premises in a non-standard analysis: ' $\exists x (x = x)$ ' secures a non-empty universe and ' $\forall x_1 \dots \forall x_k \exists y (f(x_1, \dots, x_k) = y)$ ' makes the function designated by 'f' total.

Of course, this argument cuts both ways. Thus I think it unlikely that proponents of «free logic» can show that the standard theory is wrong because it gives the wrong answers about clearly valid or invalid arguments. For again, judgments about validity are theory-re-

lative. E.g., one can't, I think, use an argument like (3)/...(4) to show that empty universes must be allowed, because the argument isn't clearly invalid. Attempts to show it is will involve us in theory at least to the extent of invoking some folk-maxim like 'You can't get existential conclusions from non-existential premisses' whose status is at least as theory-dependent as the law of the excluded middle.

There is, of course, one sort of argument whose (in)validity is not a matter of (much) theory: one with true premisses and false conclusion. If we could find one which was valid on standard analysis, we'd know that standard analysis is inadequate. But the examples that come readily to mind fail. Consider, e.g.,

- (8) 'Pegasus is a winged horse'
- (9) 'Winged horses exist'

The argument is valid on standard analysis when its form is taken to be 'W(p)'/.: ' $\exists x \ W(x)$ '. (9) is presumably false, as is ' $\exists x \ W(x)$ ' when the universe is taken to be the real one and 'W' to be true of α iff α is a winged horse. But is (8) true? Perhaps the idea is that (8) is true in fiction, in which sense (9) is also. But perhaps the claim is rather that standard analysis doesn't capture the truth conditions of (8). How, after all, should we interpret 'p'? Since Pegasus presumably isn't among the real things, it can't be made the referent; but if not Pegasus, what? (2) However, the objection shows at best only that 'W(p)' is not the logical form of (8). As far as I can see, it does not touch Quine's standard analysis as ' $\exists x \ (P(x) \ \mathscr{E} \ \forall \ y \ (P(y) \rightarrow (y = x) \ \mathscr{E} \ W(x))$ '.

In fact, such standard construals seem available in all cases that seem to demand non-standard analysis. Thus partial functions can be captured through partial functional relations, (6), e.g., being construed as $\exists x (L(x,m) \mathscr{E} \forall y (L(y,m) \rightarrow (y=x)) \mathscr{E} B(x))$. And the

⁽²⁾ This may be what Lambert and van Fraassen have in mind in writing that free logic 'enables one to measure the worth of reasoning in fictional discourse' [2]. The remark is otherwise puzzling. Surely, one wants to object, reasoning within fiction mirrors reasoning within the real world; if ordinary logic couldn't deal with some fictional argument, then there'd be a real-world argument of the same form it couldn't deal with either. But this objection presupposes that ordinary logic can express the form of fictional arguments, and that's what's at issue.

possibility of an empty universe can be had by relativizing to an undefined predicate 'U'; (3), e.g., would be construed as ' $\exists x (P(x) \mathscr{E} \forall y (P(y) \rightarrow (y = x)) \mathscr{E} w(x)$)'.

If «observation» can't decide between the standard and non-standard versions of the theory, perhaps there are «theoretical» considerations that can. My argument that the restrictions of the standard version are not in keeping with the basic approach is a consideration of this sort which favors the non-standard version. Are there perhaps «theoretical» reasons that weigh against it? For example, is the standard version *simpler*? I think not. On the contrary, while the concept of simplicity is not very precise, it seems to me precise enough to support the claim that the non-standard version is the simpler. In exploring this issue, it will be useful to break each theory into two parts: the analysis (construal of logical form) of English arguments and the formal characterization of validity in terms of argument forms.

As the examples above make clear, the standard theory can, at the level of analysis, deal with English arguments that might appear to require non-standard treatment. However, this analysis is rather cumbersome, at least by comparison with the more direct non-standard analysis. Furthermore, non-standard analysis permits a unified treatment of subject terms; we don't, e.g., have to treat some name-like expressions as names and others as disguised descriptions employing pseudo-predicates à la Quine. We can, of course, achieve a unified treatment for the standard theory by eliminating (names and) function-names entirely, but then the logical forms of English sentences become quite messy.

Turning now to the characterizations of validity for argument forms, I can't see that either account enjoys a significant advantage over the other in terms of simplicity. (Or perhaps I'm just unsure how to compare them: are partial functions simpler than total functions? are biconditional clauses simpler than two conditional clauses?) Of course, these characterizations of validity are semantic, and it might be suggested that sound judgments of relative simplicity could be made if syntactic characterizations were considered instead. But it is not clear how such a comparison should be undertaken or what it would show. There are, after all, many equivalent syntactic characterizations of standard validity. Presumably, they differ in simplicity

(although it is hard to tell: just how does one compare a (particular) Hilbert-style axiom system to a (particular) natural deduction system?). Syntactic characterizations of non-standard validity are likely to be as varied and numerous. '(3)' Which, then, of each do we pick as representative of each approach? I've no idea. And what would an answer (were it forthcoming) to the question 'Which of these two syntactic systems is the simpler?' tell us about the relative simplicity of the underlying semantic approaches? Not much, I think.

No doubt some will find this dismissal of syntactic characterizations cavalier. If they have learned their logic from some of the usual sources, they may even find the semantic approach strange. Why, they may wonder, can't we let syntactic systems stand alone? Why do we have to imagine that there are any underlying semantic theories? And if we do think of validity syntactically, isn't my argument for a non-standard account irrelevant?

The problem with thinking of validity syntactically is that it gives us only half a theory: it tells us something about validity once argument forms are established. Actually, it gives us less than that, for strictly speaking it tells us only about another concept - derivability or provability. Completing the theory in either regard requires semantics. Unlike syntactic characterizations, semantic characterizations pursue a direct analysis of validity, i.e., the characterization evolves through an analysis of the concept. In consequence, the semantic approach leads to an organic theory. Its central idea is that the logical form of sentences are to express the conditions of their possible truth. A theory of logical form is thus obtained by investigating the contribution that constituent expressions make to the truth or falsity of sentences. Such a theory in turn gives content to the notion of possibility employed in characterizing validity (it's not possible that the premisses are true and the conclusion false). On syntactic theories, the notion of logical form is also central; concepts like derivability or provability are based on it. What such theories do not

⁽³⁾ For example, Jeffrey's method of truth trees [3] for negation, disjunction, existential quantification, and identity can be revised so that open finished branches whose initial elements constitute a set Γ of sentences correspond to the non-standard satisfiability of Γ simply by restricting substituends t in the negated existential rule $[-\exists x A(x)]^r$. $[-A(t)]^r$ to (1) variables which occur free in formulae above $[-A(t)]^r$ in the branch or (2) terms which occur in atomic formulae above $[-A(t)]^r$ in the branch.

themselves provide is a sense of how such forms might be obtained for ordinary English sentences or why it might be worthwhile to do so. This, it seems to me, only a semantic theory can provide.

Are there considerations which favor the syntactic approach? Perhaps an argument like this lurks in the background: validity, as we've noted, is not a concept of the man in the street, yet such people do reason correctly; such reasoning is a matter of moving from premisses to conclusion by small steps; a study of these moves is thus the most appropriate approach to a theory of correct reasoning; but the result of such a study is a syntactic theory – some sort of natural deduction system.

This argument rests on what seems to me a very dubious supposition, namely, that logic is descriptive and not normative. One large problem with taking logic to be descriptive is deciding what the data for the theory are. People do reason correctly, but when? Presumably, the idea is that there are a number of clear cases – cases in which nobody would withold the predicate 'has reasoned correctly'; these will then be used as the links in «cartesian» chains by which we can reach any conclusion that can be reached from a collection of premisses. But the data base is still unclear. If 'nobody' really means nobody, then the number of clear cases will probably be zero. How do we deal with someone who denies that a particular instance of modus ponens is correct reasoning or with someone who asks what is meant by 'correct reasoning'? It seems to me that if we're serious about basing the theory on what people would say, such people have to be taken seriously, with the result that no theory will be forthcoming. On the other hand, if we're prepared to dismiss such people, it must be because we're convinced that there is a notion of correct reasoning which some people possess and others don't and which makes some applications of the term 'correct reasoning' correct and others incorrect. But in this case a direct «intensional» analysis of the notion (i.e., a semantic analysis) seems preferable to the indirect «extensional» analysis represented by natural deduction systems.

Nor does it seem reasonable to view the theory constructed from this data base descriptively. Saying that one reasons correctly when one moves from premisses to conclusion by a sequence of reasonings of a specified sort sounds a lot more like a stipulation of what correct reasoning is than a description of what actually happens in particular cases. I do not think we can regard it merely as an hypothesis which, as usual, goes beyond the actual evidence, for there is virtually no evidence for it. To say that reasoning can be analyzed in a certain way does not give much reason for thinking that it actually occurs in that way. If people are asked to support their conclusions, they may produce natural deductions. But what reason is there to think that they are actually reporting what really happened in reaching the conclusions? In most cases reasoning is not self-conscious: we do not put arguments into neat forms and work on them, we just reason. And my impression is that so little is known of the representational systems of the brain that a claim that natural deduction chains depict what is really going on there is wild speculation.

We might, I suppose, try to regard the theory as descriptive in some weaker sense, say, that it describes inferences people would consider correct. This, however, does not distinguish it from semantic theories. Whether there is some sense of 'descriptive' that is weak enough to be plausible but strong enough to separate syntactic from semantic theories is a question I'll leave to those with a greater investment in syntactic accounts.

A different motivation for syntactic approaches might be a desire to avoid metaphysical problems confronted by the semantic approach. One of these is the old metaphysical question of what there is. Recall that the possible meaning approach allows meanings to vary but keeps the world as it is; in particular, the universes of interpretations are to consist of real things. But now it appears that the possible meaning approach (like the possible world approach) is hostage to metaphysical questions, for just how is the world and what are the real things? Another problem is the ontological status of interpretations and their constituents. Clearly, we cannot regard all universes, functions, and extensions as real entities. (If universes were real, it would make sense to speak of the «Russell» universe whose members were those universes that didn't contain themselves as individuals; but this makes no sense. To see that functions and extensions can't be real, consider an interpretation whose universe U is whatever there is; then we can define a l-ary partial function $F: U \rightarrow U$ which differs from all those which are individuals in U by 'F(f) = f, if f: $U \rightarrow U$ is l-ary and $f(f) \neq f'$ and a l-ary extension P in U which differs from all those which are individuals in U by 'p is in P iff p is a l-ary extension belonging to U such that p is not in p'.) But if they are not real entities, what are they and how can we refer to them?

I would suggest first that syntactic approaches offer only an illusory refuge from such problems but second that these difficulties are not as threatening as they might seem. If, as I've urged above, the guiding conception of validity is semantic, problems for the semantic characterization become problems for syntactic accounts as well when one pursues their warrant. These difficulties should not be exaggerated. however. Indeed, I think the metaphysical problems encountered by the semantic approach are largely not difficulties for it. There is no doubt that the possible meaning approach makes the extension of 'valid' depend upon what there is. However, for the first-order theories I've considered here, validity depends (by Löwenheim-Skolem results) not on what the real objects are like but only on how many of them there are. The practical consequence of this is that in some cases judgments of validity will be conditional. In particular, we shall be able in some cases to reach conclusions like 'If there are at least k objects, then argument A is valid; otherwise, A is invalid'. To move to categorical judgments of validity in such cases, we need to know something about the size of the metaphysical universe. (If syntactic characterizations appear to be categorical, it is either because they directly yield no information about invalidity or because they incorporate features which correspond to an assumption that there are at least denumerably infinitely many objects.) These difficulties, however, seem not to touch the notion of validity, but our application of it. Surely they shouldn't lead us to think that the semantic characterization of validity was incoherent or imprecise or misconceived. And even in terms of application their impact seems minimal. The arguments we all know and love as valid will remain so; categorical judgments about invalidity will be confidently made only by confident metaphysicians; and (by Church's Theorem) there will remain vast numbers of arguments whose validity or invalidity is in doubt no matter what metaphysical assumptions are made. As for how we are to think of universes, functions, and extensions, it seems to me that we could reasonably require an account which supports the sort of use the semantic approach makes of them. Reality ought to constrain accounts of it, but the only reality of universes, functions, and extensions is our use of them.

By way of concluding, let me summarize the argument. I have urged that certain features of the standard semantic account of validity are not in keeping with its basic approach. In particular, the stipulations that variables can range only over non-empty sets and that functionnames can designate only total functions constitute restrictions on possible meanings that are not in harmony with the idea that such meanings shall be unrestricted except by type. What this argument establishes is at least a prima facie case against these stipulations; unless they can be justified, they ought to be discarded. But justifications are difficult to find. I have argued that there are no «crucial experiments» to be run here; we do not have clear, theory-independent intuitions about the validity of arguments which might decide between the standard and non-standard variants of the theory. Nor does the standard variant appear to be simpler than the non-standard; on the contrary, the non-standard account eases the task of construing logical forms. Thus, as the prominent criteria for deciding between theories do not favor the standard variant. I think the non-standard ought to be adopted in its stead. These arguments concern variants of the semantic approach to validity and do not apply directly to syntactic characterizations. However, I have suggested that the semantic approach is primary and controls the development of syntactic systems. Thus it seems to me that those whose logical tastes are syntactic ought to eliminate from their systems features which correspond to the restrictions of the standard semantic theory.

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