MODES OF TRANSFORMATION

William SACKSTEDER

1. Argument and Communication

All argument which we suppose to be logically compelling takes place by rearrangement of ingredients within some structure. In all our discourse, we presume that structure both rules our inferences and includes the devices which are vehicles carrying whatever external references our arguments propose. A logic is a structure so understood, that is, an order which permits rearrangements of connected elements in ways we claim correspond to rules of that order. All of our discursive communication, so far as it claims either validity or arrangements conforming to logical format, presupposes a structure so governing. Thereby we attain more than the kinds of justification for arguments which a logic compels or permits; we attain also the very possibility of convincing our interlocutors or satisfying our own critical demands for conforming our inferences to constant patterns. For in any communication, even when it is a reminder for ourselves, we need to conform our argument to some order such that its motions and rearrangements can be duplicated or retrieved by our interlocutor or ourselves: by another thinker or at a later time. We suppose that a single pattern of argument rules each instance, so that the act constituting the inference is the same in salient formal respects when another performs it or consents to it or when I retrieve it later, thinking it to be a settled argumentative step. Such possible response or feedback by repetition of steps is a condition for all human communication, and hence for any argument we suppose to be logically compelling. (1)

Any order or logistic structure which we take to govern our argumentative communication may be regarded in two ways, which ways converge on each other in a «transformation,» as I shall use the

(1) These remarks concerning the situational conditions of communication are elaborated in «Theories and Their Contexts,» *Philosophy and Phenomenological Research*, Vol. XXIV, No. 1, Sept. 1963, pp. 48-60.

term here. In the first place, the structure can be viewed as a container for any contents to which its inferences might have reference. Its immediate ingredients are place-holders or carriers for any further contents which a subsequent interpretation may designate. Since I am not here concerned with varieties of contents or with diverse possible references, I shall employ the terms «elements,» «connections,» and «arrangements,» as merely formal commonplaces covering whatever more specified materials we might try to assimilate to that structure. However it is important to see that any structure must include all three, interpreted according to some version, if it is to provide a format for arguments. For our discourse alters the patterns relating various tokens in ways which can be duplicated by another and which can be moved toward new connections of elements in arrangements in which we happen to be interested. Such alterations directly apply to arrangement and rearrangement. This capacity is the first aspect of transformations, since by transforming them some first format, which arranges elements in various connections, is carried over or replaced by another. By a transformation of structure, our arguments move from one arrangement to another, to a rearrangement.

But, in the second place, we may also regard the logistic structure which we take to govern our argumentative communication according to its capacity to rule or to provide leading principles. The transformation which we undertake as an operation on ingredients in any argument is also proclaimed to be a rule to which we suppose that the proposed argument corresponds as an instance. In advancing an argument, even one designed to convince ourselves, we claim that the alteration we undertake can be duplicated by an interlocutor, as a step which is required or permitted. As such, it is not merely an alteration: it is an instance of one rule which we suppose to be governing and to which we presume our discourse conforms. Otherwise, we are engaging only in random and kaleidoscopic juggling of the patterns in which we interrelate our tokens. When functioning as a rule, a transformation cites a *type* of rearrangement, some privileged sort of motion between arrangements. We suppose that type to be a condition whereby that step is both possible and exemplary. According to this second aspect, a transformation serves as the leading principle of inferential derivation, or as the format for compiling cases under one heading.

Thus a transformation is both instance and rule. For by a TRANSFORMATION, I shall understand here an act whereby arrangements of some structure are deliberately altered in a manner which can be duplicated as if according to a rule established or exemplified by the alteration. (2) The transformations with which I am concerned here are operations which are at once one rule exemplified in many instances and many rearrangements repeating one rule. It is in virtue of this double aspect that a transformation can form and characterize a structure which we might suppose to be ruling over our arguments after the fashion of a logical structure.

These two aspects of a transormation coincide in those cases in which we take a structure to be characterized by some one transformation. For in that case, the structure is formed *only* to include such ingredients as are subject to rearrangement by that alteration; and such rearrangements are *only* those generated by indefinite repetitions of this act of alteration. The structure is accordingly one which can govern our inferential operations: we suppose it to be a LOGIC, whose character is determined by the transformation which (as one rule) generates its rearrangements and which (as many alterations) is susceptible to its rule. (3)

It is transformations so functioning which I wish to discuss in this paper. They are themselves types for various arguments which are permitted and required by the relevant structure. But I wish here to present a sort of typology of transformations which can be understood so to form a logical structure or (to say the same thing differently) which can be taken as rules exemplified in many instances. By such a presentation, I hope to elucidate the minimum conditions in terms of which a transformation can generate a logistic structure. Since (I shall argue) these conditions can be met in different ways, I shall also derive therefrom a set of possible non-material variations according to which alternative transformations can yield alternative structures which are still logical in character. I shall show that, short of

(2) A companion paper to the present one appears in *Philosophy and Phenomenological Research*, June, 1980, Vol. XL, No. 1, pp. 474-495. It is entitled, «Transformations.» It elaborates the condensation in these preliminary paragraphs showing the governing role which transformations play in argument and communication.

(3) This rather heterodox view of the nature of logic is defended in «Logics,» *Logique et analyse*, Vol. 20, March-June 1977, pp. 42-66.

variations in content and short of highly diverse applications (neither of which concern me here), three kinds of transformation can be both rule and instance in a structure which they accordingly differentiate and characterize. There is also an important but rather eccentric fourth which can perform quasi-logical functions. I shall argue that this fourth is also best understood as a mode of transformation, albeit it is not a transformation *within* a characterized structure. It is rather that curious kind of argument whereby a new structure is created and adopted in place of a former one.

2. *Modes of Transformation*

Whenever our arguments move according to a logistic structure, they can be characterized as alterations of the ingredients of that structure. Directly, the arrangements contained by the structure are rearranged; indirectly connections are altered or inverted and elements are displaced or recomposed. However, for our consideration of the logical character of the structure, the latter changes must be considered secondary to the former. For elements and connectives are conformed to arrangements, and it is the rearrangement of these which constitutes our inferential motion. A proposed argument urges that new arrangements may be derived from a former one, or that several may be gathered together into one, or that there is a parity such that *this* one can replace *that* one. Such alterations of arrangements are transformations: each may be understood either as a ruling type or as instances.

In order for us to undertake an act of transformation within a structure, that structure must contain several arrangements: two, at the barest minimum, namely, that one from which we start and that one which results from our transformation. However, those given and derived arrangements must be such that we can suppose them inferentially connected. For our movement is proposed as an instance which can be duplicated at another time or by another person. We propose one order to govern both arrangements in such a manner that the inference we urge can be both one and repeatable in many cases. That order is a conjunction of arrangements, a partial structure. Our transformation consists in an inference joining multiple arrangements under a common order. Lacking either multiple arrangements or a single containing order, our argument could not go anywhere, so to speak. For motion within a structure requires both a distinction between point of origin and destination and a pathway between them. A transformation must be such that we can act by rule to change *this* into *that*.

Transformations in this present sense are possible only according to two types of orders conjoining arrangements. Either of these orders alone, or the two in combination, may constitute a partial structure wherein inferences moving from one arrangement to another are possible. I shall call these orders, and the modes of transformation enabled by them, «transitive» and «symmetrical.» There is a third such order, whereby rearrangements are justified by «equivalence,» thus making possible transformations which are both transitive and symmetrical. These three modes of transformation are differentiated by the types of inferences they justify, that is, by the order whereby they move from one arrangement to another. An ORDER is a conjoined grouping of arrangements, or a partial structure. A transformation is possible only by means of such an order, being the inferential act which moves according to it.

My present reduction of ruling transformations to a paucity of modes might seem skimpy, in view of the seemingly infinite variety of kinds of argumentative discourse in which we engage. For there is some order, and hence some logistic structure at least implicit in any theoretical utterance, even in the most monosyllabic transfer of information, or in a communicative act such as pointing. Also the contents of our theories are highly various and the arguments we employ for them are correspondingly diverse. Hence we should expect the ruling transformations on which these communications are based to be of incalculable variety. And indeed they are so; for some transformation supposed to be ruling is to be found in symbolic structures as diverse as speech, writing, maps, remarks, catalogues, computers, and telephone books. And by means of very different devices of communication, our attention is called to characteristic transformations among matters as diverse as a barnyard pecking order, the behavior of atoms, a legal code, a pinball machine, or a symphonic development.

However, in order that transformations be possible, only minimum and formal requirements need be supplied for them. For inferences require only that different arrangements be combined under some one unifying heading. Such an order is merely a minimal condition, without which inference is impossible. It is only an outline, which subsequent interpretations may color in diversely without altering the inferential nerve on which the transformation depends. Consider, for example, that we may speak in schematic terms about that order characteristic of a series, even though we recognize that matters as diverse as numbers, descendents, words, melodies, egg sizes, and preferences may be so arranged.

But irrespective of variant applications, so far as concerns the minimum conditions according to the present consideration of inferences, there are just two kinds of order of partial structure which are such that transformations may be permitted by them. Or, which is to say the same thing, there are two orders such that rearrangements permitted by them are repeatable as if according to a rule moving from one included arrangement to another. One of these orders is TRANSITIVE: that is, it permits movement to an arrangement whose contained elements are related by a connection common to the connection they share respecting some third item. Another order is SYMMETRICAL: that is, it permits alteration within an arrangement wherein pairs of items are so related that they can be understood to constitute a third item which includes both and their connections. The characteristics of these two orders may be combined into a single order which permits inferences equating arrangements according to an order which is *both* transitive and symmetrical. A combined mode of transformation is permitted by what I shall here call an EQUIVALENCE order.

According to these characterizations, modes of transformation are merely schematic orderings, each of which is susceptible of exemplification in quite diverse instances. The orders in a denumerable series, a causal chain, a pecking order, or a hypothetical syllogism permit transitive modes of transformations. The orders in a map, a diagram, a functioning organism, or a DeMorgan formula permit symmetrical modes of transformation. The orders in a system of equations, a personal identification, or a tautological system permit equivalent modes of transformation. Hence these modes of transformation might be said to be the inferential nerve of many very different sorts of ordered structures or logics. Therein, they may be quite diversely interpreted in preparation for very diverse application to substantive matters.

Perhaps I should make it explicit that, contrary to custom, I am deliberately using the adjectives «transitive» and «symmetrical» to modify *orders* (or partial structures, or included arrangements), rather than *relations* (or connectives). By my usage, it is only derivatively that we may apply the same characterizations to the latter substantives. But it is important to understand that any interest we have in seeing whether a relation possesses either of these characteristics is consequent to the assumption that they enter into arrangements subject to rearrangement according to the relevant transformations as rules. Characterization of orders as transitive or symmetrical is prior to consequent application of the same traits to entire structures thus characterized or to their connections and ingredients. The character of a connection is attributed to it according to the kind of inference which it can permit, rather than the reverse. Transformations are made possible by the orders in which they take place.

It is also owing to the positive role played by such orders that I omit other taxonomic variations which are usually applied to relations in the post-Russell tradition: non-transitive, non-symmetrical, asymmetrical, one-many, and the like. For my purposes, these rubrics do not characterize additional orders permissive of inferences, even though they may classify the relations subsequent to acceptance of some logic. Hence they do not enable modes of transformation. They are rather attributes resultant upon the inferences and arguments which utilize them. My quest is not for a classification of relations; I seek rather the roots permissive of inference. The opposite to order is chaos, that which has no order. In the absence of an order, no transformation is possible. (4)

(4) This shift from «secondness» to «thirdness» is so radical, and so foreign to currently fashionable expositions of logic, that I should like to cite a precedent which is respectable, though little-noted: Peirce employs the Kantian destinction between «analytic» and «synthetic»–but with an explicit recognition (appropriate to his whole scheme of logic) that he departs from the usage whereby it is *judgments* (or propositions) which are analytic or synthetic. Rather, for him, it is *arguments*–a motion through a third according to a leading principle– which are so characterized. (See *Collected Papers*, Vol. II, Paragraphs 690-93.)

It is my conviction that *only* transformations based on orders which are symmetrical or transitive or both can provide the kind of governance we expect from a logical structure. We might wish to order by logical means a massive diversity of possible materials, namely any of the furniture of the world or the contents or activities of our minds. Nonetheless, we will attain an ordering serving to support inferences which compel in some degree, however mild, only when we adopt as ruling some way of rearranging to which these contents can be assimilated. These transformations are possible according to orders which are transitive or symmetrical or both. Many analogical surrogates may do service for each of these, according to diverse material applications; but they are enabled so to serve because they order materials of one or the other or both inferential schemes. My present supposition is that no kind of order truly distinct from these is suitable for functioning in this manner. Hence it is that any argumentative rearrangement of materials entering into our communications is conformed to some one of these three modes of transformation. ([5])

There is a claim to completeness lurking in my supposition which can be properly tested only by a complete survey of all arguments professing logistic compulsions. My present remarks merely invite each of us to ruminate over the segments of argumentative usage with which we are familiar, undertaking an interpretive effort to isolate the origin of such inferential force as is displayed in each case by its characteristic arguments. Only by exercise of such interpretive recall do we test whether the inferential nerve of each sort of argument can be subsumed only under my present headings. Likewise, it is by such a trial survey that we may find any refuting negative instance, namely some workable argumentative rearrangement which is intrinsically recalcitrant to assimilation to one or the other of these orders with their consequent mode of transformation.

Omitting these latter more ranging exercises and tests for present purposes, I shall here assert as lucidly as I can the function which transformations perform in discourse, appending in as schematic as

([5]) In «Analogy: Justification of Logic» (*Philosophy and Rhetoric*, Vol. 12, No. 1, Winter 1979, pp. 21-40).

possible a manner the various guises whereby that role may be played. I thus leave for more substantive discourse the sampling and interpretive ingenuity which might render more plausible my claim to completeness. Here I propose only the schematic hypothesis that transformations as I conceive them can be implemented in only a very few modes. That scheme I shall now restate in a way which urges–although only in a verbal fashion–that it exhausts the ways in which transformations may be made possible.

Any transformation is an act whereby an arrangement of included elements and connections is altered. Hence it always includes a first arrangement of elements and connections and an altered arrangement, a rearrangement. In order for the transformation to be *one* act, these two arrangements must be differentiable and they must be brought together. The latter conjunction is accomplished by supposing a further third arrangement, namely some order or partial structure, wherein we suppose the movement from one to the other to be possible.

It is the including or conjoining third arrangement, that which a transformation presumes in order to mediate between given and derived arrangements, which makes inference possible. But being itself an order or partial structure, that third must bring together contained arrangements and elements in some manner. The formation of this third arrangement or conjoining order may be conceived in either of two ways.

First, we may assume multiple parts, and by their successive combination according to like relations among them, arrive at what we take to be a unit whole. Such a relative whole is taken to be compounded from elements and connections and subarrangements having independent identity, although they are presumed to form a whole in which all are to be located. Such formation is accomplished by accumulating new connections between distinguishable items adapted from like connection of each to a third item. Thus the relative whole is so conceived as to allow transfer through it by means of like connections. Such an order is transitive. To that extent, its contained arrangements and elements are also related by transitive connections. Items within the order are separable, however they can be brought under one purview by connections which are like, and in an including order which permits transformation according to such connections.

That is, like connections form a partial structure permitting successive and additive relations among independent items.

Or conversely, the conjoining order or partial structure which permits a transformation to move between given and derived may be conceived as formed thus: we may assume a unit whole, and by successive separations according to invertible relations within it, arrive at what we take to be multiple contents. Such relative parts are taken as composing dependent but abstractable elements and connections having characteristics dependent on their location in an order defined by that whole and its divisions. Such formation is accomplished by deriving new arrangements containing elements identifiable to the extent that the whole is understood as a third within which parts are connected by mutual or inverse relations. Thus the whole is so conceived as to allow interchange within it by means of inverse ingredients. Such an order is symmetrical. To that extent, its contained arrangements and elements are related by symmetrical connectives, or by connective pairs which are inverse to each other. The contents are dependent on the whole, except as they can be diversified by mutually correlated or inverse connections. That is, corresponding connections form an order or partial structure permitting inverse and isolating connections of dependent items.

These two ways of conceiving an order (or partial structure containing arrangements permitting an act of transformation) do not exclude each other. We may also move among arrangements by assuming both unit wholes and multiple parts forming an order. Successively, by various inverse operations, we may locate relative elements and construct relative wholes according to diverse connections. These relative parts and wholes may thence be identified and manipulated in orders and by various transformations corresponding to either of those indicated in the two previous paragraphs. In addition, a special sub-arrangement may be established having the character of both kinds of connections, thus permitting new connections or substitutions between items to be made through a third by way of equalization. Or new groupings may be formed by corresponding relocations within a third, also by way of equivalence. Each of these maneuvers uses an order which is both transitive and symmetrical. In such a partial structure, arrangements are possible in which both elements and connections are like or inverse in some respect, so that either compilation to a whole or derivation of parts may be inverse ways of conceiving one and the same order. Hence it is that orders which are transitive can be understood as reduced to those which are symmetrical and vice versa. Note, however, that such reduction is possible only to the extent that we consider both orders in purely uninterpreted terms.

These three ways of conceiving three orders or partial structures containing arrangements which permit an act of transformation are not exhaustive in any obvious way. They can be made to seem so, by employing devices which are merely verbal. By manipulating opposed dichotomies such as part/whole, like/unlike (in some respect) or combination/separation, this argument could doubtless be put into the form of a strict proof, although with some tedium. However, so to display it is not very important, for it would not thereby be sound. In part, such presentation would only derive permutations of premises which are questionable and whose application depends on an interpretive maze. Otherwise, it would depend on exhaustive alternatives so formal that one might question whether the result had any application at all.

My present argument merely presents and organizes these categories. It is properly part of a defining system, and it cites only ranging tautologies which follow therefrom. However, by means of it I invite an imaginative effort to entertain the scheme, to test its completeness, and to exercise ingenuity in seeing that the inferential possibilities which characterize any logistic structure can be understood as utilizing one or another of these modes of transformation. Such presentation will be implemented here only to the extent of more detailed exposition of the interlocking definitions in a way which merely invites their entertainment as susceptible to broader uses.

3. Transitive Transformations

By a transitive order I understand one such that from arrangements $(a \supset b)$ and $(b \supset c)$ we may repeatedly infer arrangement $(a \supset c)$. A transitive mode of transformation is a repeatable inference which proceeds according to a transitive order. It requires and is limited by the conditions making possible inference according to the formula for a hypothetical syllogism. More strictly, we should say that such an order includes two arrangements: (a R$''$ c) and (a R b and b' R$'$ c) and is such that it permits the transformation whereby the former may be derived from the latter, which transformation is possible in virtue of the identification of b and b$'$ and the likeness in some selected respect of R, R$'$, and R$''$ in an order, U, forming a common universe of discourse.

The latter more cumbersome statement makes explicit various accompaniments which are hidden among our assumptions concerning the more familiar formula. We are characterizing the order or partial structure according to the type of inferences which it permits. Accordingly, neither the transformation nor its correlate order should be understood to include any further interpretations of its ingredients or inferences. Indeed, for most of us the familiar formula for the hypothetical syllogism is itself interpretive, unless we are able to understand it as doing duty solely for that transformation and its minimum ingredients and conditions, as I cite these in the more elaborate version above. As is assumed in the common formula, it is necessary to the transformation that b be identified with b', as a third element, though further identification is not needed. Likewise the horseshoe must be disembarrassed of material interpretations such as causation or «taller than» or the word «implies.» But more importantly–and counter to our habits–it should also be understood apart from its definition as a connective by the truth table. For such a device is a secondary elaboration. We also introduce an inappropriate interpretive dimension when we raise the questions comparing material and strict implication. As used here, the horseshoe is merely that connective which appears in an ordered arrangement of elements whereby its occurrences are like in some respect which is taken as subject to such a rearrangement. «Taller than» is a case of such a connective: but it is so just because we choose to identify it as common to many arrangements, so that we may group them within an order ranging items as more-or-less-tall. That is, the transformation selects connectives conceived precisely and solely so that they will justify such inferences. The continuum of «tallness» is one we concoct in order to arrange and rearrange elements just in that respect. It is, so to speak, a universe of discourse which we form as an order within which we can shove things around exactly according to this mode of transformation. It should be noted here that we can design a transitive structure merely in order to justify inferences using the present formula, attending to a format such as «taller than» without the added interpretive sophistication of saying exactly how tall quantitatively, and also without yet adding correlated notions such as «shorter than.» Either of these complications combines transformations with orders other than with the merely transitive one defined here.

If we attend solely to the matters at hand here, a transformation may be said to be transitive to the extent that inferences of the type indicated are possible by utilizing the order of ingredients in arrangements as specified. It is this type of transformation, and its correlate order, which is common to a number series, a genealogy, a pecking order, and the various other orders in which we are enabled to draw inferences of this type. It is possible to take such transformations as the model for *all* inferences, in forming certain types of logic and supplying their philosophical explication. Hence much ingenuity may be required in order to reduce inferences which appear to have another format to this as the presumed basic transformation, or to show, as some thinkers suppose, that it is adequate for any possible inferences.

4. *Symmetrical Transformations*

By a symmetrical order, I understand one such that from $(a \,.\, b)$ we may repeatedly infer $\sim(\sim a \vee \sim b)$ and vice versa. A symmetrical mode of transformation is accordingly a repeatable inference which proceeds according to a symmetrical order. It requires and is limited by the conditions making possible inferences according to the formulae for DeMorgan's theorem. More strictly, we should say that such an order contains the arrangements$(a \text{ R } b)$ and $(a' \text{ R}' \, b')'$ and is such that it permits the transformation whereby either may be derived from the other: which transformation is possible in virtue of the location of both in U forming a common universe of discourse in which the primes are all signs of inversion, and the grouping by a parenthesis is an identifiable sub-arrangement, o, abstractable in some selected respect.

The latter more cumbersome statement makes explicit various accompaniments which are hidden among our assumptions concerning the more familiar formula. We are characterizing the order or partial structure according to the types of inferences which it permits. Accordingly, neither the transformation nor its correlate order should be understood to include any further interpretation of its ingredients or inferences. However, instances even more familiar than these formulae are possible: we readily understand points on a map to be so situated on the plane, that from «*a* is north of *b*» we may infer «*b* is not-north (i.e., is south) of *a*»; or when we pitch pennies, we take tails as implying not-heads, and vice versa. The familiar formulae of the DeMorgan theorem are themselves interpretive, unless we are able to understand them to stand solely for that transformation and its minimum ingredients and conditions, as I cite them in the more elaborate version above. Thus it is necessary to the transformation that *c* be understood as a third element, an includer, within which the other ingredients are to be found, after the manner of either arrangement. This order with its inversions is like the single glove which is still present when it has been turned inside out, even though all of the connections and locations have been inverted.

The three connections, represented by the wedge, the dot, and the curl (or prime) are just those inverse connectives which allow for correlated rearrangements holding the including container constant without remainder. The wedge and the dot are inverse to each other, and each is symmetrical as connecting the elements *a* or *b*. The curl is inverse in one usage to the whole parenthetical content, and in the others to the element to which it is joined. In either case, it is symmetrically related to its own double usage. These symbols must be disembarrassed of material interpretations such as co-existence, organic membership, or the words «and» and «or.» But more importantly–and counter to our habits–they should also be understood apart from definition as connectives by the truth table. For such a device provides a derivative explication. Nonetheless, our standard understanding of it can be inverted, so that we can regard the truth table itself as a symmetrical order or partial structure which defines symbols and runs the permutations of T and F. For such an uninterpreted understanding, we must regard these two symbols as being solely inverses of each other respecting isolated or grouped elements, and as having no interpretive connotations having to do with truth or propositions.

We assume the relevant kind of universe of discourse when we pitch pennies. That is, we assume items (or events) which are heads or tails and not both heads and tails. (We destroy that game, and at the same time, move into a totally different realm of interpreted discourse when we introduce subtleties like the possibility that a penny might stand on edge, or like the two distinct ordinary uses of the conjunction «or,» or like the influence of successive throws on our expectations or our calculations of probability. That is, we refine and elaborate a new game. However our doing so might consist in super-adding transformations involving transitive orders like numbered throws or equalizing orders like an equivalence theorem.) The game consists exactly in the conjunction of possible cases we decide to recognize. It does so because we so make it. If we attend solely to the situation thus determined by our limiting decision, a transformation may be said to be symmetrical to the extent that inferences of the type indicated are possible by adopting limitation to such an order of ingredients in arrangements. It is this type of transformation and its correlate order which is common to a map, a truthless truth table, a set of sorting bins, and the various other orders in which we are enabled to draw inferences of this type. It is possible, in certain types of logic and their philosophical explanations, to take such transformations as the model for *all* inference. The latter is seen in propositional expansions of truth tables and in current tendencies to reduce transitive inferences, or indeed the very notion of inference and validity itself, to a truth table using the assumption of material implication. These are ingenious devices for reducing all inferences of whatever format to this base, and for showing its adequacy for a whole theory of logic.

5. Transformation by Equivalence

By an equalizing order I understand one which is both transitive and symmetrical. This one order accordingly permits the inferences appropriate to either mode of transformation and also their interweaving, identification, and inversion. These transformations may be said to take place by equivalences, since they occur in an order which permits one arrangement to be identified with another in virtue of a whole which is presumed to be exhaustively composed of ingredients $a, b, c ...,$ and R, R′, R″..., and parenthetical sub-arranging. Under this presumption, we may postulate the parts, and successively compose approximation to the whole, after the manner of a transitive structure, and we may postulate the whole and successively derive ingredients, after the manner of a symmetrical structure. We are also enabled to substitute equivalent complexes for each other or to establish cases of equality. To the extent that our inferences are transformations according to an equivalence structure, we are enabled to justify and apply as rules such axioms as, «quantities equal to the same quantity are equal to each other,» and «$(a + b)$ is equivalent to $(b + a)$.» Thus arrangements can be shown to be identical in logical force in virtue of their composition and analysis within an order of equivalences. Such partial structures may be enlarged toward more inclusive orders or successively broken down into contained orders. Derivatively, it is possible to make use of connectives which are both transitive and symmetrical, such as «equals.» We may also construct one kind from the other, as is the case when mutual locations on a map are analyzed into the two inverse (transitive) relations «to the north of» and «to the south of,» or when a series of class inclusions results in a mutual exclusion.

Any complete systematization and symbolization of an order or partial structure for logistic purposes involves some equivalences, and hence aspects which are both transitive and symmetrical orders. These thereby provide for either mode of transformation alone. Thus all three modes of transformation are involved in any completed philosophical system and also in its attendant logic. Yet I believe that any such systematization is committed to selecting some one of the three kinds of order as fundamental, so that the other two and their attendant modes of transformation are derived as special sub-types needing extrapolation and individual justification. Indeed, I suspect that any vocabulary, any interpretation of symbols, and any display of instances is insidiously committed to one or the other priority, so that cases having a homeland in one type of order turn up in distorted or secondary form when transposed into another. ([6]) Consider, for example, the contortions necessary to reduce class inclusion or instantiation to truth table formulation, or the controversies over material implication and existential import. The question of the identification of overlap between logic and mathematics, and also metaphysical speculations which turn on the logistic or non-logistic character of the universe, are involved in this thicket also. However, for present purposes, I am content to characterize an equalizing order or partial structure as both transitive and symmetrical, and to suppose that we are quite familiar with obvious cases, such as elementary arithmetic, in which we perpetually engage in transformation by equivalence.

6. *Transformation by Replacement*

According to the pretentions for exhaustion which I submit for the present scheme, all orders permitting transformations are transitive, symmetrical, or both. Hence these three modes of transformation may be said to characterize possible logistic structures, and accordingly the governance of all of our argumentative communication. My present argument is that any argumentative discourse presupposes a governing structure characterized by some mode of transformation. Thereby that discourse can profess controlled rearrangements of connections and elements. Moreover, these modes of transformation are made possible by orders of three sorts, namely those which are transitive, symmetrical, or both. Insofar as our argument appeals to a pre-existent structure as supplying rules whose transformations our argument exemplifies, one of these sorts of order permits such inferences.

However, we also engage in a peculiar kind of argument which seems to evade these structures. By a piece of verbal chicanery, I shall begin by citing a mode of transformation which is *neither* transitive nor symmetrical, thereby preserving the semblance of an exhaustive scheme. But my point is rather that in such cases there is *no* order to which our argument can appeal whereby it is justified or required: it does not presuppose a structure, and it is not based on logical compulsions. In such an argument, our transformation consists in replacement of an old order with a new one. It presents a structure *de novo*, rather than justifying a movement by reference to a presumed structure. Since any argument claiming logistic force alters arrangements within a structure, that argument which presents a structure can profess logical backing only after the fact or with circularity. Yet all other arguments gain inferential force solely by reference to that structure. Hence the transformation which presents the structure must itself be proclaimed quasi-logical, since it is the origin of dependent inferential possibilities. The mode of transformation it employs I shall call «replacement.» If such a move without logical justification is a transformation at all, it may be said to attain its argumentative force by–so to speak–breaking up or dissolving one structure and replacing it with another. After the fact, the newly presented structure is proclaimed as truer, more real, or less inadequate; and we may see the former one as a distortion or truncation of it, one which had been serviceable only in virtue of a species of analogy between its orders and those which we now suppose sufficient to rule our arguments.

When neither transitive nor symmetrical orders are available, logical modes of transformation which rule in strict fashion are impossible. Hence literal procedures for identifying or reorganizing ingredients are impossible or inadequate. Nonetheless there can be a kind of rearrangement whereby $a\mathrm{U}_2b$ may replace $a\mathrm{U}_1b$, although no rule is invoked to compel that substitution. U_1 and U_2 may each form structures such that transformation by rule is possible, that is, they may derive from orders which are transitive, symmetrical or both. But the substitution of the second order for the first is a transformation by REPLACEMENT. It is a mode of transformation which appeals to neither transitive nor symmetrical orders conjoining the arrangements rearranged. The new arrangement may thence be treated as a kind of inclusive new object, replacing the earlier one, which turns out not to have been an object at all, but an image in some respect analogous to it. Such a move consists in taking this new object as showing the true nature of a, b, their connection, and even such truth as the previous arrangement contained. This kind of replacement is not strictly inferential or logical. It moves by a kind of denial or negation of transitive or symmetrical or equivalent orders.

Despite the apparent logical weaknesses of this mode of transformation, it is used continually in communication. Even though the devices it uses are not literal, it cannot be called «illogical,» for it does not violate logical rules. It might in a loose sense be said to have a «logic of its own,» namely that of transformation by replacement or analogy. When we argue in this way, we set beside each other the alternative choices, although we are unable to compel the adoption of one among them. Hence when a replacement is made, there is no rule. Or better: the transformation in the mode of replacement forms a new rule. Hence we often speak of an intuition, a leap, an *Aufhebung*, an insight: or we make vacuous appeal to some other mysterious act or faculty. Moreover, an admixture of willingness seems necessary: we make a decision to indulge in a certain understanding or to adopt a viewpoint. Our discourse does not dictate the choice, although it may persuade us to do so. But such urging is not justification by a logical structure itself, even in the cases in which it appends devices which are strictly logical. For it is the source of such logical justification as the newly presented structure provides, and it can appeal for rule to the destroyed logical structure only by an argumentative device–such as reduction to the absurd–which explodes it from within. Since the argument in effect urges us to adopt a logic, it will provide thereafter orders or modes of inferences permitting the transformations which characterize it. These will be transitive, or symmetrical, or both, although such literal orders may adopt a sort of disguise after the manner inherent in the playfulness of such a mode of transformation. Thus some analogical variation or an image both distorting and revealing may serve as way-stations in the course of such transformations by replacement.

I cite this fourth case which is «neither/nor» not only in order to pretend a scheme at least verbally exhaustive. I believe transformation by replacement to be everpresent in human discourse: it provides the very possibility of argumentative communication. I propose neither a null class nor a catch-all category. Rather the structure itself within which any communication is relatively justifiable by logic is presented by the arguments whose transformation consists in a replacement. In dialectical philosophers, some version of it is made the basis of whole schemes of communication. In more literal sciences or philosophies, the basic principles receive that justification which is appropriate to them by such presentational arguments. For more exact arguments presuppose formats so attained as principles. ([7]) The three modes of transformation which profess greater exactitude attain their own precision by borrowing a structure owing to the fourth, and by adopting or presuming its governance. Or their logistic pretentions may be so exploited that their utility is denied or they are self-destructive or we invite acceptance of more refined versions of them in the absence of any logical compulsion to do so. At any of these points communication vanishes: or else we replace our prior understanding with a new one by our own act. This act is a mode of transformation, even though it does not depend on logical compulsions. If the arguments urging such a transformation are to be justified by some appeal to rule, that rule is not one which is followed. Rather, it is one which we adopt and present and urge by our own creative insight.

University of Colorado at Boulder William Sacksteder