

REFERENCE, VARIABLES AND THE EMPTY UNIVERSE

Howard G. CALLAWAY

This paper is concerned with two important logical notions. First with 'reference' in Quine's usage, where the variables of quantification are said to refer to, or take as values, objects in a universe of discourse. I will point out some important implications of this notion of reference. I will depend upon exposition to support my interpretation. Second, I will also be concerned with 'the empty universe' and the relation between this notion and 'reference'. Finally, I will argue that this notion of reference allows us to dispense with the notion of the empty universe and that there is a considerable theoretic advantage in doing so which affects the definition of validity.

I

On Quine's view, «...the variable of quantification is ultimately to carry full responsibility for objective reference.»⁽¹⁾ Throughout his works Quine maintains that variables of quantification refer to objects. In «On What There Is», for instance he mentions reference via the variables as a virtue of the theory of descriptions:

...the burden of objective reference which had been put upon the descriptive phrase is now taken over by words of the kind that logicians call bound variables, variables of qualification, namely words like 'something', 'nothing', 'everything'. These words, far from purporting to be

(1) W. V. QUINE, «Reply to Strawson», in D. Davidson and J. Hintikka, editors, *Words and Objections*, revised edition (Dordrecht, Holland: Reidel, 1975) p. 320. Thanks are due to Professors Quine, Leblanc and Nelson for helpful comments on this paper and a near ancestor. Errors remaining are my own.

names...refer to entities generally, with a kind of studied ambiguity peculiar to themselves ⁽²⁾.

This «studied ambiguity» consists in the variables referring to objects (taking objects as values) although they do not purport to name, pick out or specify objects.

Since the variable is to carry «full responsibility» for reference, only this reference without specification will be required. Expressions purporting to specify particular objects (definite descriptions and names) receive definitions which allow them to be eliminated for the purposes of theory and reintroduced for the sake of practicality. Only variables remain as singular terms in primitive vocabulary ⁽³⁾.

In *The Roots of Reference*, Quine holds that quantification encapsulates all references:

Quantification is a welcome encapsulation of the referential apparatus. Once a theory is formulated in quantificational style, its objects of reference can be said simply to be the values of its quantified variables. This of course is explicit in the intended readings of the universal and existential quantifiers: 'everything x is such that', 'something x is such that'. And the convenience of this encapsulation becomes evident when you try to say in some other way what the objects of a theory are ⁽⁴⁾.

Not only are the variables of quantification referring expressions, there is a single answer to the question «what was referred to?» That answer is, «the values of the quantified variables.» This answer may seem disturbingly uninformative. But, if so, we must look a little further. Quine regularly answers questions concerning the referents of the variables of a parti-

⁽²⁾ QUINE, «On What There Is», reprinted in *From a Logical Point of View* (Cambridge, Harvard University Press, 1961) p. 6 Cf. *ibid.*, p. 13, and Quine, *The Roots of Reference* (LaSalle, Ill.: Open Court, 1973) pp. 98-101.

⁽³⁾ QUINE, *Mathematical Logic*, revised edition (Cambridge, Harvard University Press, 1951) pp. 146-152.

⁽⁴⁾ QUINE, *Roots of Reference*, p. 100.

cular theory or sentence. He gives a more specific answer on the assumption that the theory or sentence is true, if this assumption is plausible. Reading off the ontological commitments of a theory we determine what the values of the variables are purported to be. If the theory is true the values are what they are purported to be.

A central argument for this notion of reference without specification is that it provides for reference where specification is not to be had. Sometimes, Quine argues, the objects of a theory cannot all be specified:

For instance, under classical set theory there are, given any interpreted notation, some real numbers that are not separately specifiable in that notation. The existence sentence «there are unspecifiable real numbers» is true, and expressible as an existential quantification; but the values of the variable that account for the truth of this quantification are emphatically not objects with names ⁽⁵⁾.

In context this argument is designed to show the advantage of the objectual reading of the quantifiers over the substitutional interpretation. However, this passage also suits the present purpose since it is precisely the unspecified reference of the (objectual) quantified variable which is required to account for the truth of Quine's example. The corresponding substitutional quantification will not be true since an appropriate substitution instance cannot be produced.

Reference without specification will seem a very weak notion to those who bind reference and specification together and assume that a referring expression is one whose essential function is to pick out or specify objects. However, as we have already seen in outline, Quine seeks both to provide for specification where it is wanted and also to provide for cases where specification is beyond the power of our notation.

Quine's treatment of puzzles given rise by non-denoting

⁽⁵⁾ QUINE, «Existence and Quantification» in *Ontological Relativity and Other Essays* (New York, Columbia University Press, 1969) p. 95.

names and descriptions is in the spirit of Russell's theory of descriptions and has a prominent place in his work. We shall look at this in more detail shortly. His treatment make use of the notion of reference via the variables and the distinction between reference and purported reference. The importance of Quine's procedure with these puzzles is that it accomodates the intuition that reference is a relation so as to meet the demands of an extensionalistic interpretation of relations. For if a relation is to be interpreted as a set of ordered pairs there can be no relation to the non-existent. This is important because emphasis upon reference as a relation has sometimes brought philosophers to introduce special status ontology or complete realms of being in order to deal with non-denoting names and descriptions ⁽⁹⁾. On Quine's view no special ontology is required merely to meet the needs of a theory of reference. Rather the grounds and epistemic status of an answer to the question «what was referred to?» are the same as the grounds and epistemic status of an answer to the question «what is there?» This result is to be welcomed and expected if a true answer to the former question gives part of the answer to the latter question; i.e., if reference is a relation.

I will offer no argument with the broad lines of Quine's view of reference. The question I want to raise is this: Does the purported reference of a theory always involve reference? Quine's answer is: yes, but only so long as the universe of discourse is non-empty. A theory or sentence may purport reference to particular objects or objects of some particular kind when there are no such objects. This is to say that a theory or sentence may involve false ontological commitments. But are there cases of this sort where there are no values of the variables? Are false sentences sometimes false due to reference failure on the part of the quantified variables? To hold that

⁽⁹⁾ For instance, Leonard Linsky has held that the subject term of 'The King of France is wise' denotes or refers to an object which does not exist, partly on the grounds that «...there just cannot be such a thing as a grammatical subject which does not denote (or refer) to anything; that (by definition) is what the subject term of a sentence does.» See L. Linsky, *Referring* (New York: Humanities Press, 1967) p. 87.

they are sometimes false in this way interrupts what is otherwise an interesting and elegant uniformity of semantic theory. Otherwise we may hold that every sentence has some referents (those of the variables in it) and that it is in virtue of whether or not the relevant predicates are true of these objects that every sentence has its truth-value. It is this uniformity I will attempt to establish.

Quine's answer to this last question is evident in his treatment of the notion of the empty universe. Where Quine treats of validity in terms of set-theoretic semantics, he follows the standard practice. Schemata are counted valid if they are true under all interpretations in all non-empty universes. This may seem to make the exclusion of the empty universe a matter of logical truth and thus insure that there are always values of the variables. But this is not quite Quine's view. He regards the exclusion of the empty universe a matter of practically governed by the fact that in the usual case «...the universe relative to which an argument is carried out is already known or confidently believed not to be empty, so that the failure of a schema in the empty universe is usually nothing against it from a practical point of view.» (?)

In *Methods of Logic*, after giving this last argument, Quine provides a separate test which allows a supplementary check for validity in the empty universe. Surely there is no practical difficulty in such an account of validity. But leaving aside questions of practicality we find, from a theoretical perspective that the notion of validity is given two distinct definitions with separate tests for each. If we can eliminate the empty universe, the notion of validity and testing for validity will be thereby simplified. This objective, together with the above mentioned uniformity of reference serves to motivate the attempt to eliminate the empty universe. I will urge more conclusive considerations in the final section. Other points must first be developed.

A similar simplification of the notion of validity might be

(?) QUINE, *Methods of Logic*, 3rd edition (New York: Holt, Rinehart and Winston, 1972) p. 98.

achieved by considering valid only those schemata which are true under all interpretations in every universe, whether empty or not⁽⁸⁾. Schemata excluded on such an account of validity depend for their truth upon there being something (or other) and might be construed as a practical supplement to quantification theory for use in considering any non-empty universe. Some will find such a proposal more appealing than my own. Perhaps the most appealing argument for such an account depends, however, upon holding that logical truths are all analytic, that sentences represented by schemata such as $(\exists x)(Fx \vee \neg Fx)$ are not analytic and thus that our notion of validity should be fashioned in recognition of this, including only schemata which equally hold in the empty universe. This sort of argument has been criticized by Quine at length. I do not propose to repeat his argument here⁽⁹⁾. My point in mentioning this alternative is that Quine's criticism of this argument from analyticity appears to leave only the vigorous assertion that logic does not legislate regarding existence as a defence of this alternative simplification of the notion of validity. But from the present perspective such an argument is manifestly question begging. Whether or not logic legislates regarding existence to the extent of telling us that something or other exists is the issue at stake in deciding upon a definition of validity.

II.

Intuitively, a universe of discourse comprises the objects with which a discussion or theory is concerned. I think that this sort of definition is not quite as clear as it is sometimes taken to be. The lack of clarity is difficult to avoid. We have no alternative to construing the universe of discourse of a theory

⁽⁸⁾ QUINE describes such a system, see «Quantification and the Empty Domain» in *Selected Logical Papers* (New York: Random House, 1966) pp. 220-223.

⁽⁹⁾ QUINE, «Meaning and Existential Inference», in *From a Logical Point of View*, op. cit., pp. 160-65.

except from the perspective provided by our own best theory. But this is a limitation since the objects of our theory may turn out to be quite different from what we take them to be. A theory might involve reference to Venus without recognition of the fact that Venus is a very massive physical object orbiting the sun. Such a theory might even deny that there are such objects. Those who knew nothing of modern astronomy could only deny that their universe of discourse contained such a massive object although their theory did involve reference to Venus. The only commitment of their theory was perhaps to moving lights in the sky. The difficulty is that we cannot give the final word on what the objects of our concern may turn out to be. But we should not expect to give the final word here any more than we should expect to give the final word on what there is.

Interpreting a theory involves much concern with purported reference and less concern with reference. Deciding whether there are objects as purported by a theory *T* may involve first determining its ontological commitments and comparing these to the ontological commitments of *T'*, deciding between the two partly on grounds of economy. We need no special semantic provisions in order to interpret a theory which remains in doubt. To interpret a theory is to say what values its variables have if the theory is true. Still, since we sometimes decide to concern ourselves with a universe of ten objects, or five, or one, there may seem no reason to exclude consideration of the empty universe. The proposal to eliminate the empty universe will meet with the following sort of objection.

To accept this proposal one must reject consideration of discussions which are purely hypothetical, e.g., discussions concerning the hypothetical inhabitants of another solar system and generally any discussion where the non-emptiness of the universe of discourse is open to question. It will be argued that logic must provide rules of reasoning with application in all cases whether the universe is empty or not ⁽¹⁰⁾.

⁽¹⁰⁾ See, K. LAMBERT and BAS C. VAN FRAASSEN *Derivation and Counterexample*, (Encino California: Dickenson Publishing Co, 1972) p. 199. «Free logic is an extension of classical quantification theory that provides prin-

What I want to show first is that this sort of objection is misconceived. If ' $(\exists x)(Fx \supset \neg Fx)$ ' is a logical truth this does not exclude any discussion, whatever its special vocabulary and regardless of our assumptions concerning the denotation of the non-logical vocabulary. Descriptions and names may still be interpreted as non-denoting and any predicate may be assigned to the null set except in the obvious cases where logical truth dictates otherwise. The logical constraints introduced by exclusion of the empty universe are (1) that for each predicate either the set to which it is assigned is non-empty or the complement of the set is non-empty and (2) certain predicates are assigned to non-empty sets as a matter of logic, e.g., '...is self-identical' and '...is identical with...', since these are assigned to the universe as a whole.

I will consider some cases which might appear to demand interpretation in the empty universe. Beside answering the above objection what follows will also serve to illustrate the advantages of the notion of reference without specification. The assumption in force is that we want to make use of the relevant expressions without commitment to objects named or denoted.

The sentence 'unicorns have magical powers' may be rendered as a universal conditional:

$$(1) (\forall x)(Ux \supset Mx)$$

The sentence is not a logical truth. But it is true, given that there are no unicorns. There is no problem interpreting this sentence in a non-empty universe simply because the sentence does not logically imply that there are unicorns. There is a schema true for all non-empty universes which may seem relevant here.

$$(\forall x)Fx \supset (\exists x)Fx$$

ciples for reasoning in situations where the objects of our discourse are either non-existent or have only putative existence. So it enables one to measure the worth of reasoning in fictional discourse as well as in discourse about, say, the hypothetical entities of science.»

But this merely leads us from (1) to an existential quantification which is no more onerous than (1) itself.

$$(2) (\exists x)(Ux \supset Mx)$$

Holding (2) true merely amounts to claiming (what is equivalent) that there is something x which is either not a unicorn or has magical powers. This example sets the pattern for other cases which are sometimes thought more problematic. What it serves to emphasize is that accepting 'something exists' as a logical truth amounts to 'something or other exists' $((\exists x)(Fx \vee \neg Fx))$. The ontological commitment of (2) cannot be tied down further than a disjunction. Since the quantified variables of (2) do not purport to specify any values and the truth of (2) does not guarantee that any of the values are unicorns or are objects with magical powers, we are free to assume that the values of the variables of (2) are all non-unicorns. This is merely to recapitulate the reason for accepting (1) as true in the first place.

Now suppose we have a sentence making use of a definite description which may seem to require interpretation in the empty universe. 'The present King of France has magical powers', may, for instance be reworded as (3).

$$(3) M((\iota x)Kx)$$

This has in turn the following expansion provided by the theory of descriptions.

$$(4) (\exists x)(Kx \ \& \ ((\forall y)((y=x) \equiv Ky) \ \& \ Mx)$$

It is here that the «studied ambiguity» of the quantified variables provides answers to puzzles in the theory of reference. While (4) purports reference to the present King of France, we need postulate no such monarch or someone with magical powers to interpret (4) on any non-empty universe. Holding (4) false, the quantified variables refer to values including something which is not the King of France. Assuming there is

no present King of France the following schema (true for every non-empty universe) provides that answer:

$$\neg(\exists x)Fx \supset (\exists x)\neg Fx$$

Reference without specification together with the exclusion of the empty universe also provides a simple, straightforward answer to the problem of negative existentials. What the puzzle rests on is a semantic intuition that a true sentence always involves reference to something. This intuition is provided for here. The values of the quantified variables of negative existentials are the same objects as the values of the variables of the positive existentials which follows from them. So the variables of a sentence of the form $\neg(\exists x)Fx$, if true, refer to something (i.e. every object in the universe of discourse) of which the predicate is not true. This elegant solution is only available if the empty universe is excluded.

Individual constants receive a similar treatment. Whether a constant purports to name a unique object or does not, it receives the following sort of definition:

$$'a' \text{ for } (\exists x)Ax$$

There are a variety of ways to understand this Quinean assimilation of names to definite descriptions. We may take the non-denoting names 'Pegasus', for instance, to name the unique object which pegasizes, if any. Alternatively we may take 'Pegasus' as naming the unique object of which the predicate '...is identical-with-Pegasus' is true, if any⁽¹¹⁾. Whether or not a name names anything we may still make use of it, reflecting this consideration in the interpretation. If a name fails to name anything then the apparent reference is resolved into the non-specific reference of the quantified variables.

What has been shown is that any vocabulary may be interpreted in a non-empty universe. Holding a universe of discourse non-empty introduces no substantial restraints on an

⁽¹¹⁾ This formulation was suggested to me by some remarks of Gilbert Harman's, in private correspondence.

interpretation. Therefore, excluding the non-empty universe in an account of validity puts no restriction on the language or topic of discussion.

Quine's treatment of names is quite similar, in this respect, to the treatment rendered in so called «free logics». This being so, there is no need here to look into the comparative advantage of those free logics which follow standard practice by excluding the empty universe in the account of validity. Whatever our treatment of individual constants, no topic or vocabulary requires the empty universe. Whether objects purported by given expressions exist or not, the facts of the matter may be reflected in an interpretation in a non-empty universe.

An objection may be raised at this point which is of some importance for present purposes. It will be noted that the above account does allow for employment of expressions of the sorts illustrated. But the interpretation proceeds in the knowledge that there are no unicorns, that Pegasus does not exist and so forth. But we are not always in a position to be so confident. Physics may give up on the hypothesis of quasars or the hypothesis of neutrinos next week (for all the logician is required to know) but we cannot take the course provided above in interpreting theories which make use of 'quasar' or 'neutrino'. For such theories tell us that there are such objects and the physicists will continue to insist on the truth of such claims so long as the theories remain live and viable options. But following the procedures outlined above to eliminate the empty universe sentences such as 'a is a Quasar' or 'the unique object which caused track T is a neutrino' must be counted false. The elimination of the empty universe is thus much too costly since one also eliminates the logic needed for important areas of scientific discourse. Similar points could be made concerning the evaluation of inferences made in fictional narrative.

Again I want to urge that this criticism is misconceived. The point of the procedures outlined above is to show that the use of a predicate, description or name in a theory need not involve the theory in commitment to objects described or named by the expression in question. This is so even when the universe of discourse is held non-empty. Now the question concerns theo-

ries which do involve existential commitments, though the existence of objects as called for by the theory is left in doubt. There is no special problem here. We need only distinguish reference and purported reference. A theory may have existential implications whether or not the existentials are true. Constructing a model for such a theory these existential commitments will be reflected. We are not constrained to construct models only for theories which are in fact true. A model does not commit us to any claims regarding referents of a theory; rather it exhibits what the referents are, if the theory is true. Claims involving such predicates as '...names...', '...is true of...' and so forth are merely hypothetical.

Though a theory *T* is not true, one still make use of the vocabulary of the theory of reference in working through an interpretation. Such claims serve to indicate the purported reference of expressions of the object language on a given interpretation. A schema is valid if it turns out true on each interpretation is every non-empty universe. Thus on this account a schema is valid no matter what exists. Excluding the empty universe involves no assumptions concerning what exists. The valid schemata are those which remain true regardless of what we take the purported reference of the object language to be. This is a powerful pragmatic consideration favoring a uniform construal of validity excluding the empty universe.

III.

It still might be urged that 'something exists' is not a logical truth, even if this does not introduce substantial restraints on the interpretation of non-logical vocabulary. The intuition may yet remain. But this is not a sufficient objection in this context. Even if elaborated by expressions of concern for the «purity» of logic, such an objection merely begs the question. For what is in question is what to count as a logical truth. On the other hand there is no purely logical objection to the view that 'something exists' is not a logical truth which will not equally beg the question. Such a standoff on logical grounds serves to

direct attention to pragmatic considerations and systematic philosophical considerations. If the question is to be settled it will not be settled on purely logical grounds. Some readers may think that the question cannot be settled. In view of these points I will advance some further considerations of a different character.

Let us suppose a theory *T* has been shown false. None of the predicates are true of anything; all the individual constants and definite descriptions are non-denoting. My argument is that even in this extreme case we do not know enough to identify the universe of discourse with the null set. Rather we must allow that the variables of *T* do have existing objects as values.

In some such cases it may be quite evident that the quantified variables do not fail of reference even if none of the non-logical predicates are true of anything. We may know that the variables do not fail of reference in virtue of background information. Concerning all cases generally, my point is that one can never show that new information will never be had sufficient to convince us that the variables of *T* take as values the objects of another theory *T'* (which might only be formulated long after *T* has been disregarded). Here it is crucial to keep in mind that two theories *T* and *T'* may involve reference to the same objects although the purported reference or ontological commitments of the two theories are totally different. Thus even if the non-logical predicates of *T* are all null this never serves to guarantee the universe of discourse empty. The exclusion of the empty universe is then justified in that this procedure allows for such epistemic possibilities. If we assume the universe of discourse empty on the evidence that the non-logical predicates of *T* are all null, this is to rule out such possibilities and hold *T* irrelevant to any theory whose universe is not empty. But logical procedures are not sufficient for judgments regarding relevance.

T might be a theory whose non-logical vocabulary includes only predicates such as '...is a witch', '...is a sorcerer', '...has magical powers' and others with null extension. Although there are no witches, there were (and in some cultures still

are) people held to be witches. Although no mere analysis of the theory T will show that certain people are regarded as witches and that these people are among the values of the variables of T, we might know that T was formulated to account for the activities of a certain group of people. No analysis of T will show that these people are not values of its quantified variables unless we assume that the universe of discourse is empty on the grounds that there are no witches or sorcerers and so forth. In view of this and given the notion of reference without specification there seems no reason to hold the universe of T empty and thus fly in the face of common sense. From our perspective it is evident that the theory T does involve reference to certain people regardless of the purported reference of T and regardless of what the witch-theorists might hold.

In general if 'F' is a predicate of T which is true of nothing, there is no guarantee that background information will never be provided sufficient to show that F's are nothing but G's (where 'G' is a predicate of non-empty extension of a true theory T'). But in order to make sense of this sort of claim we must suppose that T and T' involve reference to the same objects although the purported reference or ontological commitments of T are totally different from those of T'. To assume the universe of discourse of T empty on the evidence that its predicates are all null is to assume that T is referentially incommensurate and thus totally irrelevant to any true theory T' with a non-empty universe of discourse. Such an assumption is never justified.

My conclusion then is that purported reference can never safely be regarded as involving no reference and that our definition of validity should exclude the empty universe in recognition of this point. I am therefore quite willing to regard 'something exists' as a logical truth.

Drexel University Florida State University

H. G. CALLAWAY