TOWARDS A LOGIC OF RELATIVE IDENTITY

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§1. *Introduction*

Identity statements in natural language come in two syntactic varieties. Some are of the form '*a* is the same as *b*' or '*a* is identical with *b*' and may be symbolized, after the manner of classical identity theory, as '$a = b$'. These will be called absolute identity statements. Others have the form '*a* is the same Φ as *b*' where 'Φ' is some general noun. Statements of this form will be called relative identity statements and will be formalized (following Wiggins [1], p. 2) as '$a \underset{\Phi}{=} b$'. In such statements 'Φ' is called the covering concept of the identity statement.

In seeking a logical analysis of natural language identity statements a number of policies are open. The most Draconian is that proposed by the early Wittgenstein (*Tractatus*, 5.53-5.534) which would eliminate identity statements altogether from the canonical language in favour of special constraints on the singular terms of the language, so that no more than one singular term is assigned to each item. This unworkable policy is not to be considered here. There are two alternative reductionist policies: the classical theory (see Perry [1], [2]; Nelson [1]) which reduces all natural language identity statements to absolute identity statements (usually glossed by Leibniz' Law) (1); or the relativist policy which reduces them all to relative identity statements (see Geach [1], [2]). The fourth policy, which we shall term *mixed*, is the non-reductionist policy which maintains both absolute and relative identity statements in the canonical language (see Odegard [1]).

(1) For this paper Leibniz' Law is read definitionally:

(LL) $$x = y =_{Df.} (\forall \Phi)(\Phi(x) \equiv \Phi(y))$$

As usual the class of predicates Φ admits is subject to some constraints (e.g., to exclude at least those involving quotation).

Two theses are of central concern in considering relative identity. The first is the claim (R) that two items may be the same with respect to one general noun but distinct with respect to another; the second is the claim (D) that absolute identity statements are semantically incomplete. It is difficult to be clear about the precise import of (D) since its advocates agree neither on the nature of the semantic incompleteness absolute identity statements suffer from — whether it is some kind of ambiguity (between a range of background relative identity statements), or an indeterminancy of truth-value, or a lack of clear sense — nor on the source of the incompleteness. To add to the difficulty, the cases presented for the incompleteness are wanting. Arguments that (R) entails that absolute identity statements are ambiguous are invalid; and claims that absolute identity statements lack clear sense or have indeterminate truth-conditions depend upon the verification principle (2). Every combination of these two theses, (D) and (R), has been held by someone: Geach [1], [2], [3] accepts (R) and (D); Odegard [1] accepts (R) but rejects (D); Stevenson [1], [2] accepts (D) but rejects (R) (3); Feldman [1], Perry [2] and Nelson [1], [2] reject them both. Of these four positions we shall be concerned here only with theories which, like those of Geach and Odegard, adopt (R). The Nelson-Perry-Feldman theory is the orthodox absolute identity theory of the logic books. The third theory which includes (D) but not (R) is in many ways obscure. Despite (D)'s claim that absolute identity statements are semantically incomplete, current formalizations (see Stevenson [2]) of this type of relative identity theory easily collapse into the classical absolute theory once a two-place identity predicate is added by the definition: (4)

(2) The issues are discussed at length in Griffin [1] Chapters 6 and 7.

(3) This has generally been taken to be Wiggins' position. However, the second edition of *Identity and Spatio-Temporal Continuity* makes it clear that he rejects (D) as stated above. To our knowledge this misinterpretation has been universal and we are grateful to Professor Wiggins for pointing out his true position in a personal communication.

(4) Differently, by using (LL) as a definition of absolute identity and adopting

$$(1)\ x = y =_{Df.} (\exists \psi)(x =_{\psi} y)$$

In this case the relative identity theory becomes a mere notational embellishment of the classical theory. Alternatively, Perry [2] has argued (successfully we believe) that the grounds which are adduced in favour of (D) can equally well be used to support the classical theory. The upshot of these criticisms is that this type of relative identity theory is not so much a theory of relativized identity as a theory of absolute identities restricted in their fields to given categories of items (see Griffin [1], pp. 123-9, for further comment). The theories to be considered here do not succumb in this way to the temptations of absolutism.

In this paper we propose (roughly in order of increasing plausibility) a variety of logics for relative identity theories of the first and second types, that is, theories with both (D) and (R) and theories with (R) but not (D). In doing this we refute two claims about relative identity which have gained currency: firstly, the claim made by Nelson [1] and Ayers [1] that identity theories in which (R) is satisfiable are incoherent; secondly, the claim, implied by Wiggins [1], p. 27, that (R) entails (D). In the case of each theory proposed, the satisfiability of (R) can be demonstrated by adding to the theory appropriate constants which can be used to form an example of (R). Although we do not consider here the questions of whether such examples occur in natural languages and, if so, how they may best be analysed, we believe that natural language is replete with examples (e.g., the one noted by Heraclitus or the frequently cited case of two distinct word-tokens being the same word-type) and that a theory of relative identity supplies a plausible and consistent analysis together with a resolution of their associated 'paradoxes' (e.g., the ship of Theseus problem) (5). It

(DLL) $$x =_{\Phi} y \supset . (\forall \Psi)(\Psi(x) \equiv \Psi(y))$$

as an identity axiom.

(5) For a contrary view see Wiggins [1], Part I; and Griffin [1], Chapter 10, for a reply.

is this fact which gives relative identity theories their interest.

The theories proposed satisfy (D) rather more tenuously since the two-place absolute identity predicate is left undefined in them. In each case, however, the classical Leibnizian definition could be added, giving a natural extension of the theory in which (D) would fail (6). This is not unduly worrying since arguments for (D) are scarce and the most frequently used one (that (D) is entailed by (R)) is, as we show, invalid. Those who believe that (D) is true could presumably use whatever arguments they have in its favour to reject the (LL) definition of absolute identity and thus prevent the extension which falsifies (D). However, we do not believe that they have any good reasons for pursuing this policy.

A standard presentation of absolute identity is gained by adding (LL) to classical second-order logic. The remaining formal properties of absolute identity (viz. reflexivity, symmetry, transitivity and substitutivity) follow by well-known proofs. It is desirable to approach relative identity theories in a similar way, both because it enables similarly easy proofs for appropriately formulated formal principles of relative identity — and, in fact, our theories of relative identity will turn out to be structurally similar to the theory of absolute identity — and further because we will clearly need some substitutivity principle for relative identity statements to validate certain inferences of the form: $a =_{\Phi} b,\ \psi(a) \Rightarrow \psi(b)$. The principle

$$\text{(DLL)}\quad x =_{\Phi} y \supset .\ (\forall \psi)(\psi(x) \equiv \psi(y))$$

is not the principle we want, firstly because, together with a

(6) There is one exception to this. In an extension of Theory S, within the second-order significance logic $2Q_5$, we can state and prove as a theorem the claim that 'x = y' is always non-significant, given (LL). However, this does not prove as much as the (D)—theorist might have hoped since it is plainly inappropriate to define '=' by means of a two-valued connective in a three-valued logic and a more suitable definition is easily provided.

relative identity principle we would want to accept, it is incompatible with (R) (see Wiggins [1], pp. 3-4) and secondly, because it obviously validates too much. If *a* is the same Φ as *b* it doesn't follow for any property ψ that $\psi(b)$ provided $\psi(a)$. A natural proposal, and one which we follow here, is to limit the range of the second-order quantifier in (DLL), the idea being that each relative identity relation '$=_\Phi$' carries a commitment to indiscernibility among a range of properties determined by the covering concept of the identity relation in question. Thus for each relative identity relation, '$=_\Phi$', there will be a set of properties, Δ_Φ, such that Φ-identity implies indiscernibility with respect to the properties in Δ_Φ, or Δ_Φ-indiscernibility. The complete specification of Δ_Φ for given Φ is fairly difficult (as is the specification of the precise range of the quantifier in (LL) if the well-known paradoxes of referential opacity are to be avoided; see Hintikka [1], pp. 132-6). However we can give at least a partial specification (see Griffin [1], pp. 140-1) since Δ_Φ contains Φ and is closed under negation, conjunction, and implication. The Δ sets will play a central role in the construction of our theories.

We begin with theories in what is essentially an applied second-order classical logic. In § 3 we move on to consider theories based on second-order significance logics which enable us to obtain principles not available in the theories of § 2. In what follows the universal and particular quantifiers, U and P respectively, are interpreted non-referentially as in Goddard and Routley [1], pp. 123-52, with individual variables ranging over possibilia and impossibilia as well as actual objects. Apart from the general desirability of such a wide interpretation of the quantifiers, such an interpretation has distinct advantages here since natural language identity and distinctness claims are not restricted to actual items. Greek letters are reserved for syntactical predicate variables (predicate constants are denoted by Greek letters surmounted by a bar); *x, y, z* are syntactical individual variables (*a, b, c* syntactical individual constants) and upper case letters, *A, B,* are well-formed formulae. It is assumed that defined formulae are also wffs.

§ 2. *Theories within an Extended, Second-order Classical Logic*

To form these theories we add to classical second-order logic a new constant (relation) symbol, Δ, with the *formation rule*:

If Φ and ψ are 1-place predicates then $\Delta_{\Phi}(\psi)$ is a wff.

Constant Δ is intended to represent a function from properties or, more simply semantically, a relation on properties. In what follows we choose the simpler, and more general option. Relation Δ provides the restriction in terms of which relative identity is logically characterised, the basic characterisation taking the following determinable form:

$$x =_{\Phi} y \text{ iff, for every } \psi \text{ such that } \Delta_{\Phi}(\psi),\ \psi(x) \text{ iff } \psi(y).$$

Thus the formal theory of relative identity which we shall develop is logically an outcome of the theory of restricted (second-order) quantification. Different theories emerge according to the way the restricted quantifiers and the equivalence (iff) are cashed out. Classically, restricted quantifiers are eliminated through extensional connectives; accordingly for a classical theory of relative identity we define:

$$\text{D0: } (U\psi \in \Delta_{\Phi})A =_{Df.} (U\psi)(\Delta_{\Phi}(\psi) \supset A).$$

Intuitively, $\Delta_{\Phi}(\psi)$ can be read 'ψ is a member of the set of properties Δ_{Φ} determined by Φ'.

§ 2.1. *Theory 1*

Theory 1 is obtained by adding:

$$\text{D1} \quad x =_{\Phi} y =_{Df.} (U\psi \in \Delta_{\Phi})(\psi(x) \equiv \psi(y))$$

to introduce the identity relations of the theory.

D1 plays the same role in Theory 1 that (LL) plays in classical identity theory, with its help and with D0 the following are theorem schemata:

T1 $\vdash x =_{\Phi} x$ (*reflexivity*)

Proof: From $(U\psi)(\psi(x) \equiv \psi(x))$ by D0, D1.

T2 $\vdash x =_{\Phi} y \supset . \ y =_{\Phi} x$ (*symmetry*)

Proof: From $(U\psi)(\psi(x) \equiv \psi(y)) \supset . \ (U\Psi)(\psi(y) \equiv \psi(x))$ by D0, D1

T3 $\vdash x =_{\Phi} y \ \& \ y =_{\Phi} z \supset . \ x =_{\Phi} z$ (*transitivity*)

Proof: From $(U\psi)(\psi(x) \equiv \psi(y)) \ \& \ (U\psi)(\psi(y) \equiv \psi(z)) \supset .$ $(U\psi)(\psi(x) \equiv \psi(z))$ by D0, D1.

T4 $\vdash x =_{\Phi} y \ \& \ \Delta_{\Phi}(\psi) \supset . \ \psi(x) \equiv \psi(y)$ (*substitutivity*)

Proof: From $(U\psi \in \Delta_{\Phi})(\psi(x) \equiv \psi(y)) \supset . \ (U\psi)(\Delta_{\Phi}(\psi) \supset .$ $\psi(x) \equiv \psi(y))$ by D0, D1.

Semantics. The semantics of Theory 1 and the other classically-based theories are similarly simple. We add to Henkin's second-order semantics (as set out in Church [1]) a two-place relation, $v(\Delta)$, on properties. Then

$$v(\Delta_{\Phi}(\psi)) = t \text{ iff } \langle v(\Phi), v(\Psi) \rangle \in v(\Delta).$$

The standard consistency and completeness proofs for second-order logic can be trivially extended for Theory 1, since we have added no new axiom schemata for identity.

Some Interesting Non-Theorems. Since every valid wff of

Theory 1 is a theorem we can exhibit some non-theorems by showing interpretations under which they fail. The interest of these non-theorems lies in the fact that they exhibit the differences between Theory 1 and the classical theory of identity.

Our first rejection:

$$(2)\quad \dashv x =_{\Phi} y \supset .\ \psi(x) \equiv \psi(y)$$

demonstrates the failure of the unrestricted substitutivity of identity in Theory 1. Various counter-models are available but the following is good as any. Let $\bar{\Theta}$ be a property such that, for given individuals a and b, $\bar{\Theta}(a)$ and $\sim\bar{\Theta}(b)$, let a and b agree as to all other properties, i.e., for every ψ in $\Delta_{\bar{\lambda}}$, where $\Delta_{\bar{\lambda}}$ is the set of all properties with the exception of $\bar{\Theta}$. The wff $a =_{\bar{\lambda}} b\ \&$ $\sim(\bar{\Theta}(a \equiv \bar{\Theta}(b))$ is then satisfiable.

Similarly

$$(3)\quad \dashv (\Phi(a)\ \&\ \psi(b)\ \&\ a =_{\bar{\lambda}} b) \supset (Px)(\Phi(x)\ \&\ \psi(x))$$

fails. Suppose we have three properties $\bar{\Theta}$, $\bar{\varsigma}$, and $\bar{\lambda}$ and two items a and b such that $(Ux)(\bar{\Theta}(x) \equiv \sim\bar{\varsigma}(x))$ and $\Delta_{\bar{\lambda}}(\psi)$ for all ψ except $\bar{\Theta}$ and $\bar{\varsigma}$, and such that $\bar{\Theta}(a)$ (so $\sim\bar{\varsigma}(a)$) and $\bar{\varsigma}(b)$ (so $\sim\bar{\Theta}(b)$). Then $(\bar{\Theta}(a)\ \&\ \bar{\varsigma}(b)\ \&\ a =_{\bar{\lambda}} b)\ \&\ \sim(Px)(\bar{\Theta}(x)\ \&\ \bar{\varsigma}(x))$ is valid in the interpretation. By contrast two relativized versions of (3) are provable in Theory 1:

$$\text{T5}\quad \vdash (\Phi(x)\ \&\ \psi(y)\ \&\ x =_{\bar{\lambda}} y) \supset .\ \Delta_{\bar{\lambda}}(\Phi) \supset (Pz)(\Phi(z)\ \&\ \psi(z))$$

$$\text{T6}\quad \vdash (\Phi(x)\ \&\ \psi(y)\ \&\ x =_{\bar{\lambda}} y) \supset .\ \Delta_{\bar{\lambda}}(\psi) \supset (Pz)(\Phi(z)\ \&\ \psi(z))$$

More importantly, (R) is satisfiable in Theory 1 since

(4) $\dashv x =_{\Phi} y \supset . \; x =_{\psi} y$

fails. Let there be five constants a, b, $\overline{\Theta}$, $\overline{\chi}$, $\overline{\varsigma}$ subject to the following conditions:

(i) $\overline{\Theta}(a)$, $\sim\overline{\Theta}(b)$, $\Delta_{\overline{\varsigma}}(\overline{\Theta})$, $\sim\Delta_{\overline{\chi}}(\overline{\Theta})$

(ii) $\overline{\chi}(a)$, $\overline{\chi}(b)$, $\Delta_{\overline{\chi}}(\overline{\chi})$

(iii) $\overline{\varsigma}(a)$, $\overline{\varsigma}(b)$, $\Delta_{\overline{\chi}}(\overline{\varsigma})$

From (i) we get $\Delta_{\overline{\varsigma}}(\overline{\Theta})$ & $\sim(\overline{\Theta}(a) \equiv \overline{\Theta}(b))$, and so $\sim(a =_{\overline{\varsigma}} b)$; (ii) and (iii) ensure $a =_{\overline{\chi}} b$. We can thus construct an example of (R) in Theory 1, which was our original intention.

A mixed Extension of Theory 1. Theory 1 satisfies (D) in the weak sense that the two-place absolute identity predicate '=' is so far undefined in it: there are no theorems involving '=' in Theory 1 because there are no wffs containing '=' in the theory. This can be remedied by adding to Theory 1 the classical definition of absolute identity:

(LL) $x = y =_{Df.} (U\psi)(\psi(x) \equiv \psi(y))$

This gives:

T7 $\vdash x = y \supset . \; x =_{\Phi} y$

Proof: From $\vdash (U\psi)(\psi(x) \equiv \psi(y)) \supset . \; (U\psi \in \Delta_{\Phi})(\psi(x) \equiv \psi(y))$

But not, of course, its converse:

(5) $\dashv x =_{\Phi} y \supset . \; x = y$

which fails if $(P\psi) \sim (\Delta_{\Phi}(\psi))$. The consistency of this extension of Theory 1 is sufficient to demonstrate that (R) does not entail

(D). The attraction of this type of extension is that it enables us to combine the advantages of relative identity with those of the classical theory. Absolute identity appears at one end of a spectrum of identity relations of differing degrees of strictness.

Defects of Theory 1 (and its Mixed Extension) Theory 1 is about the simplest identity theory we can frame which satisfies (R). However, it does not capture all our intuitions about relative identity. One such intuition is that if a and b are the same Φ they must both be Φ's. Yet in Theory 1

(6) $\dashv x =_{\Phi} y \supset . \Phi(x) \vee \Phi(y)$

fails. Let $\bar{\Theta}$ be a property such that $(U\psi \in \Delta_{\bar{\Theta}})(\psi(a) \equiv \psi(b))$ and $\sim\bar{\Theta}(a)$ and $\sim\bar{\Theta}(b)$ (which is consistent with $\Delta_{\bar{\Theta}}(\bar{\Theta})$). We then have $a =_{\bar{\Theta}} b \;\&\; \sim(\bar{\Theta}(a) \vee \bar{\Theta}(b))$. Reflection along these lines shows a similar defect with the form of reflexivity given by Theory 1 since we should not expect $a =_{\Phi} a$ to hold in those cases in which $\Phi(a)$ fails. These defects are fatal for Theory 1, and carry over into its mixed extension; but their source is easy to diagnose since we did not include $\Phi(x)$ and $\Phi(y)$ as necessary conditions for $x =_{\Phi} y$. An obvious correction, still within classical extended second-order logic, gives Theory 2.

§ 2.2 *Theory 2.*

In Theory 2 we have D1 of Theory 1 by

D1* $x =_{\Phi} y =_{Df.} (U\psi)[\Phi(x) \;\&\; \Phi(y) \;\&(\Delta_{\Phi}(\psi) \supset . \psi(x) \equiv \psi(y))]$,
i.e., $x =_{\Phi} y \equiv . \Phi(x) \;\&\; \Phi(y) \;\&\; (U\psi \in \Delta_{\Phi})(\psi(x) \equiv \psi(y))$.

Theory 2 has the same semantics as Theory 1. Theory 2 gives us a more acceptable form of reflexivity:

T1* $\vdash \Phi(x) \supset. x =_{\Phi} x$ (*reflexivity*)

Proof: Since $(U\psi \in \Delta_{\Phi})(\psi(x) \equiv \psi(x))$ we have $\Phi(x) \supset. \Phi(x) \& (U\psi \in \Delta_{\Phi})(\psi(x) \equiv \psi(x))$

Symmetry, transitivity and substitutivity are unchanged:

T2* $\vdash x =_{\Phi} y \supset. y =_{\Phi} x$ (*symmetry*)

T3* $\vdash x =_{\Phi} y \& y =_{\Phi} z \supset. x =_{\Phi} z$ (*transitivity*)

T4* $\vdash x =_{\Phi} y \& \Delta_{\Phi}(\psi) \supset. \psi(x) \equiv \psi(y)$ (*substitutivity*)

as are T5 and T6 of Theory 1. The ill-fated (6) of Theory 1 becomes

T7* $\vdash x =_{\Phi} y \supset. \Phi(x) \& \Phi(y)$.

Moreover, T1* gives us another desirable theorem schema which was not available in Theory 1:

T8* $\vdash \Phi(x) \supset (Py)(y =_{\Phi} x)$.

(2) and (3) remain non-theorems in Theory 2, and

(7) $\dashv x =_{\Phi} x$

becomes one. Theory 2 is consistent with (R) since (4) remains a nontheorem. Suppose we have two items *a*, *b* and three properties $\bar{\chi}$, $\bar{\Theta}$, $\bar{\varsigma}$. Let $\bar{\chi}$ be such that

(i) $\bar{\chi}(a), \bar{\chi}(b), \Delta_{\bar{\chi}}(\bar{\chi}), \sim\Delta_{\bar{\chi}}(\bar{\Theta}), \sim\Delta_{\bar{\chi}}(\bar{\varsigma})$

Thus $(U\psi \in \Delta_{\bar{\chi}})[\bar{\chi}(a) \& \bar{\chi}(b) \& (\psi(a) \equiv \psi(b))]$, and so $a =_{\bar{\chi}} b$.

Let $\bar{\varsigma}$ and $\bar{\Theta}$ be such that:

(ii) $\bar{\Theta}(a)$, $\sim\bar{\Theta}(b)$, $\Delta_{\overline{\varsigma}}(\bar{\Theta})$

so that $\sim(a =_{\varsigma} b)$. Moreover, suppose $\bar{\varsigma}(a)$ and $\bar{\varsigma}(b)$. In this case we not only reject (4) as a theorem schema but also

(8) $\dashv \Phi(x) \,\&\, \Phi(y) \,\&\, \psi(x) \,\&\, \psi(y) \supset . \, x =_{\Phi} y \supset x =_{\psi} y$

This is significant since it is mainly these cases of (R) which are controversial, as many absolutists (e.g., Wiggins [1], p. 6) are prepared to accept cases of (R) in which $a =_{\chi} b \,\&\, \sim(a =_{\varsigma} b) \,\&\, \sim(\bar{\varsigma}(a) \,\&\, \bar{\varsigma}(b))$. Theory 2 thus permits the strongest and most controversial cases of (R). Moreover, we can form a mixed extension of Theory 2 by adding (LL) to it and thus preserve the advantages of the absolute theory (7).

Defects of Theory 2. We objected to Theory 1 on the grounds that in it

(9) Chomsky is the same physicist as Chomsky

is true even though Chomsky is not a physicist. Theory 2 surmounted that difficulty by making (9) false. However, Theory 2 fails to distinguish between (9), which is correctly treated as false, and

(10) Chomsky is the same number as Chomsky

which is more correctly treated as non-significant. Since Theory 2 assigns the same value to (9) and (10) there is good reason for considering alternatives which don't. Moreover,

(7) That this is possible even when 'strong' cases of (R) can be modelled is worth noting, since it is only these cases which lead the absolutist to fear a conflict between (R) and the classical theory. For an account of how the absolutist can cope with the weaker cases of (R), see Griffin [1], pp. 187-93.

since the thesis (D) claims that absolute identity statements are semantically incomplete there is a suspicion that we have not yet done (D) justice since the (D)-theorists' claim seems to be that absolute identity statements are neither true nor false but non-significant. For these reasons we turn from classical quantification logics to second-order significance logics.

§ 3 *Theories based on Second-order Significance Logic*

§ 3.1 *The Significance Logic* $2Q_5$.

The logic $2Q_5$ (studied in Goddard and Routley [1], pp. 590 ff) is essentially the significance transcription of standard second-order logic (as formulated, e.g., in Church [1]). The logic results by adding standard ($\supset$ — $\forall$) quantificational postulates for second-order logic — but with the 3-valued material implication '$\rightharpoonup$' replacing ordinary material implication '$\supset$' (and the quantifier '$\forall$' replaced by 'U') — to a functionally complete 3-valued sentential logic S_6, with the values *t*, *f* and *n* interpreted respectively as *true, false* and *non-significant.* Apart from the classical connectives ('&', '$\vee$' and '$\sim$') which take the value *n* whenever one of their components takes the value *n* and otherwise behave truth-functionally, the connectives of $2Q_5$ (and S_6) which we shall use are characterised, matrix-wise, as follows:—

$\rightharpoonup$	*t*	*f*	*n*	*S*	*T*	$\neg$
t	*t*	*f*	*n*	*t*	*f*	*f*
f	*t*	*t*	*t*	*t*	*f*	t
n	*t*	*t*	*t*	*f*	*f*	t

$\simeq$	*t*	*f*	*n*
t	*t*	*f*	*f*
f	*f*	*t*	*f*
n	*f*	*f*	*t*

'*S*' and '*T*' may be read 'it is significant that' and 'it is true that', respectively. It is evident from the matrices that detachment holds for $\rightharpoonup$ with *t* the only designated value, and also that

(11) $C \rightharpoonup D \rightharpoonup . \; A \,\&\, B \,\&\, C \rightharpoonup A \,\&\, D$ and

(12) $(D \rightharpoonup . A \simeq B) \,\&\, (D \rightharpoonup . B \simeq C) \rightharpoonup . D \rightharpoonup . A \simeq C$

are valid. For suppose, to illustrate a shortcut method, (11) is not valid. Then for some assignment of values to the variables $C \rightharpoonup D$ must have value t and $A \,\&\, B \,\&\, C \rightharpoonup A \,\&\, D$ value $\bar{t}$ (i.e. f or n). But by the last $A \,\&\, B \,\&\, C$ must have value t, and so each of A, B and C have value t, and $A \,\&\, D$ must have value $\bar{t}$. Since A has value t it must be D that has $\bar{t}$. But this is impossible as both $C \rightharpoonup D$ and C have value t.

A final distinctive feature of $2Q_5$ that we shall need (the feature that distinghuishes $2Q_5$ from the system $2QS_6$ of Goddard and Routley [1]) is that an unrestricted second-order abstraction schema holds, i.e., given any formula schema $B(x_1, \ldots, x_n)$ of the language there is some relation f such that for every $x_1, \ldots, x_n$,

$$f(x_1, \ldots, x_n) \simeq B(x_1, \ldots, x_n).$$

(Equivalently, full substitution of wffs for predicate formulae holds.) Using the abstraction schema it is straightforward to establish the following theorems of $2Q_5$

(13) $(Ux)(P\Phi) \sim S\Phi(x)$

(14) $(Ux)(P\psi)\, \psi(x)$

§ 3.2 *The Theory S.*

Theory S is formed by adding Δ to $2Q_5$. Its formation rule is the same as in Theories 1 and 2, and its interpretation is simply the 3-valued analogue of that for Theories 1 and 2. D0 is amended to

DS0. $(U\psi \in \Delta_{\Phi})\, A =_{Df} (U\psi)(\Delta_{\Phi}(\psi) \rightharpoonup A)$.

D1 is replaced by the definition

DS1. $x =_{\Phi} y =_{Df} (U\psi)(\Phi(x) \,\&\, \Phi(y) \,\&.\, \Delta_{\Phi}(\psi) \rightharpoonup.\, \psi(x) \simeq \psi(y))$; thus $x =_{\Phi} y \simeq.\, \Phi(x) \,\&\, \Phi(y) \,\&(U\psi \in \Delta_{\Phi})(\psi(x) \simeq \psi(y))$.

The following are theorem schemata of Theory S:

TS1 $\vdash \Phi(x) \rightharpoonup.\, x =_{\Phi} x$ (*reflexivity*)

Proof: Since $\Delta_{\Phi}(\psi) \rightharpoonup.\, \psi(x) \simeq (U\psi \in \Delta_{\Phi})(\psi(x) \simeq \psi(x))$.

Note that this result would not follow had D0 been retained, as $\Delta_{\Phi}(\psi)$ may be non-significant for some ψ. (Of course D0 could be retained were it postulated that $S\Delta_{\Phi}(\psi)$.) TS1 now follows from DS1 using the theorem $(A \simeq B \,\&\, C) \rightharpoonup.\, C \rightharpoonup.\, B \rightharpoonup A$. Thus Theory S preserves the desirable features of reflexivity in Theory 2.

TS2 $\vdash x =_{\Phi} y \rightharpoonup.\, y =_{\Phi} x$ (*symmetry*)

Proof: By the symmetry of $\simeq$ and & using DS1.

TS3 $\vdash x =_{\Phi} y \,\&\, y =_{\Phi} z \rightharpoonup.\, x =_{\Phi} z$ (*transitivity*)

Proof: By (11) and (12),

$$\Phi(x) \,\&\, \Phi(y) \,\&\, [\Delta_{\Phi}(\psi) \rightharpoonup.\, \psi(x) \simeq \psi(y)] \,\&\, \Phi(y) \,\&\, \Phi(z) \,\&\, [\Delta_{\Phi}(\psi) \rightharpoonup.\, \psi(y) \simeq \psi(z)] \rightharpoonup.\, \Phi(x) \,\&\, \Phi(z) \,\&\, [\Delta_{\Phi}(\psi) \rightharpoonup.\, \psi(x) \simeq \psi(z)].$$

TS3 then follows by generalising and distributing quantifier U and applying DS1.

TS4 $\vdash x =_{\Phi} y \,\&\, \Delta_{\Phi}(\psi) \rightharpoonup.\, \psi(x) \simeq \psi(y)$ (*substitutivity*)

Analogues of T5 and T6 of Theory 1 are also preserved in Theory S. T7* of Theory 2 becomes:

TS7 $\vdash x =_{\Phi} y \rightharpoonup.\, \Phi(x) \,\&\, \Phi(y)$

TS8 $\vdash S(x =_{\Phi} y) \supset . \ S\Phi(x) \ \& \ S\Phi(y)$

Proof: From $S[\Phi(x) \ \& \ \Phi(y) \ \& \ (U\psi \in \Delta_{\Phi})(\psi(x) \simeq \psi(y))] \supset S\Phi(x) \ \& \ S\Phi(y)$.

Since $2Q_5$ contains the theorems $(Ux)(P\Phi) \sim S\Phi(x)$ and $(Ux)(P\psi)\psi(x)$ we can prove an example of (R) in Theory S:

TS9 $\vdash (P\Phi,\psi)(x =_{\Phi} x \ \& \ \neg \ (x =_{\psi} x))$

Proof: $(P\Phi) \sim S\Phi(x)$ gives $\sim S\bar{\Theta}(x)$ and thus by contra-position of TS8 $\sim S(x =_{\bar{\Theta}} x)$ and hence $\neg (x =_{\bar{\Theta}} x)$. From $(P\psi)\psi(x)$ we get $\bar{\chi}(x)$ and then, by TS1, $x =_{\bar{\chi}} x$.
PG gives TS9.

TS9 embodies the principle that x is not self-identical with respect to every property Φ, and naturally follows from the principle that for every item x some property may not significantly be asserted of x. Both are desirable principles to have in a relative identity theory.

A Mixed Extension of Theory S: Simply adding (LL) makes (D) provable in Theory S. Since $(Ux)(P\Phi) \sim S\Phi(x)$ in $2Q_5$ it follows that $\sim S(x = y)$. However, adding a relation defined by a two-valued connective in a three-valued logic is not a very intelligent policy and one which the prudent absolutist is not likely to accept. A more reasonable alternative is to add '=' to Theory S by means of the definition:

DS2 $x = y =_{Df.} (U\Phi)(\Phi(x) \simeq \Phi(y))$

With DS2 we can prove

TS10 $\vdash x = y \rightharpoonup . \ (\Phi(x) \ \& \ \Phi(y)) \supset x =_{\Phi} y$

but not, of course, its converse.

Moreover, we can extend Theory S further by adding a constant property, Θ, such that $(U\Phi)(\Delta_{\bar{\Theta}}(\Phi))$. This gives us: (8)

TS11 $\vdash x =_{\bar{\Theta}} y \simeq. \bar{\Theta}(x) \& \bar{\Theta}(y) \& (U\psi)(\Phi(x) \simeq \Phi(y))$.

If we make the further stipulation that $\bar{\Theta}$ applies universally, i.e., that $(Ux)\bar{\Theta}(x)$, the first two conjuncts on the right-hand side of TS11 drop off and we have

TS12 $\vdash x =_{\bar{\Theta}} y \simeq x = y$.

TS12 is important since it is reasonably claimed that in natural language 'item' satisfies the conditions imposed on $\bar{\Theta}$. The claim that if *a* and *b* are (absolutely) identical they are one and the same item occurs in much of the philosophical literature on absolute identity. It also explains the (D)-theorists' reluctance to accept '*a* is the same item as *b*' as a properly relativized identity statement.

§ 4 *Conclusion and a further Suggestion.*

We have shown that, contrary to widespread belief, formal theories of relative identity can be consistently formulated even within a classical second-order logic. Moreover, such theories can be extended to encompass absolute identity, again contrary to widespread belief. Within classical quantification logics Theory 2 seems to be the best we can do. However, by considering relative identity theories within second-order significance logics we get results much more in line with philosophical thought on relative identity.

We have already mentioned the difficulties of providing a significance logics gives rise to the hope that some adequate complete specification of Δ_{Φ}, for given Φ. The introduction of characterization of Δ_{Φ} may be possible in terms of the significance range of Φ. Unfortunately, the most obvious attempts to

(8) Similar extensions are possible with Theories 1 and 2.

do this result in too wide a range of properties being included in Δ_{Φ} and thus in the exclusion of plausible natural language examples of (R). It is not easy to see what additional constraints can justifiably be imposed on membership of Δ_{Φ}. This is not to suggest that there is no connection between Δ_{Φ} and the significance range of Φ, but that the precise specification of the connection is not a simple matter and requires further work. As already noted, relative identity theory is not at a unique disadvantage in this respect since restrictions of a similar nature on substitutable predicates are required in the absolute theory if the modal paradoxes are to be avoided.

(D) has not fared well in any of our theories but this is not worrying since we know of no good arguments in its favour. However, there is a more plausible claim which nonetheless has certain affinities with (D). This is the claim, made by Wiggins (9), that in natural language certain types of general nouns (viz., sortals, as distinct from mass nouns, and dummy sortals) play a fundamental role in identity statements since they are necessary to individuate items (10). It is claimed, and at least one of the authors agrees with the claim, that the concept of an individual item is incoherent unless some sortal is available to provide principles for individuating the item. This claim cannot be accomodated within the theories presented in this paper, and meeting it is almost certain to require substantial changes within the semantics of quantification (11).

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(9) It is made much more clearly in the forthcoming second edition of his [1].

(10) Recognition of the different types of general nouns which may be substituted for Φ also eases (though it does not solve) the difficulty of specifying Δ_{Φ} adequately. See Griffin [1], pp. 140-1.

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