PROPOSITIONS AND THE LIAR PARADOX

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The liar paradox has been treated in recent years as setting a problem about the construction of languages or the relations among sentences (1). This paper treats the liar as raising a problem about the existence of propositions: are there propositions whose content is that they themselves are false ? The paper shows that there are no such propositions, at least when propositions are construed according to the recent 'possible-worlds' analysis (2). Thus, this paper supports a so-called 'Chrysippian' solution to the liar paradox, a solution that says that liar sentences are meaningless (3).

I.

I shall begin by offering some preliminary considerations about propositions and truth-values. I view propositions, for the purposes of this paper, as functions from the set of possible worlds to the set of truth-values. Here I shall consider the set of truth-values to be {T, F, I}, where *I* is the truth-value *indeterminate*. I consider such a three-valued approach to be well within the boundaries of the ordinary notion of truth, but whether this is correct is of little import, for the subsequent arguments go through *mutatis mutandis* when the set of truth-values is taken to be {T, F}.

(1) See, e.g., Dorothy Grover, «Inheritors and Paradox», *Journal of Philosophy*, Vol. LXXIV, No. 10 (October, 1977), pp. 590-604; Saul Kripke, «Outline of a Theory of Truth», *Journal of Philosophy*, Vol. LXXII, No. 19 (November 6, 1975), pp. 690-716; Brian Skyrms, «Return of the Liar: Three-Valued Logic and the Concept of Truth», *American Philosophical Quarterly*, Vol. 7, No. 2 (April, 1970), pp. 153-61.

(2) This analysis is discussed in Robert Stalnaker, «Possible Worlds», *Nous*, Vol. X, No. 1 (March, 1976), pp. 65-75.

(3) See SKYRMS, *op. cit.*, for a discussion of Chrysippianism.

A proposition p has a truth-value at (or: on, in) every possible world w. That is, p(w)=T or p(w)=F or p(w)=I, for every world w. Since p is a function, it has exactly one truth-value at any given possible world.

The ordinary notion of truth is such that truth can be meaningfully predicated of any proposition, even a proposition with an indeterminate truth-value. But the notion of predicating truth of a proposition is ambiguous. Let p be any proposition. Then the proposition that p is true can be either of two propositions. First, it can be the proposition p(@)=T, where @ is the actual world. Secondly, it can be the proposition τ(p), where τ is a function from the set of propositions to the set of propositions (i.e. a propositional operator) defined by the truth-table following.

P	τ(p)
T	T
F	F
I	F

That is to say, τ is defined by the following rules. For any possible world w,

if p(w)=T, then [τ(p)](w)=T;
if p(w)=F, then [τ(p)](w)=F;
if p(w)=I then [τ(p)](w)=F.

In the same way, the ordinary notion of truth is such that falsity can be meaningfully predicated of any proposition, although the predication is ambiguous. The proposition that p is false can be either the proposition p(@)=F or the proposition ø(p), where ø is the propositional operator defined by the truth-table following.

P	ø(p)
T	F
F	T
I	F

If we are given a proposition p, it is clear how to construe $\tau(p)$ and $\emptyset(p)$. But how shall we construe the propositions $p(@)=T$ and $p(@)=F$? Let us answer this question by answering the more general question of what proposition $p(w)=X$ is, for any proposition p, any possible world w, and any truth-value X. I would like to argue that $p(w)=X$ will itself be either the necessarily true proposition (i.e. the unique proposition that maps every possible world to T) or the necessarily false proposition (i.e. the unique proposition that maps every possible world to F).

We have three possibilities:

(a) $p(w)=T$,
(b) $p(w)=F$,
(c) $p(w)=I$.

In case (a) it seems clear that $[p(w)=T](w)=T$, $[p(w)=F](w)=F$, and $[p(w)=I](w)=F$. In case (b), $[p(w)=T](w)=F$, $[p(w)=F](w)=T$, and $[p(w)=I](w)=F$. In case (c), $[p(w)=T](w)=F$, $[p(w)=F](w)=F$, and $[p(w)=I](w)=T$. Hence, for every possible case we have $[p(w)=X](w)=T$ or $[p(w)=X](w)=F$. That is to say, *at w* the proposition that p has a given truth-value on w is itself either true or false, not indeterminate.

Moreover, whatever truth-value $p(w)=X$ has *at w*, it should have exactly the same truth-value *at any other possible world w'*. This seems intuitively clear for the reason that $p(w)=X$ is itself a claim about the truth-value of a given proposition at a given world, and the truth-value of *this* claim should not vary from world to world. For the skeptical, however, a more precise argument can be given (4). Any proposition p, as a function, is a set of ordered pairs (w,X), where the first member is a world and the second a truth-value. The identity of the function p is given by the identity of these pairs. If $p(w)=X$, then (w,X) is one of the ordered pairs of p. Hence, no matter from what

(4) I am indebted to John Stevens for suggesting this way of sharpening up the argument.

world we 'view' p, when we find w as the first member of an ordered pair of p, X must be its second member. For otherwise we would not be viewing p but rather some other function p'.

It follows from the preceding two paragraphs that p(w)=X is itself either necessarily true or necessarily false. For, if [p(w)=X](w)=T, then [p(w)=X](w')=T for all worlds w'. And if [p(w)=X](w)=F, then [p(w)=X](w')=F for all worlds w'. But it is either the case that [p(w)=X](w)=T or that [p(w)=X](w)=F.

II.

Let us now turn to the liar's paradox itself. There are, of course, a number of versions of the paradox, and each raises its own special problems [5]. The central version of the paradox, however, can be put in the following form.

The statement in box A is false.

A

I shall call this situation (i.e. a box labelled «A» with a sentence «The statement in box A is false.» in this box) the Initial Situation of the paradox. From this initial situation a contradiction is alleged to result. It is contended that we are forced to this unhappy end by the following argument.

(1) The statement in box A says that the statement in box A is false.

Therefore,

(2) If the statement in box A is true, then it is false; and if the statement in box A is false, then it is true.

(1) is usually assumed to be obvious from the Initial Situation,

[5] For example, the versions of the paradox that involve the notion of «lying» seem to raise questions about the phenomenon that Zeno Vendler calls «illocutionary suicide».

and the validity of the transition from (1) to (2) is usually thought be elementary. I shall argue, however, that none of the plausible ways of construing (1) are such as both to make (1) true and to lead validly to (2).

(1) displays an initial problem. It presupposes that there is something, a statement, in box A. But what kind of a thing is a statement? Is it a proposition (i.e. a function from possible worlds to truth-values) or is it a sentence (i.e. a certain sort of sequence of words in a language) ? The enemy of the paradox might say that neither answer is satisfactory. Propositions, he might say, are not capable of being located in boxes. Sentences, he might say, are not capable of possessing truth-values. To argue this way, however, is sophistry. As long as we do not deny that propositions are capable of being expressed by sentences in certain sets of circumstances, there need he nothing mysterious about saying that propositions can be located in boxes, and there need be nothing odd about saying that sentences can have truth-values.

There is a more substantial difficulty, however, with the presupposition that there is a proposition in box A. For, let us make this assumption and see where it leads. In order to carry through the argument to a contradiction, (1) must be read as (3).

(3) The proposition in box A says that the proposition in box A is false.

But what does (3) mean ? It is initially tempting to suggest that for a proposition p to say that q is for p to *entail* the proposition that q. On this reading, (3) becomes (4).

(4) The proposition in box A entails the proposition that the proposition in box A is false.

But this interpretation of (1) is too weak to enable a contradiction to be derived. For, propositional entailment is not a symmetrical relation. From (4) and the truth of the proposition in box A we can derive the falsity of the proposition in box A;

but from the falsity of the proposition in box A we cannot derive its truth. Something stronger than (4) is needed. What is needed seems to be an appeal to propositional identity. (On my view of propositions, this simply amounts to the identity of functions.)

Thus, let us try to explicate (1) as (5).

(5) The proposition in box A is identical with the proposition that the proposition in box A is false.

But what is the proposition that a given proposition p is false ? This is either the proposition ø(p) or the proposition p(@)=F. Thus, (5) claims that there is a proposition b such that either (6) or (7) is true of it.

(6) b=ø(b).
(7) b=(b(@)=F).

But, I want to argue, there is no such proposition as b.

That there is no proposition such as the one (6) is about follows at once from the definition of ø. Two propositions are identical if they have the same truth-values at all possible worlds. And obviously ø(p) will never have the same truth-value at a possible world as p. If one assumes that there is a *b* such as (6) is about, then one can derive a contradiction. But there is nothing paradoxical about this: the derivation of the contradiction simply amounts to a *reductio ad absurdum* proof of what we knew already, namely that there is no such proposition. The situation here is exactly like that in the barber's paradox. The assumption that there is that certain barber leads to a contradiction; but that merely constitutes a *reductio* proof that there is no such barber.

In the same way it is easy to see that there is no such proposition as (7) is about. For suppose that there were a proposition p such that p=(p(@)=F). Then, from part I we know that p must be either necessarily true or necessarily false, since it is equal to p(@)=F. Assume p to be necessarily true. Then, in particular p(@)=T, and this implies that [p(@)=F](@)=F. But by substituting into p(@)=T, we get that [p(@)=F](@)=T, which is impossible. Now assume p to be necessarily false. Then, in particular p(@)=F, and this implies that [p(@)=F](@) =T. But by substituting into p(@)=F we get that [p(@)=F] (@)=F, which is impossible. Hence the supposition that p= (p(@)=F) leads to contradiction and is in error. Accepting (7) does, indeed, lead to a contradiction. But again this merely constitutes a *reductio* proof that there is no such proposition as (7) is about. The situation again is exactly like that of the barber's paradox.

The defender of the paradox might object at this point that the difficulties of (6) and (7), and especially the contradiction deduced from (7), merely confirm the paradoxical nature of the situation initiating the liar's paradox. But this objection misses the point. The defender of the paradox must show that the Initial Situation of the paradox leads inevitably to a contradiction. What I have shown is that each of (6), (7) leads to a contradiction. But what has not been shown is that the Initial Situation leads inevitably to either (6) or (7). The assertion that (6) or (7) is implied by the Initial Situation was assumed, not proved, in the above exposition. In the absence of proof to this effect, (6) and (7) have no more supporting them than does the proposition that there is that certain barber who shaves all and only those who do not shave themselves.

III.

The paradox might be resuscitated, however, if its defender could offer good reasons why the Initial Situation does imply a proposition such as (6) or (7). It is for this reason that the idea that in box A we have a *sentence* is more attractive than the idea that we have a proposition in it. For it is a certainty that the Initial Situation contains the fact that in box A there is a sentence. And it is quite plausible to hold that the Initial Situation implies that what this sentence *says* is that this sentence itself is false.

We can follow out this line of thought by explicating the intuitive idea that a sentence in a particular set of circumstances says (or: asserts, means, expresses) something. I define a *grammar* as a function G from ordered pairs of sentences and sets of circumstances ([6]) to propositions. With this notion of a grammar, one can define what it is for a sentence s to express (or: assert, mean) a proposition p in circumstances c, as follows:

Def. s in c expresses p if and only if $G((s,c))=p$.

The proposition that s in c is false can then be construed as either $\emptyset(G((s,c)))$ or else as $[G((s,c))](@)=F$.

The defender of the liar's paradox requires to say that the Initial Situation of the paradox implies that there is a sentence s and a set of circumstances c such that

(8) s in c expresses the proposition that s in c is false.

By the above explications, (8) means either

(9) $G((s,c))=\emptyset(G((s,c)))$

or else

(10) $G((s,c))=([G((s,c))](@)=F)$.

But from the examination in Part II of (6) and (7), we already know that there can be no such proposition as $G((s,c))$. That is to say, there is no possible grammar that makes (8) correct. If we *assume* that (8) is correct, then it is not surprising that contradictions result. But that they do result simply amounts to a *reductio* proof of the falsity of (8).

A defender of the paradox might say that (8), unlike (6) and (7), has actually been *shown,* and not merely assumed, to be implied by the Initial Situation of the paradox. But he is laboring under an illusion. There might be, I suppose, some weak sense of «expresses» (call it «W-expresses») in which it follows

([6]) An adequate notion of a grammar needs further specification, of course. But, I am arguing that no possible grammar can do what the defender of the paradox requires it to do; and, because I can do this without appealing to any further conditions on a grammar, I omit further specifications of the notion.

from the Initial Situation that there is a sentence s and a set of circumstances c such that

(11) s in c W-expresses that s in c is false.

But W-expression so far has no semantical import. That is, so far it does not have any implications for the existence of any propositions, much less for the truth and falsity of any propositions. From (11) there is no reason to think that (8) follows. Indeed, if (11) is correct, then (8) could not follow from (11), because (8) is not possibly correct.

On the view of propositions and truth that I have employed here, it therefore seems impossible to derive a contradiction from the Initial Situation. It does not matter whether we construe what is in box A as a proposition or as a sentence. One can, of course, proceed to *assume* that the Initial Situation implies some such claim as (6) or (7) or (9) or (10), and to deduce a contradiction from such a claim. But to do this is not to defend the paradox. It is only to show that such a claim could not possibly be correct, and hence that it could not possibly be implied by the Initial Situation.