REMARKS ON PRESUPPOSITION, TRUTH, AND MODALITY IN SUPERVALUATIONAL LOGIC

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I

In recent years there has been some attention paid to supervaluational techniques both by linguists and philosophers in dealing with various kinds of semantic presuppositions. Whether these methods are adopted or not, most writers on presupposition seem to believe that van Fraassen's original definitions in [3] and [4] lead to superlanguages that have the properties that he purports them to have. Specifically, it is claimed that we can define a language admitting truth-value gaps such that any sentence that is bivalent in an intuitive sense remains bivalent in the formal semantic interpretation. Failure of this requirement could amount to assigning truth-value gaps to completely innocent sentences like *it is raining.* But this is exactly what van Fraassen's formulation of presuppositional languages forces us to do.

In order to demonstrate the validity of this claim, let us turn to the commonly accepted definition of a «conservative superlanguage», i.e. those partial interpretations of a formal language that «admit as few truth-value gaps as possible» ([3], p. 73). Obviously, this is what we are looking for to prevent presupposition-free sentences from being truth-valueless. All we need in advance is a formal language L, a set V of (bivalent) valuations for L and a (necessitation-)relation N exhibiting the presuppositional structure on the formulas of L.

To develop my argument in a natural and well-known setting, let us assume that L is any language of propositional logic having a functionally complete set of connectives, and V is any set of functions from the sentences of L into the truth-set {T, F} that behave in the usual manner with regard to connectives. Accordingly, if «$\neg$» represents negation, $\neg$A is true iff (if and only if) A is false; If «.» represents conjunction, A.B is true iff A and B are true etc. A set X of sentences is *satisfiable* in V iff there is a valuation $v \in V$ such that $v(A) = T$ for each $A \in X$; X *entails* (classically) A in V iff A is true for all $v \in V$ that satisfy X (in symbols: $X \Vdash_V A$). N can be any two-place relation on the sentences of L, but it is reasonable to assume that no tautological sentence has a contingent presupposition. We therefore require N to be *normal* with respect to V which means that the following condition must always be fulfilled: if A N B and $\Vdash_V A$ (A is a consequence of the empty set), then $\Vdash_V B$.

Definition 1-1: A set X of sentences is *saturated* iff:

(a) X is satisfiable in V
(b) if $X \Vdash_V A$, then $A \in X$
(c) if $A \in X$ and A N B, then $B \in X$.

Saturated sets form the basis of supervaluations in a superlanguage. If X is such a set, the supervaluation s represented by X is that partial function from sentences of L into $\{T, F\}$ such that $s(A) = T$ iff $A \in X$, $s(A) = F$ iff $(\neg A) \in X$, s(A) is undefined ($s(A) = *$) iff neither $A \in X$ nor $(\neg A) \in X$. We therefore identify a *superlanguage* with the tupel $\langle L, V, N, V^* \rangle$ where V^* is an arbitrary set of saturated sets (or the set of corresponding supervaluations). Conditions (b) and (c) above express the requirement that X should be closed under $\Vdash_V$ and N. Writing «*Cl(X)*» for the smallest set containing X that is closed in this sense, we can reformulate (b) and (c) as: $X = Cl(X)$.

Of course, we have not yet delineated exactly which saturated sets should be contained in V^*. This is the point where «presuppositional policies» come in. Each of these policies is a way to fix V^* once L, V, and N are given. Implementing the so-called «radical policy», which allows all contingent sentences to lack a truth-value, $\langle L, V, N, V^* \rangle$ is a *radical superlanguage* iff V^* is the set of all possible saturated sets. To establish the class of conservative superlanguages, we need an auxiliary definition:

Def. 2-1: A saturated set X is called *maximal* iff there is some valuation $v \in V$ such that X is maximal (by set-inclusion) among the saturated sets satisfied by v.

Def. 3-1: $SL = \langle L, V, N, V^* \rangle$ is a *conservative superlanguage* iff V^* is the set of all maximal saturated sets.

We now say that a set of sentences X *entails A in SL* (in symbols $X \Vdash_{V^*} A$) iff for all $s \in V^*$, $s(A) = T$ whenever s assigns T to all formulas in X. Clearly, N and $\Vdash_{V}$ are subrelations of $\Vdash_{V^*}$. Moreover, if SL is radical or conservative and L is the language of propositional logic, one can prove that $\Vdash_{V} A$ iff $\Vdash_{V^*} A$ for all formulas A of L.

Having restated van Fraassen's definition of a conservative superlanguage, we are now in a position to construct the announced counterexample to his claim that

> (R) «all reasons for truth-value gaps must be reflected in V and N». ([4], p. 157).

Like van Fraassen we regard the reflection condition (R) as an informal criterion for an adequate implementation of the conservative policy. But note that our example — if it is to impeach the validity of (R) — must not be flawed by any intuitive doubts concerning the presuppositions of the sample sentences themselves.

For the sake of simplicity, let us restrict the atomic sentences of L to just *A*, *B*, and *P*, where *A* stands for the *king of France is bald*, *B* for *it is raining*, and *P* for the *king of France exists*. N will, of course, be $\{\langle A, P \rangle, \langle \neg A, P \rangle\}$ and V is to be the set of all possible valuations for *A*, *B* and *P*. Table 1 gives the four valuations that falsify the presupposition *P* of

A together with four admissible supervaluations. The lines between a supervaluation s_n and a valuation v_m indicate that the saturated set corresponding to s_n is maximal with respect to v_m.

	P	A	B	A≡B	A≡B	B	A	P	
v_1	F	T	T	T	—	T	—	F	s_1
v_2	F	F	T	F	F	—	—	F	s_2
v_3	F	T	F	F	—	F	—	F	s_3
v_4	F	F	F	T	T	—	—	F	s_4

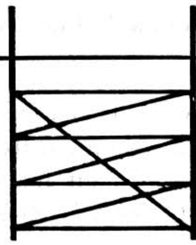


table 1

The following argument shows that $s_4(B)$ cannot be true or false. If B were true, it follows logically from the truth of $A \equiv B$ that A would also be true; since A presupposes P, P must be true in s_4, which is contradictory. Similarly, if B were assigned false, then A would have to be false, but since $\neg A \,\mathrm{N}\, P$, then once again P would be true in s_4. An analogous argument works for s_2(B); thus we get the unexpected result that *it is raining* comes out truth-valueless in s_2 and s_4, which clearly runs counter to the intuitive content of (R) !

But how to exclude s_2 and s_4 from the set of admissible valuations ? To see how this can be done in general, we have to examine the way saturated sets are built up. In fact, we can start with *any* set of sentences, construct its closure and eventually arrive at a saturated set. In the above example we simply start with $A \equiv B, \neg P\}$ and arrive at the saturated set Cl $\{A \equiv B, \neg P\}$. Excluding such unwanted supervaluations would, however, amount to barring the truth-values of such complex formules as $A \equiv B$ from arising deus ex machina. The proper remedy will therefore be a modification of the definition for saturated sets.

If X is a set of sentences of L, then B(X) (*the basis of* X) is the set of all sentences A∈X such that either A is an atomic formula or A≡¬B and B is an atomic formula. Intuitively, the basis of X represents the truth-values of all elementary propositions in X. A compound sentence A is *generated* by B(X) iff $A \in Cl(B(X))$.

Def. 1-2: A set X of sentences is *optimally saturated* iff:

(a) X is satisfiable in V
(b) $X = Cl\,(B(X))$.

Since neither $A \equiv B$ nor $\neg(A \equiv B)$ is generated by the basis of s_2 or s_4, neither supervaluation is optimally saturated. The definitions for radical and conservative superlanguages apply as above: *Def. 2-2 and* 3-2 result from 2-1 and 3-1 replacing «saturated» by «optimally saturated». And again it can be proved that $\Vdash_{V}$ A iff $\Vdash_{V^*}$ A.

As might have been expected missing generations from a basis may in themselves cause counterintuitive truth-values. Applying definition 3-1 with

$$N: = \{ <A, P>, <\neg A, P>, <B, Q>, <\neg B, Q> \}$$

we get the following truth-table in the case when both *P* and Q are false:

A	B		A	B	A∨B	A⊃B	A≡B	
T	T							
T	F		—	—	T	—	F	s_1
F	T		—	—	—	T	T	s_2
F	F							

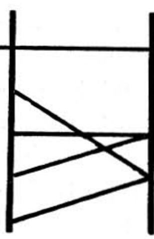


table 2

By s_1 a sentence like *the king of France or the king of Germany is bald* will come out true although both presuppositions are false ! But no sentence in table 2 is generated by a basis and consequently none can have a truth-value if we adopt 3-2 as the correct definition.

Note, by the way, that this last example should be carefully distinguished from the following situation: Suppose that A and P are false and B presupposes P. Although P is false it is consistent to say that (1) A.B is false and (2) A.B presupposes P. (1) is justified by two-valued logic as a consequence of $\neg$A, and (2) by the fact that P can be validly inferred from A.B in presuppositional (as opposed to two valued) logic. Thus the truth-value of A.B can be justified, whereas the value for A$\vee$B in table 2 comes out of the blue. Clearly, our present formulation of a conservative superlanguage is closer to the fulfilment of condition (R) than the original one. But aside from inspecting the new definitions and convincing ourselves that they are in accord with (R), we might want to have a formal counterpart of (R) that makes the modified proposal subject to a rigorous proof. The rest of part I of the paper will be devoted to the discussion of one such criterion that has recently been proposed by H. G. Herzberger. In [1] he introduces the notion of a presuppositionally complete language as follows:

Def. 4: If $SL = \langle L, V, N, V^* \rangle$ is a superlanguage, then SL is *p-complete* if for each $A \in L$, there is a set $X \subseteq L$ such that for all $s \in V^*$, s satisfies X iff $s(A) \neq *$.

Accordingly, if $s(A) = *$, there must exist a presupposition $C \in L$ such that $s(C) \neq T$ while $s'(C) = T$ whenever $s'(A)$ is defined. p-completeness captures the intuition that a language is able (at least to some extent) to «express its own presuppositional structure» ([1], p. 140). If I understand Herzberger correctly, p-completeness should figure as a condition of adequacy for any definition of a conservative language in the sense that p-incompleteness implies a violation of condition (R).

He first tries to show by example that 3-1 defines a class of SL's some of which are not p-complete and then he somehow makes the best out of this undesirable situation by using the criterion itself in the definition of a «policy» whose SL's will trivially fulfil his criterion. Apart from the questions that one could pose concerning the legitimacy of such a procedure, one must in addition observe that his counterexample has a serious defect. It is based on a language L that is itself «semantically incomplete», because L doesn't have a functionally complete set of connectives. As soon as we drop this implausible restriction, his example no longer constitutes a genuine counterexample to presuppositional completeness and it is in no way obvious how to construct one. Now, by comparing the example I have depicted in table 1, we see that any sentence C true in s_1 and s_3 must also be true in v_1-v_4 (follow the lines from right to left). But then C will be true in all valuations that satisfy s_2 (or s_4), hence C must be true in s_2 by V-entailment and it is impossible to find the desired presupposition for B.

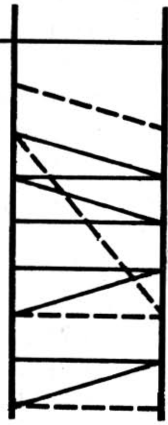
This opens up the possibility that p-completeness is indeed a plausible candidate for a precise formulation of (R). We then have to ask whether def. 3-2 always leads to p-complete interpretations, but the answer to this question is still negative. Let me give two relevant examples. The first smacks of paradox because N is defined as $\{<\neg(A \vee B), A \vee B>\}$. Since N must be normal, there are valuations $v, v' \in V$ with $v'(A \vee B) = T$ and $v(A \vee B) = F$. Consider two supervaluations s_1 and s_2 such that $s_1(A) = s_2(B) = F$ and $s_1(B) = s_2(A) = *$. Both are maximal with respect to v and it is easy to see that neither *A* nor *B* can have the desired presupposition. The second example is more instructive but admittedly more complex. Assume that

$$N := \{<(A \,.\, B) \mathrm{v} C, P>, \; <\neg((A \,.\, B) \mathrm{v} C), \; P>\}$$

and V is the set of all possible valuations over L. Table 3 lists all situations in which *P* is false.

Even if we restrict the admissible supervaluation to those maximal simply by set-inclusion (s_2 to s_6), we cannot find the required presupposition for *A* in s_5. To see this, follow the unbroken lines from right to left and then apply the same argument given above in the case of table 1. (The same procedure will work analogously for *B* and s_2 of table 3.) We must conclude that def. 3-2 still admits p-incomplete interpretations. What has happened is that any truth-value for *A* in s_5 would have generated $(A \, . \, B)\mathrm{v}C$ or its negation from the basis of s_5 and thereby would force a truth-value upon it. But in this example A does not seem to be as innocent as sentence B of table 1 and I believe that the supervaluations in question do *not* overtly conflict with the reflection condition.

	A	B	C
v_1	T	T	T
v_2	T	T	F
v_3	T	F	F
v_4	T	F	T
v_5	F	T	T
v_6	F	T	F
v_7	F	F	T
v_8	F	F	F



A	B	C	
—	—	—	s_1
T	—	F	s_2
T	F	—	s_3
F	T	—	s_4
—	T	F	s_5
F	F	—	s_6
—	—	F	s_7

table 3

To sum up, tables 1 and 2 should have convinced us that we are far better off with the modified definitions. Table 3 should have raised some doubt about the proposal that p-completeness, as an appropriate criterion, is on a par with the informal reflection condition (R). Unfortunately I have no idea which criterion can provide us with a rigorous proof or disproof in such a complicated matter but I suspect that any suitable formal condition to exhaust the content of (R) will turn out to be conceptually more complex than the proposed definitions 1-2 to 3-2. As a tool for testing adequacy, such a condition would of course be inappropriate.

II

In this part of the paper I want to show how sentential operators can be interpreted in a presuppositional superlanguage extending the framework of van Fraassen (1969 or 1971). A general solution of this problem should be completely independent from any special choice of a presuppositional policy discussed in part I, and we will use «SL» to refer to an arbitrary superlanguage based on propositional logic, its supervaluations being at least saturated in the sense of definition 1-1. But in what follows I cannot proceed «too generally», i.e. without any restriction on the logical behaviour of the operators to be examined; we therefore focus our attention on the truth-operator T and the modal operator $\Diamond$. The former will obey the usual truth-table matrix in three-valued logic:

(T) TA is true iff A is true, and TA is false iff A is false or undefined.

As for the latter, its semantics is designed to meet the challenge of Karttunen ([2], p. 171, fn. 3) who has observed that in ordinary language (I) is a valid scheme of inference:

(I) A presupposes B
A is possible
therefore, B.

In addition, I believe that (II) agrees with the reading of possible» in (I):

(II) $\Diamond$A is undefined iff A is undefined.

An alternative intuition could leave *A is possible* defined even if A is undefined in the actual world. We can keep in tune with (II) and then express *A is possible* by $\Diamond$TA, but note that following this path right from the outset destroys (I) as a valid mode of inference. In the terminology of Karttunen (1973) «possibly» is a «hole» which lets through all presuppositions of its arguments and this is exactly what will be achieved by requirement (II) as it stands.

Requirements (III) and (IV) ensure that $\Diamond$ is really «modal» in character:

(III) If $\Diamond A$ is true in a world i, then there is a world j accessible to i and A is true in j.

(IV) $\Box A := \neg\Diamond\neg A$ and the characteristic modal laws
$\Box A \supset A$ for T
$\Box A \supset A$, $\Box A \supset \Box\Box A$ for S4
$\Box A \supset A$, $\Diamond A \supset \Box\Diamond A$ for S5 ...
are valid in the respective modal systems.

So much for the intuitive foundations we have to mirror in the formal definitions to come. Before we start to work, some further motivating considerations are perhaps in order. First assume that T is the truth-operator in L. Any reasonable semantics for T in two-valued logic will make $TA \equiv A$ valid. This implies that $TA \equiv A$ and therefore $A \supset TA$ must also be true for all supervaluations in SL. Condition (T) requires that TA is false if A is undefined, but then $A \supset TA$ comes out undefined in supervaluational logic, which is contadictory. If T is not interpreted by V in L we have the trouble of making $TA \vee \neg TA$ valid in SL. By the equivalence, stated in part I, of V-validity and V*-validity for some SL's, $TA \vee \neg TA$ has to be valid in V, which is impossible if we adopt the truth-functional matrix-approach to handle truth-value gaps induced by sentences of the form TA. Of course, the assumption that L itself is a superlanguage is the first step in an infinite regress. A similar argument given later in the text applies to $\Diamond$. We conclude that these operators cannot be integrated into L and construe the appropriate extension L^+ of L, taking «$\neg$» and «v» as primitives in L and L^+, and «T» and «$\Diamond$» as primitives in L^+. The syntax of L^+ is as expected; in addition we introduce the following abreviations in L^+:

$FA := T\neg A$
$+A := TA \vee FA$
$—A := \neg + A$
$\Box A :\neg\Diamond\neg A$

Next consider a saturated set X of SL. We will extend this set (or supervaluation) to a set (or supervaluation) $X^+ \supset X$ of

SL^+ according to the «rules» laid down above. For instance, if $A \in X$, then $TA \in X^+$, if $\neg A \in X$, then $FA \in X^+$; if neither $A \in X$ nor $\neg A \in X$, then $-A \in X^+$. Moreover we want to retain classical propositional logic for X^+: if $A \in X$, then $AvO \in X^+$ and $\neg OvO \in X^+$, where O is a formula containing an operator; if $A = BvC$ and $B \notin X$, $C \notin X$ (the truth of A in X being necessitated by some further element in X) we must also have $(BvO)vC \in X^+$ etc.

These remarks suggest that the classical valuations V have to be extended as well. Definition 5 will prepare the ground for the necessary extensions

Def. 5: Let W be an indexing set for $V*$ (W is the set of possible worlds, and if $X \in V*$, X_i is the set of all sentences that hold in world i). For each $i \in W$, V_i is the set of all valuations $v \in V$ that satisfy X_i, and $\Vdash_{V_i} A$ iff $v(A) = T$ for all $v \in V_i$.

Since each X_i is closed under V-entailment, we have $X_i \Vdash_V A$ iff $\Vdash_{V_i} A$, and consequently $X_i = \{A \in L : \Vdash_{V_i} A\}$. For each $i \in W$ we define the bivalent function V_i^+ recursively as follows:

Def. 6: (1) $V_i^+(A,v) = v(A)$ for all $v \in V_i$ and atomic formulas $A \in L$

(2) $V_i^+(TA,v) = T$ iff $V_i^+(A,v') = T$ for all $v' \in V_i$

(3) $V_i^+(\neg A,v) = T$ iff $V_i^+(A,v) = F$

(4) $V_i^+(AvB,v) = T$ iff $V_i^+(A,v) = T$ or $V_i^+(B,v) = T$

We will add a fifth clause for the modal operator later on. (Note that the truth-operator behaves like a necessity-operator in S5, which might cast some light on the connection between supervaluations and modal logic.) As usual, $\Vdash_{V_i^+} A$ holds iff $V_i^+(A,v) = T$ for all $v \in V_i$. Clearly, $\Vdash_{V_i} A$ implies $\Vdash_{V_i^+} A$; thus

V_i^+ is an extension of V_i. Define

$$X_i^+ := \{A \in L^+ : \Vdash_{V_i^*} A\}$$

By the remarks above X_i is a subset of X_i^+ and it is easy to see that X_i^+ has the desired properties: If $A \in X_i$, then $v(A) = T$ for all $v \in V_i$; hence $V_i^+(TA,v) = T$ for all $v \in V_i$ and consequently $TA \in X_i^+$. Likewise $FA \in X_i^+$ if $\neg A \in X_i$. If neither $A \in X_i$ nor $\neg A \in X_i$ there are valuations $v,v' \in V_i$ such that $v(A) = T$ and $v'(A) = F$. Hence, $V_i^+(TA,v) = V_i^+(T\neg A,v) = F$ for all $v \in V_i$, therefore $\neg TA . \neg T\neg A \in X_i^+$ and by definition $—A \in X_i^+$. Since V_i^+ is a bivalent mapping all propositional tautologies are in X_i^+ together with all instances of the subsequent schemes:

(1) $TA \supset A$
(2) $TA \supset TTA$
(3) $\neg TA \supset T\neg TA$
(4) $T(A \supset B) \supset (TA \supset TB)$
(RT) TA if A is a tautology.

By analogy with modal logic we infer that (1)-(4) as axioms and (RT) as a rule are in fact all we can have for superlanguages in general. In this sense we can claim to have found a complete answer to the problem of interpreting a truth-operator in superlanguages.

Let us now turn to the final clause giving the semantics of the modal sign $\Diamond$. As always we need a two-place accessibility relation R on W that, in accordance with (IV), should be thought of as being at least reflexive in the modal system T. To guarantee the validity of $\Box A \supset A$, we need further auxiliary relations r on each V_i that must be reflexive as well. In case R is in addition transitive (and symmetric) all r's must be transitive (and symmetric) in order to match up with S4 (and S5), as will become clear in a minute. In the definition itself we will distinguish two cases, one where the operand A is defined and one where A is undefined in a world i. The former can be expressed by: $V^+_i(A,v) = V^+_i(A,v')$ for all $v,v' \in V$, the latter is just the negation of the former.

Def. 6: (5) $V^+(\Diamond A,v) = T$ iff:

a) $\begin{cases} V^+_i(A,v) = V^+_i(A,v') \text{ for all } v' \in V_i \text{ and there is some} \\ j \in W \text{ such that iRj and } V^+_j(A,v) = T \text{ for all } v \in V_j \end{cases}$

or

b) $\begin{cases} V^+_i(A,v) \neq V^+_i(A,v') \text{ for some } v' \in V_i \text{ and there is some} \\ v'' \in V_i \text{ such that } vrv'' \text{ and } V^+_i(A,v'') = T. \end{cases}$

Def.7: (1) $\Vdash_{V^+_i} A$ iff $V^+_i(A,v) = T$ for all $v \in V_i$ and all possible r on V_i

(2) $X^+_i := \{A \in L^+ : \Vdash_{V^+_i} A\}$

(3) $V^*_+ := \{X^+_i : i \in W\}$

(4) $SL^+ := \langle SL, L^+, W, R, V^*_+ \rangle$

(5) A is valid in SL^+ iff $A \in \bigcap \{X^+_i : i \in W\}$

It is now routine to check whether the resulting language SL^+ is as it should be with regard to (I)-(IV) above. For example, if A is defined in i, $\Box A$ is defined in i and $\Box A \supset A$ is true by the reflexivity of R. Assume A to be undefined; then $\Box A$ is undefined, $\Box A \supset A$ is undefined by quantification over all possible r. As an analogue of (II)

(5) $+\Diamond A \equiv +A$

will turn out valid in SL^+. Further

(6) $T\Diamond A \supset \Diamond TA$

(7) $T\neg\Diamond A \supset \neg\Diamond TA$

(8) $-(A \supset B) \supset (\Box(A \supset B) \supset (\Box A \supset \Box B))$

are valid in SL^+ and it is easily seen that, if A N B and $\neg$A N B, then the truth of $\Diamond$A implies the truth of B. As a consequence of the above definitions we observe that modal operators do not necessarily distribute over sentential connectives. Consider the following two examples:

Example 1: Assuming that *A* is the sentence *the king of France exists, B* is *the king of France is wise*, and

N:= { <*B, A*>, <$\neg$*B, A*> }, the sentence *A.B* is defined in all possible situations as a consequence of (II) above $\Diamond(A \, . \, B)$ will also be defined. Since $\Diamond$(A . B) expresses merely the possibility that there is a king of France who is wise, it is in accord with my intuition about failure of presupposition that $\Diamond(A \, . \, B)$ can be true even if *A* is false and *B* is undefined. But if *B* lacks a truth-value, $\Diamond B$ is undefined and since «true implies undefined» is again undefined, $\Diamond(A \, . \, B) \subset \Diamond B$ cannot be valid in our modal logics based on supervaluations (although it is in two-valued modal logics).

Example 2: Suppose that Q is A and P is B$\supset$A with A and B of example 1. P will be true iff Q is, and if Q is false, P is undefined. Note that necessary truths are no longer truths in all possible worlds, because it is sufficient for the truth of $\Box$P in i$\in$W that P is true in i and P is merely not falce in all j accessible to i. Again, this feature of $\Box$ seems to be in accord with natural language use: if A is said to be necessary, then we normally disregard those possible worlds where a presupposition of A does not hold. Thus, $\Box$P and $\Box$(P$\supset$Q) will be true if Q is, but $\Box$Q may be false in the same situation. This shows that $\Box(P \supset Q) \supset (\Box P \supset \Box Q)$ can be falsified, which is implausible in two-valued logic.

As was the case with the truth operator we now see that it is impossible to provide $\Diamond$A with a reasonable semantics within L. Any such attempt should make the crucial formulas of examples 1 and 2 valid, but then we inevitably get into conflict with the above stated conditions (I) to (IV) imposed on the logical behaviour of modal operators in partial interpretations.

In special circumstances however, the validity of these formulas can be saved, e.g. if P and Q are necessarily bivalent. Let us say that a formula A is *covered* iff all variables in A are in the scope of truth-operators occurring in A. We can then ascertain the validity of

(9) $\Box(A \supset B) \supset (\Box A \supset \Box B)$ if A and B are covered.

It is an interesting question whether (1)-(9), $\Box A \supset A$, and the syntactic rules $A \vdash TA$ and $A \vdash \Box A$ together with a complete axiomatic version of propositional logic themselves form a complete system in the sense that any formula not derivable from it can be invalid for some L^+ with R at least reflexive (and N possibly empty). But what seems more important to me is the following observation. Imagine some further one-place operator K! in L^+ that presupposes the truth of its complements. K! may be read *you know that A* and its logical properties might be the following:

(V) K!A is true in $i \in W$ iff A is true in i and there is no (epistemic alternative) $j \in W$ such that $iR^e j$ and A is false in j
K!A is undefined in $i \in W$ iff A is not true in i
K!A is false in $i \in W$ otherwise.

Since we do not want to restrict the arguments of K! to formulas of L, we cannot interpret K! within L and a moment's reflection will reveal that it is indeed impossible to represent (V) in our framework without introducing some additional machinery for a suitable definition of V_i^+ for K!A. What is needed, I propose, are functions f_i that choose arbitrary values T or F for K!A in the case that A is not true. We might then accept the provisionary (V') as an extension of definition 6:

(V') $V_i^+(K!A) = T$ iff
a) $V_i^+(A,v) = T$ for all $v \in V_i$ and there is no $j \in W$

such that $iR^e j$ and $V^+_i(A,v)=F$ for all $v \in V_j$; or

b) $V^+_i(A,v) \neq T$ for some $v \in V_i$ and $f_i(A)=T$.

Now, $A \in X_i^+$ iff $\Vdash_{V^+_i}$ for all possible choices of f_i. But (V') still has its drawbacks since K!*A* v ¬K!*A* is valid, while K!*A* v ¬K!*B* can be undefined even if *A* is logically equivalent to *B*. As this outcome is undesirable — at least in a formulation of epistemic logic that comes close to Hintikka's approach — we should rather take propositions, i.e. partial functions as arguments of f_i and then impose some restrictions on the possible choices of values for f_i. Nevertheless these modifications seem to be a bit ad hoc and we are back to the problem of representing presuppositions in a universal and homogeneous formalism.

In conclusion, I believe that there are no insurmountable technical difficulties in the way of keeping to the spirit of van Fraassen's ideas, but as soon as our formal language has a sufficient degree of expressive power to be useful, e.g. in linguistic inquiries, the theory's virtues (form a logical point of view) will lose much of its attractiveness, and considerable technical complexity will be the price to pay in any application of supervaluational techniques in a logical grammar. (*)

(*) The present paper evolved out of my Master's Thesis on presuppositions in a logical grammer, delivered at the University of Düsseldorf in June 1977. Thanks are due to J. Edmondson, P. Lutzeier, and D. Wunderlich, who helped to clarify my thoughts, and to Chr. Saunders who corrected my English.

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