ON THE INTERPRETATION OF DEONTIC LOGIC

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I

Deontic logic, we are told, is the logic of normative sentences and in particular sentences containing expressions like «obligatory», «forbidden» and «permitted». In all systems of deontic logic these words are treated as operators. A seemingly trivial but fundamental problem is *what* they are operating on. Some authors maintain that the deontic expressions operate on *actions*. But normally, logic deals with *sentences*, not with actions. Consequently, most authors let the deontic expressions operate on sentences to form new sentences (1). It is taken for granted that the sentences in the range of the deontic operators in some way describe actions, but, except for von Wright who has extensively treated the logic of action sentences, very little has been said on this topic by deontic logicians (2).

The models, which have been suggested in the semantic approaches to deontic logic have been strongly influenced by the current models of (alethic) modal logic. (3) One might hope that these semantic considerations would shed some light on the sense in which deontic sentences deal with actions. But I cannot see that the ideas about deontically better worlds are relevant to this problem (except in the trivial sense that it is assumed that in a deontically perfect world no action is performed which is morally bad).

It has been suggested that there are two kinds of deontic sentences. According to Castañeda,

> «deontic statements divide neatly into: (1) those that in-

(1) In his first paper (13) on deontic logic, von Wright lets the deontic operators operate on (generic) actions, but in later writings he lets them operate on sentences. Cf. the quotation in section 7.

(2) Von Wright's most extensive treatments can be found in (14) and (17).

(3) Cf. Hansson (7) and Hintikka (10).

volve agents and actions and support imperatives, and (ii) those that involve states of affairs and are agentless and have by themselves nothing to do with imperatives. The former belong to what used to be called the *Ought-to-do* and the latter to the *Ought-to-be.*» [4]

An example of an Ought-to-be statement is «There ought to be no pain». Castañeda claims that the traditional possible world semantics for deontic logic is applicable to Ought-to-be sentences, but not to the Ought-to-do. However, Ought-to-do statements seem much more interesting for moral philosophy than Ought-to-be statements. Castañeda gives a semantic analysis which «is not suited for the Ought-to-be; but, we hope, ...is adequate for the Ought-to-do.». [5]

In this paper I will present an alternative semantic approach to deontic logic which is more directly based on actions. Models or courses of action will be outlined which are somewhat more complicated than the kinds of models used so far in connection with deontic logic. I hope that this complication will be compensated by an improved understanding of the logic of normative sentences. The models will be used as a basis for a discussion of how the various formal languages of deontic logic can be interpreted and thus contribute to the understanding of what deontic logic is about.

II

If the deontic expressions operate on sentences that describe actions, then an analysis of action sentences seems important for the foundations of deontic logic. Donald Davidson's paper «The Logical Form of Action Sentences» provides an excellent starting point. I agree with his criticism of earlier attempts to analyse action sentences and I find his own analysis very illuminating. One of the main ideas in his paper is

(4) Castañeda (2), p. 452.
(5) *Ibid.* For a similar attempt, cf. Hilpinen (9).

that «there are such *things* as actions». (6) Davidson treats actions as logical individuals and, as a consequence, actions have properties and may occur within the scope of quantifiers. That actions should be treated as things or individuals in the logical sense is supported by examples like «He did *it* with his secretary» or «She did *it* on purpose».

When analysing action sentences, Davidson suggests that

> «verbs of action — verbs that say 'what someone did' — should be construed as containing a place, for singular terms or variables, that they do not appear to. For example, we would normally suppose that «Shem kicked Shaun» consisted in two names and a two-place predicate. I suggest, though, that we think of 'kicked' as a *three*-place predicate, and that the sentences be given in this form:
>
> (17) $(\exists x)$ (Kicked(Shem,Shaun,x))». (7)

In the sequel I will not need the details of Davidson's analysis; only the insight that actions should be treated as individuals on a par with 'ordinary' things will be used.

Individual actions may have different properties, or, what amounts to the same thing, be of different kinds. A class of individual actions which constitute all instances of a property of actions is called a *generic* action. Loosely spoken, generic actions are talked about by action verbs. «To jump» is a name of a generic action and the sentences «This morning he jumped into the swimming-pool» describes an individual action which is an instance of jumping. An individual action is normally an instance of several generic actions just as an ordinary thing has many properties. For example, a person may by a single action jump into the river, commit suicide, destroy her hair-do and beat up a flight of ducks.

As we will see, deontic logic deals mainly with generic ac-

(6) DAVIDSON (4), p. 84.
(7) *Ibid.*, p. 92.

tions rather than individual actions. I believe that much confusion comes from not distinguishing between individual and generic actions when interpreting deontic formulas.

Some of the things that humans do is better named *activities* than actions. Actions are performed at certain points of time — activities go on for some time. For simplicity I will confine the discussion to actions, although I am aware of the fact that also activities can be normatively regulated.

III

We have found that in the world there are, among all other things, individual actions. But which actions do in fact exist? Some actions are already performed, other actions will be but are not yet performed, and still other actions could have been performed but noone did and they are now no longer available and will never be available again because something else was done. Than an individual action exists is the same as it has been performed. An action which is done remains so, no matter how much we wish it undone. If an action comes to existence by being performed, it remains performed, i.e., it never ceases to exist. In this respect individual actions differ from 'ordinary' objects which may cease to exist.

Which set of individual actions there will be depends on what theory of action-identity you are willing to accept. I will here avoid the problem of identifying actions since my discussion is not dependent on which theory is chosen.

Let there, for the moment, be only one agent. This agent finds himself in different *acting situations*. In each acting situation he can choose to perform one action from the set of available actions. That the actions which are not performed are available means that the agent might have performed any one of them, had he chosen to do so. Which actions are available in a given acting situation is dependent on which actions he has chosen to perform in earlier situations and what else has happened in the world. I assume that the concept of action is general enough to warrant the conclusion that in every acting situations, no matter what the agent does, he must perform one of the available actions. Even the extremest form of passivity will count as acting.

In each acting situation *one* individual action is performed. The remaining available actions may not be available in the next acting situation (if you adher to a very discriminating theory of act-individuation, they will never be available anymore).

When the agent performs an action the world changes and a new acting situation comes up. Actions have consequences — if nothing else, the performance of an action entails that the action now exists which it did not do before. Furthermore, actions take time. I will avoid discussing how long it takes to perform an action, but in order to talk about individual actions and acting situations it is convenient to assume that actions are performed discretely (in contrast to continuously).

If the agent, in a certain acting situation, had chosen to perform another action than the one he did in fact perform, the resulting acting situation would probably have been different from the situation that did in fact arise. In this counterfactual acting situation a new set of actions are available which (most probably) is different from the set of actions available in the actual situation.

If we start from a particular acting situation, the future branches out due to the presence of alternative actions. Which branch will become the actual depends on what the agent does now, what he does next, and so on. The complete picture would therefore be a tree where each branch represents a possible *course of action*. Such a tree will be called an *action tree*.

So far I have only considered the acting of one agent. As a matter of fact the things done by other agents influence the course of events and some even say that the freaks of Nature determine what is true and false in different acting situations. If we take other agents' (including Nature's) actions into consideration, then the resulting tree of possible courses of action will be a *game tree* in the classical game theoretical sense, where an action by an agent corresponds to a move of a player. Making things simpler for myself, I will in the sequel assume that Nature is deterministic in the sense that only the actions of the agents determine what is true and what is not. Furthermore, I will not attempt at an analysis of normative statements which refer to two or more agents, and it will therefore not matter much whether we assume that there be one or many agents as long as only one agent at a time is acting. These simplifications of course weaken the claim that deontic logic is the logic of normative sentences, but they seem to be more or less standard.

IV

After this preparatory discussion about actions, acting situations, and action trees I am now ready to turn to the normative aspects of conduct. The action trees will be complemented in order to cope with notions of obligatory, forbidden and permitted actions.

In each acting situation there is a number of individual actions to choose between. Some of these actions may have consequences which are to be condemned on moral reasons, and some may not. The individual actions, available at an acting situation, which are morally acceptable I call the *deontic alternatives* at that situation. Since this is not a paper on morality I have nothing to say as to which actions in fact are deontic alternatives and which are not. I will simply assume that for each acting situation the set of deontic alternatives is given.

Also in situations where you have not done what you ought to there is a moral. The norms which apply to such situations are called 'contrary-to-duty imperatives'. To account for such norms it is assumed that there are deontic alternatives also in situations which have arisen from an acting situation where the agent did not choose one of the deontic alternatives at that situation.

In a secondary sense we may say that an acting situation y is a deontic alternative to another acting situation x, if there is some deontic alternative in x such that when it is performed the resulting acting situation is y.

It is possible that for some acting situation *all* of the available actions are deontic alternatives. In such a situation the norms in force do not rule out any of the actions as morally unacceptable. But is it possible that there are acting situations where *none* of the available actions is a deontic alternative ? Such a situation will be called a *predicament*. Von Wright has the following words on predicaments: (8)

> «From a 'practical' point of view this *is* an interesting case. It is a case which *can arise* and in which the agent *can do* various things. But whichever course of action he chooses he will of logical necessity stay outside the permitted region (since there is no such region). ...we can raise the question: is this (really) logically possible»

I will not *a priori* exclude predicaments from being possible situations in an action tree. If it really (no matter what happens) is obligatory not to kill it seems possible that one may end up in a situation where all the available actions are instances of (the generic action) killing. I will return to this problem in connection with the axioms for deontic logic.

V

In the next sections I will try to show how the action trees together with the division of actions into deontic and non-deontic alternatives can be helpful for the understanding of normative sentences. But before doing that, I will recapitulate a bit more formally the models of actions which have been outlined in the preceeding sections.

An *action tree consists* of the following components:

(i) A set of *acting situations*. I will denote individual situations x, y, and z. Each of the acting situations can be regarded as an ordinary model of standard predicate calculus, i.e., a universe consisting of individuals together with properties of

(8) Von Wright (17), p. 67.

and relations between individuals. The subset of the universe of an acting situation x which consists of all individual actions will be denoted A_x.

(ii) For each acting situation x, a set C_x of *consequent acting situations*. If y is a consequent situation to x, I will write x R y. If x R y, then I demand that A_y contains all individual actions of A_x and *one* additional individual action. This action is the one which has to be performed in x in order to make y the consequent situation. A consequence of this demand is that the transitive closure of R will be irreflexive, which means that one will never return to the same acting situation again, no matter what is done.

(iii) For each acting situation x, a subset D_x of C_x consisting of the *deontic alternatives* (or rather, the individual actions which correspond to the consequent situations in D_x) to x. I demand neither that D_x be a proper subset of C_x, nor that it be non-empty.

The relation R which connects acting situations yields a tree-like structure on the set of acting situations. If your favorite theory of act-individuation is not too demanding, it may happen that two branches of the action tree which start from some acting situation later connect again, e.g., if you first do *a* and then *b* you may end up in the same situation as when you first do *b* and next *a*. However, I do not think it matters very much for the interpretations of deontic logic if you assume that the relation R in fact is a proper tree relation. It will not be assumed, however, that action trees always have 'roots', i.e., first acting situations.

If x R y, then x preceeds y in time. I do not demand that the time interval between any two consecutive acting situations is always the same, but in other respects I have nothing to say about the temporal aspects of the action trees.

In general, I do not demand that the set of consequent situations of an acting situation be non-empty. Some action may cause the death of the agent, so in the resulting situation there is no action he can perform. Such situations are, in a sense, predicaments. If it is required that in every acting situation there be a non-empty set of available actions, then the action trees will necessarily be infinite and non-ending.

In the literature on the logic of action the idea of using tree-like structures is, of course, not new. Von Wright (17) calls his models «life-trees», Åqvist (1), who considers several agents including Nature, naturally calls his structures «game-trees». However, neither of these authors starts out from individual actions as primitive when defining the tree structures. In fact, Åqvist does exactly the opposite — he defines an individual action as an ordered pair of two consecutive nodes in the game tree and a generic action as a set of such ordered pairs. From a purely formal point of view it may not matter much in which end one starts, but I believe that the meaning or function of normative sentences will be better understood, if one starts out from individual actions.

Now we have a semantics — the action trees are designed for being used when understanding the logical form of normative sentences concerning obligation, prohibition and permission. But so far we have no formal logic because that presupposes a formal language. And I believe that it is by no means trivial how to interpret the components of a formal language for deontic logic. This is the main reason why I have started with semantic considerations — before evaluating the merits of an axiomatic system based on a formal language it is necessary to know what the formal expressions are about, and, when deontic logic is considered, this is more problematic than the logic itself.

VI

Before turning to the problems connected with the formal languages for deontic logic I will briefly discuss how the concepts exploited to construct the action trees can be used to interpret deontic sentences.

Take as an example the syntactically simple norm «Thou shalt not kill». We can reformulate it in the following way:

«In all acting situations it is forbidden to perform an action which is a man-slaughter». The expression «an action which is a man-slaughter» refers to the *generic* action «man-slaughter» and not to any particular individual action. Rewriting similar (unconditional) deontic sentences in this manner indicates that *what* is obligatory, forbidden or permitted is a generic action rather than any individual action. This supports the claim that deontic sentences are about generic actions.

However, in a secondary sense individual actions may be forbidden or permitted. An individual action is forbidden if it is an instance of *some* forbidden generic action. Similarly, an individual action is permitted if it is not an instance of any forbidden generic action. On the contrary, individual actions are seldomly obligatory. In an acting situation there is normally a large number of available individual actions. The set of deontic alternatives consists of those individual actions which are instances of *all* obligatory generic actions. Only when there is merely one deontic alternative in an acting situation it is possible to say that an individual action is obligatory. How often this happens depends on what criteria you have for identifying individual actions.

In the discussions of the interpretation of deontic sentences the distinction between individual and generic actions is not always upheld. In most systems of (monadic) deontic logic the formula «$P(p \vee q) \leftrightarrow Pp \vee Pq$» is a theorem , «P» being interpreted as «it is permitted that» and «p» and «q» being sentential variables which in some way describe actions. Sometimes, however, the equivalence «$P(p \vee q) \leftrightarrow pq \,\&\, Pq$» is proposed instead. Von Wright has the following motivation: (9)

> «If we are told that we may do this thing or that thing, we normally understand this to mean that we may do the one thing *but also* the other thing»

But «doing a thing» seems to correspond to performing an indi-

(9) Von Wright (18), p. 160. Cf. also von Wright (17), p. 21 and the discussion in Føllesdal and Hilpinen (5), pp. 22-23.

vidual action and I believe that von Wright is thinking of individual actions in the above quotation, although he asserts that the deontic formulas are about generic action. (10)

The distinction between generic and individual actions can be used to resolve the so called Ross' paradox. The formulas «$Op \rightarrow O(p \vee q)$» and «$Op \rightarrow Pp$», where «O» is interpreted as «it is obligatory that», are generally considered as logically valid. From these one can derive «$Op \rightarrow P(p \vee q)$», and since it is easy to slip from «$P(p \vee q)$» to «Pp & Pq», as we saw above one is tempted to conclude that «$Op \rightarrow Pq$» is logically valid, i.e., if anything is obligatory, then everything is permitted. But if it is remembered that the first two formulas are interpreted in terms of generic actions, while the derivation of «Pp & Pq» from «$P(p \vee q)$» must be interpreted in terms of individual actions to be valid, then the paradox disappears.

VII

Only now will I turn to a discussion of what deontic *logic* is about. Hitherto I have mainly been presenting some semantic concepts which I believe are useful when interpreting normative sentences formulated either in natural or formal languages.

A next to necessary prerequisite for a deductive system of logic is a formal language. The main justification for this seems to be a desire to limit and isolate the concepts at focus.

As regards deontic logic I believe that the choice of an appropriate formal language is more problematic than for many other kinds of so called intensional logics. Three kinds of languages for deontic logics have been suggested, viz. (i) monadic propositional logic where the deontic operators are unary, (ii) dyadic propositional logic where the deontic operators are binary, and (iii) quantified monadic logic where the

(10) As far as I can see this is not explicitly stated in (18), but von Wright stresses the distinction between individual and generic actions in (14) and (17).

deontic operators are unary. In this section I will concentrate on the monadic propositional logic and return to the others in the sequel.

Any formal language for deontic logic must contain means for expressing «obligatory», «forbidden» and «permitted». The most common symbols for these elements have been «O», «F» and «P» respectively. Often only one of these is taken as primitive and the others are then introduced via definitions.

It will be assumed that the deontic symbols operate on *propositional variables* which will be denoted «p», «q» etc. Propositional variables can be combined by the standard truth-functional connectives to form new well-formed expressions.

An expression of the form «Op» is normally read «it is obligatory that p». This reading, however, is problematic, since for many sentences, if substituted for «p», it is utterly meaningless. Here we encounter one of the basic problems of the interpretation of deontic logic: Which sentences may meaningfully be combined with the deontic operators ?

When Ought-to-do normative sentences are considered it is clear that the sentences which can be combined with the deontic operators should be about actions. But are all action sentences allowed ? As was hinted at in the previous section, it is not sentences that are obligatory, forbidden or permitted but rather generic actions. In fact, when von Wright constructed his first system of deontic logic, he let the deontic symbols operate on generic actions. (11) However, since the deontic operators have generally been considered as some kind of modal operators, taking sentences as arguments to form new sentences, von Wright later changed the reading of the deontic expressions. He remarks that: (12)

> «against this reading, however, it may objected that it does not accord very will with ordinary usage. Only seldom do we say of a state of affairs that *it* is permitted,

(11) Von Wright (13).
(12) Von Wright (17), p. 16.

> obligatory or forbidden. Usually we say this of actions. But it is plausible to think that, when an action is permitted, etc., then a certain state of affairs is, in a secondary sense, permitted, etc., too.»

If one wishes to maintain that the deontic operators have sentences as arguments it is thus most natural to let these be sentences which describe generic actions. A propositional variable «p» will be read «an action of the sort A_p is done» and, consequently, «Op» will be read «it is obligatory that an action of the sort A_p be done».

This reading amounts to a restriction of the scope of the deontic operators — only sentences of the above form are allowed as arguments of the operators. I believe, however, that the suggested reading is compatible with most views on what the Ought-to-do deontic operators operate on. As was mentioned above, some authors have argued that the arguments of the deontic operators should be sentences and some that they should be actions. The compromise presented here, not because it is a compromise but because it seems to be the simplest way of giving a coherent interpretation of the deontic expressions, is to use sentences that describe generic actions.

Castañeda (2) suggests that

> «deontic operators of the Ought-to-do type (of course, not perhaps those of the Ought-to-be type) are operators on prescriptions».

Prescriptions are defined as structures consisting of an agent and a generic action. Prescriptions cover a wider range of expressions than those of the form «an action of the type A is done» since different agents may be considered. I have here made the simplifying assumption that only one agent at a time is acting, so the norms in a given acting situation apply only to one person. And if the prescriptions are restricted to a fixed agent they will correspond to expressions of the form «an action of the type A is done».

If we try to extend the suggested reading of the deontic expressions by means of propositional connectives we encounter another problem. The formule «p & q» would, if read straightforwardly, mean «an action of type A_p is done and an action of type A_q is done». But, as Hintikka points out, [13] this may mean either (i) that an action of the sort A_p is done in some acting situation and an action of the type A_q is done in some (perhaps different) acting situation, or (ii) that in some acting situation an action is done which is an instance of both the generic actions A_p and A_q. The interpretation (i) gives rise to insurmountable problems. The most blatant example is that «p & —p» may be true. The other interpretation has the consequence that we have to take all formulas to be concerned with *one* particular acting situation. This seems to out down considerably the applicability of the formulas of monadic propositional deontic logic and correspondingly abate the interest in the topic. I believe, however, that if a consistent and meaningful interpretation of the formulas of the monadic language is aimed at, then the only way is to restrict the reference of the formulas to particular acting situations.

Once this restriction is accepted, the interpretation of compound propositional formulas present no problems. Since generic actions are conceived of as sets we can create new generic actions by the ordinary set-theoretical operations. A convenient name of the generic action $A_p \cap A_q$ is $A_{p \text{ and } q}$. Similarly, $A_p \cup A_q$ will be called $A_{p \vee q}$ and the complement of A_p will be called A_{-p}. This naming allows us to extend the above reading of propositional variables to any propositional expression whatsoever.

VIII

We have now obtained an interpretation in words of the deontic formulas of the monadic language. In this paper the discussion of the validity of deontic formulas will not be based on an axiomatic system of deontic logic. Instead I will use the

(13) Hintikka (10), p. 65.

action tree models to formulate truth conditions for such formulas. Let us begin with formulas which consist of one of the deontic operators followed by a purely propositional expression.

The intended meaning of the deontic alternatives in an acting situation is that one of these individual actions *ought to* be performed. A generic action A_p is obligatory in a given acting situation iff every deontic alternative is an instance of A_p. This leads us to the following formal truth condition:

(O) A formula «Op» is *true* in a given acting situation x iff *every* deontic alternative in x is of the sort A_p.

Similar considerations motivate the following truth conditions for «permitted» and «forbidden».

(P) A formula «Pp» is *true* in a given acting situation x iff *some* deontic alternatives in x is of the sort A_p.

(F) A formula «Fp» is *true* in a given acting situation x iff *no* deontic alternative in x is of the sort A_p.

These definitions make the familiar equivalences «$Op \leftrightarrow - P - p$», «$Pp \leftrightarrow - Fp$» and «$Fp \leftrightarrow O - p$» true in all acting situations.

Using the standard interpretation of the propositional connectives it is easy to derive truth conditions for formulas which are built up by connectives from formulas of the above kind. I will postpone the discussion of formulas of mixed modalities, i.e., purely propositional expressions combined with deontic formulas, and formulas containing iterated deontic operators.

We will say that a formula is *valid* iff it is true at all acting situations (in all action trees). It can be noted that for this definition we need no assumption concerning the tree structure of the acting situations.

Before directly turning to the problem of which formulas are valid, I will turn back to a seemingly minor point in the description of action trees. There I left open the question whether it is possible that there be *no* deontic alternatives in some acting situations (either because there are no available actions at all or because of a predicament). In such situations some of the formulas traditionally considered valid in deontic logic will not be so, if the above truth conditions are accepted. Any generic action will be obligatory (because all deontic alternatives are trivially instances of any generic action) but noone will be permitted (because it is not true that some of the deontic alternatives fulfils the requirements). Hence, the formula «Op → Pp» will not be true in such situations and therefore not valid. However, it is true that if anything at all is permitted, then anything obligatory will also be permitted, and thus the formula «Pq → (Op → Pp)» is valid even if predicaments or action-less situations are accepted.

For the same reasons the formula «— O(p & — p)» is not true in acting situations where there are no deontic alternatives. However, the weakened formula «Pq → — O(p & — p)» is valid.

If it is assumed that in every acting situation there is at least one deontic alternative, then the reader who is familiar with the techniques of modal logic can easily show that the set of valid sentences are exactly the theorems of the logic which has been called the 'standard system of monadic deontic logic'. This system is axiomatized by some standard axioms for propositional logic together with e.g., the following axioms:

(A0) O(p ∨ — p)
(A1) O(p & q) → Op & Oq
(A2) — O(p & — p)

The rules of inference are substitution and modus ponens.

As we have seen, if the above assumption is not made, (A2) has to be replaced by:

(A2') Pp → — O(p & — p)

These results are gratifying. Our semantic considerations give rise to a logic which on the whole is the standard system for monadic deontic logic.

Some authors advocate a formal language for monadic deontic logic with contains formulas with iterated deontic operators, as e.g., «OOp», or formulas of mixed modalities, as e.g., «p & Op». As regards formulas with iterated operators I do not see how they could be interpreted in a way that is consistent with the reading of formulas proposed here. Deontic operators are applicable on sentences that describe generic actions in a particular acting situation, but a formula «Op» does not describe any generic action and thus an expression of the form «OOp» is meaningless under the suggested reading.

Is it possible to give an interpretation of a mixed formula where propositional connectives are used to combine purely propositional expressions with deontic expressions ? We have taken propositional variables to mean «an action of the sort A is performed» and thus, for example, the formula «p & O — p» would read «an action of the sort A_p is (was) performed, but it is (was) obligatory that an action of the sort A_{-p} be performed». Here again it should be pointed out that, if troubles are to be avoided, this reading must be relative to some particular acting situation.

If we try to provide truth conditions for mixed formulas, we observe that there is nothing in the action tree models to inform us on which action is in fact performed in a given acting situation. In order to obtain the truth values of the propositional variables, it is therefore necessary to complement the action trees with an *assignment* of 'true' of 'false' to each variable in each acting situation in the tree. (It is interesting to observe that we need not know anything about the truth of purely propositional expressions in order to determine the truth values of (purely) deontic expressions). Once this complementation of the action trees is done, I see no problems in extending the language to mixed formulas. However, as far as the deontic expressions are concerned, we learn nothing new about them in this way.

IX

It soon turned out that the language of monadic deontic logic is insufficient for representing the logical structure of certain normative statements. In particular so called 'contrary-to-duty imperatives' have attracted the attention of moral philosophers. These norms are of the form «if an action of the type A has been performed, then it is obligatory that an action of type B be performed». Here «A» is supposed te be a forbidden generic action.

In order to cope with the problems created by the contrary-to-duty imperatives and some other types of normative sentences, a more sophisticated formal language was introduced. This language is based on a *dyadic* deontic operator «O(—/—)» where the blanks are supposed to be replaced by purely propositional expressions. It is customary to read an expression of the form «O(p/q)» as «it is obligatory that p under circumstances q».

The above reading is, as in the monadic case, not free of problems. Which kinds of sentences can be substituted to the left and to the right in the dyadic deontic expressions ? I will take it, without further argument, that the sentences in the left place are of the form «an action of the sort A is done», i.e., the same kind as in the monadic case. But what, then, are 'circumstances' ?

Some circumstances are 'factual' truths about the world — sentences describing facts which are independent of the agent's actions. Some norms are obligations (permissions or prohibitions) which apply only under certain circumstances, e.g. «If you are taken ill with a contagious disease, it is obligatory that you avoid contact with other people». Such 'factual' circumstances may of course occur to the right in the dyadic deontic expression. As far as I see, they present no problems of interpretation.

Other circumstances arise because of actions performed by the agent. Consider, for example, the following contrary-to-duty imperative: «If you have hurt somebody's feelings, then you ought to apologize». This norm applies when the circumstances are that you have hurt somebody's feelings, which has become true because of an earlier action in your course of actions. When circumstances are described by action sentences, it therefore seems natural to interpret the expression «O(p/q)» as «If in an earlier acting situation, an action of the type A_q has been done, then, in the present acting situation, it is obligatory that an action of the type A_p be done». This reading, however, suffers from the same kind of ambiguity as we encountered in the monadic case. The if-clause can mean (i) that in some earlier acting situation or another an action of the sort A_q has been done or (ii) that in some particular acting situation, e.g., the one preceding the present acting situation, an action of the sort A_q has been done.

For most conditional norms the interpretation (ii) seems to be the more natural. Again, consider the norm «If you have hurt somebody's feelings, then you ought to apologize». Suppose you hurt somebody's feelings in the acting situation x. Then, if y is the consequent acting situation, you ought to apologize in y. But suppose you do not apologize in y and your action in that situation leads to the situation z. Is it still obligatory that you apologize in z ? This may be the case, but I believe there are exceptions. An extreme example is that in the situation y you may have killed the person whose feelings you have hurt.

Although I have no conclusive argument, I believe that the interpretation (i) of dyadic deontic expressions leads to difficult problems as regards the reference of different circumstance-describing sentences. In order to avoid a lengthy discussion I will in the sequel stick to the interpretation (ii) and outline what dyadic deontic logic will be like when this interpretation is accepted.

Disregarding problems of consistency, it should be noted that both of the interpretation (i) and (ii) refer to *two* acting situations, i.e., some situation taken to be the present situation and some situation preceding the present situation. This is a fundamental feature of dyadic deontic logic and the reason why the monadic variant is essentially weaker than the dyadic language. This insight also explains why all attempts to formalize contrary-to-duty imperatives and other conditional norms within the language of monadic deontic logic have proven to lead to absurdities.

X

I will now turn to truth conditions for dyadic deontic expressions. It was pointed out earlier that 'circumstances' may be either 'factual' or 'action describing'. Since I believe that truth conditions for factual circumstances are comparatively innocuous, I will here concentrate on circumstances that refer to generic actions. This means that in an expression «O(p/q)» both «p» and «q» will be interpreted as sentences of the form «an action of type A is done».

The basic idea for a general truth condition is that if an action of type A_q is performed in the acting situation preceding x, then a formula «O(p/q)» is true in x, if every deontic alternative in x is of the type A_p, and otherwise the formula is false. However, this condition does not tell us anything about the status of the formula when the action performed in the preceding situation is not of the type A_q. Because of this we do not formulate truth conditions for 'true in an acting situation' but turn directly to the more general 'true in an action tree'.

A formula «O(p/q)» is *true in an action tree* iff in all acting situations in the action tree where an action of the type A_q is performed in the preceding acting situation, every deontic alternative is of the type A_p. A formula «O(p/q)» is *false* in an action tree iff it is not true in the action tree.
Corresponding truth conditions can be given for formulas of the type «P(p/q)» and «F(p/q)» or such expressions can be introduced by definitions from «O(p/q)» in the standard way. Truth-functional compositions of deontic expressions are regulated by standard truth conditions for the connectives. We say that a formula is *valid* iff it is true in all action trees.

Owing to the assumed tree structure of an action tree there is always, for each acting situation, at most one 'preceding' acting situation. A small snag is that if the action tree has a beginning, there will be one situation, the 'root', which has no preceding acting situation. This problem can be circumvented by stipulating that the only generic action that was performed in the situation preceding the 'root' was the one described by a tautology.

A consequence of the truth conditions for «O(p/q)», which may not seem pleasant, is that «O(p/q & — q)» is (vacuously) valid for all formules p, i.e., everything is obligatory under contradictory circumstances. But, since such circumstances never arise under the interpretation of dyadic formulas chosen here, this can be dismissed as a logical oddity.

I will not give an axiomatization of the logic corresponding to the definition of validity given here, but merely discuss the validity of a few formulas which have caused some controversy among deontic logicians. Before that, however, the deontic structure of formulas with fixed circumstances will be investigated.

If predicaments or acting situations where no action is available are possible, then the same kind of problems arise as in the monadic case and these can be treated in a similar way. For simplicity, I will therefore assume that in every acting situation there is some deontic alternative.

The following formulas are valid:

(B1) $O(p \vee - p/q)$
(B2) $O(p/q) \,\&\, O(r/q) \leftrightarrow O(p \,\&\, r/q)$
(B3) $-O(p \,\&\, - p/q)$, if q is not a contradiction.

Thus, if we keep the circumstances fixed and if they are noncontradictory, then the dyadic system will satisfy the principles of the monadic system.

I will next turn to formulas in which different circumstances are mentioned. The strength of the assumption that the interpretation (ii) of dyadic formulas be used will now become apparent.

As can be easily checked, the following formula, which is an axiom in von Wrights first system for dyadic deontic logic (cf. (14)), is valid:

(B4) $O(p/q) \,\&\, O(p/r) \leftrightarrow O(p/q \vee r)$

The formula (B4) has encountered severe objections from logicians who, following Hansson (6), base their semantic analysis of dyadic deontic logic on an ordering of possible worlds into more or less morally perfect worlds. If substitution and propositional logic is accepted we can derive the following formula from (B4):

(B5) $O(p/q) \rightarrow O(p/q \,\&\, r)$

David Lewis criticizes this formula as follows: [14]

> «Several aziomatic or semantic treatments of conditional obligation are open to serious criticism because they validate inferences from '*Given that ø, it ought to be that* ψ' to '*Given that* $ø_1$*, and* $ø_2$ *it ought to be that* ψ', or conversely. Neither direction ought to be valid, since it seems that we can have consistent alternating sequences ... For instance: '*Given that Jesse robbed the bank, he ought to confess; but given in addition that his confession would send his ailing mother to an early grave, he ought not to; but given that an innocent man is on trial for the crime, he ought to after all...*'.»

I take this example not as a counterexample to (B5), but as an example of the difference between 'ought' as used in ordinary language in the Ought-to-be sense and 'obligatory' as used in Ought-to-do norms. If some action is *obligatory* under circumstances q, then it is obligatory no matter what else we come to know about the circumstances. Lewis seems to be aware that a distinction is needed since he adds in a footnote in connection with the above quotation: [15]

> «'obligation' is here used in a special, impersonal sense. What is obligatory (conditionally or unconditionally) is

(14) Lewis (11), pp. 102-103.
(15) *Ibid.*, p. 100.

what ought to be the case, whether or not anyone is obligated to see to it.»

On the whole, it seems as if Hansson's idea of constructing semantic models of deontic logic from more or less morally perfect worlds is useful when analysing Ought-to-be norms, but gives misleading consequences when applied to Ought-to-do norms.

I believe that when using this kind of semantics for deontic logic, which on the whole is transferred from the semantics used when analysing modal logic, one misses some of the essential features of actions. In particular, the temporal aspects of actions (in which situation an action is performed) are overlooked. (16)

Von Wright has held vascillating opinions about his own axioms. It was pointed out to him that the following formula is a consequence of his original axioms: (17)

(B6) $O(p/q) \rightarrow - O(-p/r)$

Von Wright says about this in (16): (18)

> «This is manifestly absurd. Generally speaking: From a duty to see to a certain thing under certain circumstances nothing can follow logically concerning a duty or non-duty under entirely different, logically unrelated circumstances.»

Under the interpretation of dyadic formulas suggested here the formula (B6) is true in an action tree if and only if the circumstances «q» and «r» both are true for some acting situation in the tree. To see this, we note that if p is obligatory under circumstances q then an action of type A_p is obligatory

(16) For further arguments against the semantics based on more or less morally perfect worlds, the reader is referred to Goldman (6). As far as I can see, the semantics presented in this paper evades Goldman's criticism.

(17) By P. Geach. The derivation can be found in (16).

(18) VON WRIGHT (16), p. 104.

in all acting situations where an action of the type A_q has been performed. If an action of the type A_r has been performed in any of these acting situations, i.e., if the circumstances in some situation is q & r, then p is obligatory in this situation and —p cannot be obligatory under circumstances r. However, «under entirely different, logically unrelated, circumstances» (B6) is not valid under the interpretation suggested here.

A final example exhibits a formula which is valid under the Ought-to-be interpretation, but not under the interpretation presented here. The following formula comes out valid in the Hansson-type semantics: ([19])

(B7) $P(p/r) \,\&\, P(q/p \,\&\, r) \rightarrow P(p \,\&\, q/r)$.

This formula is one of the axioms in Rescher's system of dyadic deontic logic in (12). The following example, essentially due to A. R. Anderson, shows that, under the interpretation of the deontic operators given here, the formula is not valid. Let «p» stand for «he is smoking», «q», «he is not smoking», and «r», «he goes to the smoking car». Then «P(p/r)» means that «if he in the previous acting situation went to the smoking car, he is now permitted to smoke» and «P(q/p & r)» means that «if he in the previous acting situation were smoking and went to the smoking car, then he is now permitted not to smoke». Both these formules seem to express reasonable norms. «P(p & q/r)», on the other hand, means that «if he in the previous acting situation went to the smoking car, then he is now permitted to (simultaneously) smoke and not smoke». The last sentence is never true and thus (B7) is not a valid formula.

The reason why (B7) comes out valid in the Hansson-type semantics is that the temporal features of the dyadic operator disappear when one thinks of possible worlds instead of acting situations. A «p» in the right position of a dyadic expression refers to an action that is already performed, but a

([19]) Cf. HANSSON (7), p. 145.

«p» in the left position refers to an action that is not yet performed.

The results of the investigations in this section suggest that the appropriate dyadic deontic logic is similar to von Wright's original system. An exception is that (B3) is only valid for non-contradictory circumstances. I have no completeness proof, however, so I do not know which formulas could be used as an axiomatic base.

XI

I have now, to some extent, studied monadic and dyadic propositional deontic logic. In this section I will turn to quantificational deontic logic. My discussion will, however, be confined to a few brief remarks.

I have argued that in order to obtain a satisfactory interpretation of monadic propositional deontic logic it is necessary that the formulas be concerned with one particular acting situation. The formulas of dyadic propositional deontic logic should be interpreted as being about two acting situations and, I believe, they are best understood if they are taken to refer to two consecutive situations.

There are, however, norms that are not covered by these formal languages. Consider, for example, «For an orthodox Muslim it is obligatory that he at least once in his life go on a pilgrimage to Mecca». There is no acting situation about which we can say that it is obligatory in this situation that the Muslim go on a pilgrimage, but the content of the norm is that in *some* acting situation he ought to go on a pilgrimage.

In (10), Hintikka argues that in order to codify such norms satisfactorily it is necessary to use a quantificational language as a basis for the deontic concepts. Hintikka allows the deontic operators to operate on any kind of formulas, not only formulas describing singular actions. Thus both «(x)OAx» and «O(x)Ax» should be meaningful according to his analysis. He gives a few examples of how some norms can be translated into a language of this kind, attempting to show that the order between the quantifier and the deontic operator is important. He does not, however, provide any general rules for how quantified deontic formulas ought to be interpreted. I suppose that he takes his semantic rules to account for the interpretation of these formulas, but I find these rules too much influenced by corresponding rules for (alethic) modal logic. Hintikka's description of 'deontic alternatives' does in no way take into account the order in which actions are performed. He explicitly rejects the assumption that individual actions which exist in a situation necessarily also exist in the deontic alternatives to that situation. This assumption is central for the description of action trees as presented in this paper.

Here I will not discuss how quantified deontic formulas ought to be interpreted. The tree structure of the action trees, which has not been used to the fullest extent when interpreting the monadic and dyadic languages, will become important now. It may very will be that the ordinary language for predicate calculus together with the deontic operators is not sufficiently strong to codify the logical structure of different kinds of norms, but has to be supplemented with further intensional operators e.g., expressions referring to the temporal ordering of actions.

XII

Now, finally a summary of the story presented here.

The theorems of a formal system of deontic logic are not interesting until you know what they mean. This paper has dealt very little with theorems and derivations and instead concentrated on the interpretation of the formulas of deontic logic.

I started from the assumption that normative sentences, which are the objects of deontic logic, are about actions. On the basis of an elementary understanding of actions, I then outlined semantic models for norms. Not until this was done I turned to the problem of interpreting the different formal languages for deontic logic.

The formulas of monadic propositional deontic logic were found to consider only one acting situation at a time. The so called paradoxes of contrary-to-duty imperatives and commitments arise when the meaning of these kinds of norms, which essentially refer to more than one acting situation, is squeezed into the monadic language. With the aid of dyadic logic one is able to handle two different actions — what has been done and what is to be done. Using a quantificational language it may become possible to give a unified analysis of still other kinds of norms.

In our days, the *logic* of normative sentences, i.e., the class of theorems in a certain formal language, is not interesting *per se*. The powerful techniques, developed by modern modal logicians, for proving the completeness and decidability of a formal system make it more a matter of patience than of ingenuity to find the appropriate axiomatic system once an elaborate interpretation of the language is given and truth conditions for the formulas are put forward.

I do not claim that the interpretations presented here are the only possible ones, but at least they provide a consistent way of reading deontic formulas. However, I hope that this paper has shown that, as far as deontic logic is concerned, problems related to interpretation and semantic models are far more important and intricate than axioms in a (badly understood) formal language.

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