RENOUNCING EXISTENTIAL PRESUPPOSITIONS*

A. MARRAS

It is well known that the «existential presuppositions» of traditional quantification theory with respect to the (free) individual terms of a system cannot the *stated* in the object-language of the system. The object-linguistic expression '$(\exists x) x = a$,' as it occurs in a system with existential presuppositions (a P-system), is not a symbolic translation of the (presumably) contingent statement that *a* exists, but, given a nonempty domain, is instead a logically true statement derivable from the axiom of identity '$(x)x = x$'.

Although the existential presuppositions of a P-system are *inexpressible* in the sense just explained, it is commonly thought that they are nonetheless *shown* by the validity of statements of the form

(1) $\psi a \rightarrow (\exists x) \psi x$

sanctioned by the rule of existential generalization. And some have thought that the natural way to eliminate the existential presuppositions of a system is to deny the validity of (1), and therefore the rule of existential generalization. A result of thus allowing nondesignating terms in a system is that the expression '$(\exists x)x = a$' is no longer logically true but, given the ordinary interpretation of the existential quantifier, can be taken to provide a satisfactory translation of the ordinary language statement that *a* exists. This, indeed, is one of the virtues of systems of «free logic»: as Hintikka has pointed out, the expression '$(\exists x)x = a$' enables us to give formal expres-

(*) Some of the ideas developed in this paper originated in an earlier paper of mine entitled «Properties and Beliefs about Existence,» *Logique et Analyse* 13 (1970), 438-451. I am grateful to W. Demopoulos for his helpful comments on an earlier version of this paper.

sion to Quine's dictum that to be is to be a value of a bound variable. (1)

An outright rejection of (1), however, seems to me to have unacceptable consequences for a semantical interpretation of a system. The *mere truth* of the statement that Socrates is wise would no longer suffice to warrant the inference that (there exists) someone (who) is wise. To warrant such an inference we would be required to have the *further* premise that Socrates exists. And the fact that Pegasus does *not* exist, though it forestalls the unwanted inference that something flies from the statement that Pegasus flies, does nothing to prevent us from construing 'Pegasus flies' as *true*. Indeed, proponents of free logics have claimed that such statements as 'Pegasus flies' or 'Moby Dick is a white whale' are no less true than statements like 'Socrates is wise'. (2) This result I find undesirable, at least in so far as we wish to take the classical (Aristotelean-Tarskian) conception of truth seriously. For to say that a statement is true, according to this conception, is to say that it asserts something true about the world: and there is nothing in the world that would make the statement 'Pegasus flies' *true* (what Greek mythology speaks of is not part of the actual world). And to say that 'Socrates is wise' is true is to say that some one thing in the world, namely the space-time object we call 'Socrates', has the property of being wise. In other words, classical semantic theory has taught us that a subject-predicate statement is true when, and only when, the predicate (open sentence) from which the statement results (by replacing the free variables by individual terms) is *true of* the objects designated by the individual terms — such objects being within the range of values of the quantifiable variables. But what can 'x is a winged horse' or 'x flies' be true *of* ? Unless we allow Pegasus among the objects over which the quantifiers are allowed to range (thus allowing Pegasus the same ontological status as Bucephalus and Napoleon's horse), the first predicate is true of nothing, the second predicate is true of something but not of

(1) J. Hintikka, *Knowledge and Belief* (Ithaca, N.Y., 1962), p. 130.
(2) *Knowledge and Belief*, p. 131.

Pegasus, as there is no Pegasus. Hence 'Pegasus is a winged horse' and 'Pegasus flies' cannot, in classical semantic theory, be counted as *true*. (3) And this much of classical semantic theory is, I think, as it should be.

But how can we have a system without existential presuppositions and yet retain the validity of a schema like (1) ? It seems to me that we can, with certain qualifications. To see this we need only consider that in a system with Russellian descriptions a statement of the form '$\psi(\iota x \varphi x)$' entails '$(\exists x)\psi x$'; i.e. the following analogue of (1)

(2) $\psi(\iota x \varphi x) \rightarrow (\exists x)\psi x$

(where the scope of the description does not extend past '$\rightarrow$') is valid, and yet, as is well known, a system with Russellian descriptions is free of existential presuppositions with respect to descriptions. That is, vacuous descriptions, albeit definitionally eliminable, are allowed in the object language of the system.

Why then do we suppose that the validity of (1) guarantees the existential presuppositions of a system in which it occurs, whereas the validity of (2) does not ? The following considerations, I suggest, will provide an answer to our question.

When in a P-system a term 'a' occurs in a sentence 'ψa' which is part of a larger sentence '$\chi(\psi a)$', the larger sentence, provided it is *extensional*, (4) can always be taken to be of the same form as 'ψa' in any theorem in which 'ψa' occurs, e.g. in (1), and can therefore be substituted for 'ψa' in (1). Thus the following schema

(3) $\chi(\psi a) \rightarrow (\exists x)\chi(\psi x)$

can be taken to be of the same form as (1) and is therefore valid. An instance of (3) could be:

(3) In a two-valued logic, that means they must be counted as *false* if admitted as meaningful subject-predicate statements at all.

(4) Extensional instances of '$\chi(\psi a)$' may be '$\sim\psi a$', '$p \vee \psi a$', '$(\exists y)(p \,.\, y = a)$', but not 'It is possible that ψa'.

(3a) $\sim\psi a \rightarrow (\exists x)\sim\psi x.$

What justifies the validity of (3), intuitively, is the idea that in a P-system any (extensional) open sentence whatsover (e.g. '$\sim\psi x$') can be construed as a predicate which says something, truly or falsely, about some or all of the values of the variables; and since such values are (pre) supposed to be actually existing objects, *whatever* an open sentence says, truly or falsely, about any given object, it must also say, truly or falsely, about *some* actual object. Thus (3) may be interpreted as resulting from (1) by substituting '$\chi(\psi x)$' for 'ψx', and therefore as being just an *instance* of (1). Thus in a P-system (1) may be said to be valid for *any* substitution value of 'ψx', where *any* (extensional) open sentence containing a free 'x' can be a substitution value for 'ψx'.

Analogous considerations do not apply, in general, to non-P systems. In a system with Russellian descriptions, for example, it is in general *not* the case that whatever we say, truly or falsely, about the so-and-so, we can also say, truly or falsely, about some actual object. When a description occurs in a sentence '$\psi(\iota x\varphi x)$' which is part of a larger (extensional) sentence '$\chi\{\psi(\iota x\varphi x)\}$', we cannot in general infer that the open sentence '$\chi(\psi x)$' is true (or false) of some object. There is no straight-forward analogue of (3) in description theory. The following sentence

(4) $\chi\{\psi(\iota x\varphi x)\} \rightarrow (\exists x)\chi(\psi x)$

is systematically ambiguous, for it could mean either

(5) $[\iota x\varphi x]\chi\{\psi(\iota x\varphi x)\} \rightarrow (\exists x)\chi(\psi x)$

or

(6) $\chi[\iota x\varphi x]\psi(\iota x\varphi x) \rightarrow (\exists x)\chi(\psi x)$

depending on whether the scope of the description is the whole of '$\chi\{\psi(\iota x\varphi x)\}$' or only the part '$\psi(\iota x\varphi x)$'. (5) is of course valid; (6) is not. (The following sentences

(5a) $[\iota x\varphi x] \sim \psi(\iota x\varphi x) \rightarrow (\exists x) \sim \psi x$
(6a) $\sim [\iota x\varphi x]\psi(\iota x\varphi x) \rightarrow (\exists x) \sim \psi x$

may be taken to be instances of (5) and (6) respectively.)

It is precisely the kind of ambiguity involved in contexts like (4), and the *absence* of any such ambiguity in contexts like (3), that, I am suggesting, unequivocally reveals whether a system is presupposition-free or not with respect to its individual terms. The ambiguity involved in (4) is essentially an ambiguity of scope, and when, and only when, the scope is the larger, as in (5), then (4) (as can be seen if we unpack the sentence in accordance with Russell's theory) may be taken as an *instance* of (2) itself, just as (3) is an instance of (1). This last consideration reveals that, when nonatomic predicates are allowed as substituends for 'ψ', (2) is itself ambiguous, for it does not tell us, in general, which statements are proper instances if it. Hence (2) should be rewritten, more properly, as

(2*) $[\iota x\varphi x]\psi(\iota x\varphi x) \rightarrow (\exists x)\psi x$

which unambiguously indicates that any nonatomic predicate '$\chi(\psi)$' replacing 'ψ' will generate a sentence where the description has the larger scope. (When the predicate is atomic, i.e. when the description occurs in a sentence which is not part of a larger sentence, the scope symbol is of course superfluous.)

No ambiguity of scope, on the other hand, affects the validity of (1); and since the ambiguity of scope involves essentially an ambiguity of logical form, we can say that no ambiguity of logical form affects the validity of (1). (3) is an unambiguous instance of (1); that is, substitution of 'ψ' by '$\chi(\psi)$' in a valid schema like (1) does not alter the basic structure of the schema.

To conclude this part of the discussion we may then say that the existential presuppositions of a system with respect to its individual terms are revealed by two considerations: (A) that schemas like (1) are valid in the system, and (B) that schemas like (3) are *unambiguous instances* of the valid schema (1). It follows that in order to free a system of its existential presuppositions one needs *either* to reject (A) *or* to reject (B). The usual tack by recent «free logicians» has been to reject (A). Rejection of (A), however, is only a sufficient, not a necessary, condition for rejecting existential presuppositions. Maintaining (A) while rejecting (B) instead seems to me an equally effective and, in view of the previous considerations, semantically preferable alternative.

Let us see, briefly, what is involved in this alternative. In the presuppositions-free system we are envisaging, the logic of Russellian descriptions will be taken as a *model* for the logic of *all* individual terms (names and descriptions). Unlike Russellian descriptions, however, we shall not require that individual terms be *eliminable* from the object language of the system. Although the existence-condition will not be *presupposed* (but explicitely *entailed,* as for Russell's descriptions) by the use of individual terms, the *uniqueness* condition *will* be (i.e. we shall presuppose that individual terms refer, or purport to refer, to exactly one individual).

We shall let free variables be place-holders for any individual term, vacuous or nonvacuous. The statement that 'a' is nonvacuous, i.e. that *a* exists, shall be expressed in the object language by '$(\exists x)x = a$', which in turn shall be abbreviated by 'E ! a'. In addition, I shall stipulate that a truth-condition for a statement of the form 'ψa' is that 'a' be nonvacuous. (Thus 'Pegasus is a winged horse' will turn out false, just as in Russell's theory of descriptions 'The present king of France' turns out false.) The other truth-condition, of course, will be that the predicate 'ψx' is true of *a*. These truth-conditions may be expressed in the system by means of the following equivalence schema:

(7) $\psi a \leftrightarrow (\exists x)\,(x = a\,.\,\psi x)$ (5)

(It is an obvious consequence of this schema that the following presupposition-free analogue of (1)

(5) It may be noted that in P-systems (7) is a derived theorem (cf. W.V.

(1a) $\psi a \rightarrow (\exists x)\, \psi x$

is valid.)

It follows from (7) that a sentence of the form 'ψa' can be *false* either if 'a' is vacuous, or, if 'a' is nonvacuous, if 'ψx' is not true of *a*. Thus the contradictory of a sentence of the form 'ψa' will be ambiguous precisely in the way the contradicory of a Russellian descriptive statement is ambiguous. In general, whenever 'a' occurs in a sentence which is part of a larger sentence '$\chi(\psi a)$', the same sort of ambiguity will arise as in the case of descriptions. That is, a question will arise as to whether the whole of '$\chi(\psi a)$' or only the part 'ψa' is to be taken as the 'ψa' in (7). The ambiguity will be avoided by introducing a scope distinction for (free) individual terms, as Russell does for descriptions, by prefixing a bracketed instance of the individual term to the sentence which is to be taken as its scope. A sentence like 'ψa' will thus unambiguously be written as '$[a]\psi a$', and the equivalence schema (7) will be rewritten, in a more general form, as

(7*) $[a]\psi a \leftrightarrow (\exists x)\,(x = a\,.\, \psi x)$

where 'ψx' may be replaced by any extensional open sentence containing no quantifiers. ([8])

Given (7*), we can now unambiguously distinguish between the two contexts '$[a]\chi(\psi a)$' and '$\chi[a]\psi a$' as follows:

(8) $[a]\chi(\psi a) \leftrightarrow (\exists x)\,\{x = a\,.\, \chi(\psi x)\}$
(9) $\chi[a]\psi a \leftrightarrow \chi(\exists x)\,(x = a\,.\, \psi x)$

Quine, *Word and Object* (Cambridge, Mass., 1960), p. 178). Here it is a definitional equivalence.

([8]) The stipulation that 'ψx' contain no quantifiers is required in order to avoid nontheorems like '$[a]\sim(\exists y)y = a \leftrightarrow (\exists x)\,\{x = a\,.\sim(\exists y)\ y = x\}$ A similar stipulation is one that Russell must make in description theory in order to avoid such nontheorems as '$[\iota x\varphi x]\{\sim(\exists y)y = (\iota x\varphi x)\} \rightarrow (\exists x)\sim(\exists y)y = x$', which would result from substitution of '$\sim(\exists y)y = x$' for 'ψx' in (2*).

(8) is merely an instance of (7*), from which it follows by replacing 'ψx' by '$\chi(\psi x)$'; (9) follows from (7*) and the extensionality axiom '$(p \leftrightarrow q) \rightarrow [f(p) \leftrightarrow f(q)]$' by letting 'p' be the LHS of (7*) and 'q' the RHS.

The following

(8a) $[a] \sim \psi a \leftrightarrow (\exists x)(x = a \, . \sim \psi x)$
(9a) $\sim [a]\psi a \leftrightarrow \sim (\exists x)(x = a \, . \, \psi x)$

are instances of (8) and (9) and resolve the ambiguity of scope arising in negative contexts. It is easy to see that '$[a] \sim \psi a$' implies '$\sim [a]\psi a$', but not conversely. That is, the following is valid:

(10a) $[a] \sim \psi a \, . \rightarrow \sim [a]\psi a$

However, with the additional premise that *a* exists, the converse implication also holds. Thus we have

(11a) $E \, ! \, a \rightarrow (\sim [a]\psi a \leftrightarrow [a] \sim \psi a)$

(10a) and (11a) are instances of the general schemas

(10) $[a]\chi(\psi a) \rightarrow \chi[a]\psi a$
(11) $E \, ! \, a \rightarrow \{\chi[a]\psi a \leftrightarrow [a]\chi(\psi a)\}$

which can be shown to be valid by an inductive proof which we shall not give here. [7] (11) shows that, when 'E ! a' is given, the scope distinction collapses. In such a case, the individual terms will behave exactly as they do in P-systems. A P-system can thus be viewed as an *extension* of our system, and the consistency of the former will guarantee the consistency of the latter.

Let us now briefly consider two consequences of our system.

1. It will be noted that 'a = a' (e.g. 'Pegasus = Pegasus') turns out *false* when a does not exist. This does not appear to

[7] The proof involves showing that since 'χ' contains no quantifiers, '$\chi\{\ \}$' is reducible to one of the forms: (1) '$p \vee \{\ \}$', (2) '$p \vee \sim \{\ \}$', (3) '$\sim p \vee \{\ \}$', (4) '$\sim p \vee \sim \{\ \}$'. The rest of the proof requires the use of the rules of confinement for the quantifiers.

me, as it would e.g. to Hintikka, to be an undesirable result, for there is a well known and to me philosophically plausible sense of existence according to which to exist, to be an entity, is to be self-identical; and a nonentity such as Pegasus ought not to be expected to be self-identical. This view of existence as self-identity can be shown to be a formal consequence of (7*): replace 'ψ' by 'a =', simplify, and by the definition of 'E ! a' we get:

(12) $E!a \leftrightarrow a = a$

2. Our distinction between '∼[a]ψa' and '[a]∼ψa' serves essentially the same purpose as V. Wright's distinction between *weak* and *strong negation* ('It is *not* the case that *a* is ψ' and 'It *is* the case that *a* is *not*-ψ' respectively) — at least when the latter distinction is used to distinguish the case in which 'a is ψ' is false because of the nonexistence of *a* from the case in which it is false because 'ψ' is not true of *a*. When *a* does not exist, neither 'a is ψ' nor its *strong* negation are true, while the *weak* negation *is* true. On our account, when *a* does not exist, neither '[a]ψa' nor '[a]∼ψa' are true, while '∼[a]ψa' *is* true. Thus our account captures precisely the sort of distinction with which Von Wright is concerned, without, however, postulating *two distinct concepts or senses* of negation, as Von Wright does. For the negation symbol in Von Wright's weak negation functions as a *sentential* operator, whereas in strong negation it functions as a *predicate* operator. But *our* negation symbol functions both in '∼[a]ψa' and in '[a]∼ψa' as a sentential operator. Our distinction is thus not between two *kinds* of negation but between two *occurrences* of the same kind of negation: in one case it occurs outside the scope operator, in the other case it occurs inside. Thus, given an independent introduction of the scope operator (e.g. to distinguish 'χ[a]ψa' and '[a]χ(ψa)', our distinction recommends itself over Von Wright's on grounds of economy in the use of primitive notions.

University of Western Ontario A. Marras