QUANTUM LOGIC AND THE STATUS OF CLASSICAL LOGIC

Edward ERWIN

Logic is often thought to be an a priori discipline: its principles are not confirmed or disconfirmed by laboratory experiments, or by the use of empirical techniques of any kind. Instead, logicians come to know truths of logic in an a priori manner: through reflection or calculation; or at least, they could do that. Even if truths of logic are sometimes known through experience, they are at least *knowable* a priori. What explains this fact, it is often said, is that principles of logic are necessarily true when true and necessarily false when false.

I shall refer to the above view of logic as «the traditional view» or «the classical view», and shall assume that it is expressed by a conjunction of the following two principles:

1) Correct principles of logic are necessarily true (i.e., they hold in all possible worlds).
2) Correct principles of logic are a priori, i.e., they are knowable a priori (roughly, they are knowable without support of empirical evidence).

In recent years, the traditional view of logic has been challenged by a group of writers, some of whom appeal to quantum mechanics in support of their view. Quine, for example, has referred to the proposal that the law of excluded middle be abandoned as a means of simplifying quantum mechanics (1) and Putnam has argued that the development of quantum logic makes it reasonable to reject the distributive laws and with them the traditional view of logic. (2)

(1) W.V. QUINE, «Two Dogmas of Empiricism», reprinted in *Necessary Truth,* edited by L.W. Sumner and John Woods (New York: Random House, 1969), p. 137.

(2) Hilary PUTNAM, «Is Logic Empirical ?», *Boston Studies in the Philosophy of Science, Vol. V,* edited by R. Cohen and M. Wartofsky (Dordrecht, Holland: D. Reidel, 1969).

In what follows, I shall try to determine what implications quantum mechanics has for the classical view of logic. Since Putnam has developed the most widely known and most detailed account of these matters, I shall concentrate mainly on his argument.

I

Putnam compares logic to geometry and argues that just as the General Theory of Relativity shows that we live in a non-Euclidean world, Quantum theory shows that we live in a world with a non-classical logic. (3) The statements we encounter in every day life obey a classical logic, but that is because the corresponding subspaces of H(S), where «H(S)» represents a certain infinite dimensional vector space, form a very special lattice, a so-called «Boolean lattice». Furthermore, quantum mechanics, according to Putnam, explains the approximate validity of classical logic «in the large», i.e., in everyday life, just as a non-Euclidean geometry explains the approximate validity of Euclidean geometry «in the small».

Putnam's main reason for contending that quantum mechanics refutes classical logic is that quantum logic, the logic recommended for quantum mechanics by Birkoff, Von Neumann, Finkelstein and others, is said to be non-distrbiutive. It contains analogs of the classical distributive laws,

$$p \,.\, (q \vee r) \equiv (p \,.\, q) \vee (p \,.\, r)$$

$$p \vee (q \,.\, r) \equiv (p \vee q) \,.\, (p \vee r)$$

and these analogs turn out to be false. What is unclear, however, is whether the analogs and the distributive laws are identical. Putnam's main argument for interpreting them that way is that by so doing we can avoid certain quantum mechnical paradoxes, in particular the two-split and orbital-electron

(3) Ibid., p. 218.

paradoxes. The key idea is that each of the derivations of the pertinent paradoxical conclusions employs at least one of the distributive laws. Hence, by giving up the distributive laws, we can short circuit the derivations and hence avoid the unwanted conclusions.

One natural reply to the above argument is to claim that the meaning of the word 'or' is partially expressed by certain logical laws including the distributive laws; consequently, in any alternative logic in which analogs of the distributive laws turn out to be false, the symbol 'v' in these analogs must not mean what it does in standard truth functional logic. Thus, these analogs can be denied without giving up the distributive laws. Putnam, however, claims to bypass this «change of meaning» objection. [4] I think he means to argue that the paradoxes of quantum mechanics are evidence against the assumption that the meaning of 'or' is partially expressed by these laws.

One might be dissatisfied with the above reply because one doubts that Putnam has shown that the paradoxes are evidence against the distributive laws. To show that, he would presumably have to: (1) provide reason for thinking that rejecting these laws are sufficient for avoiding the paradoxes; and (2) show that taking this way out is superior to alternative routes. Concerning (2), we would not want to reject the distributive laws unless we were given an explanation of why they can be safely used in our reasonings about macro-phenomena when they cannot be used in our reasonings about quantum phenomena. Without such an explanation, a good many of our seemingly sound experimental reasonings about macro-phenomena would be rendered dubious, which would probably make it simpler to retain the distributive laws and reject one of the non-logical assumptions leading to the paradoxical conclusions. Putnam does say that quantum mechanics provides such an explanation, but he does not say what it is; consequently, his argument is inconclusive. Furthermore, there are reasons for

(4) Ibid., p. 232.

doubting that rejecting the distributive laws would be sufficient for eliminating the paradoxes. (5)

In light of the above, one might reasonably doubt that Putnam has shown that evidence from quantum mechanics refutes the distributive laws. Perhaps, however, the above mentioned difficulties are minor; perhaps Putnam's argument can be supplemented and made convincing. If it can, what implications does that have for the traditional view of logic ?

II

Suppose we accept the following assumption, (A): if empirical evidence can disconfirm the distributive laws, they are a posteriori and contingent. We might also assume (B): if the distributive laws are a posteriori and contingent, it is likely that most or all of the other principles of logic will be so as well. If (A) and (B) are true, we can refute the traditional view of logic if we can demonstrate the antecedent of (A) by showing that empirical evidence from quantum mechanics does in fact disconfirm the distributive laws.

It is not clear that Putnam is relying on the above argument: but if he is, there is a rather obvious way to render assumption (A) implausible.

Consider the following: «148 + 371 + 814 + 541 = 1874»

Even if the above statement is knowable a priori and is necessarily true, someone might obtain empirical evidence which would make it reasonable for him to believe that it is false. For example, consider a man who has evidence that he is not very good at doing sums. He finds that he gets different answers for the same sum when he adds more than a few figures and needs to re-check any such addition in order to be reasonably sure that he is correct. This same man might have substantial evidence that if a certain computor does similar sums, it will not err. The man might then get empirical evidence that

(5) On this point, see Michael R. Gardner's interesting article, «Is Quantum Logic Really Logic ?», *Philosophy of Science* (1972).

the above statement of arithmetic is false if he learns that the computor, which unknown to him has malfunctioned, has printed out an answer of «1873» for the above sum. Since all of us might get evidence that we are inept in doing sums, all of us might get empirical evidence against the same statement in the above described manner. Once we had reason to reject the above statement, we might subsequently get evidence that the computor in question had malfunctioned and, by using other computors, get empirical evidence that the statement is true.

The same kind of argument can be used to show that a law of logic, such as one of the distributive laws, can be empirically disconfirmed or confirmed, even if it is necessarily true and knowable a priori. We might, first, get empirical evidence that we often make mistakes when we reflect upon logical laws having the degree of complexity of the distributive laws. When we accept or reject such laws after doing a truth-table analysis we do somewhat better, but even then we often make mistakes, even after re-checking our calculations. Under such conditions, we might obtain empirical evidence that one or both of the distributive laws are false by looking at the output of a computor. We might have good evidence that the computor is much more likely to be right than we in any disagreement between it and us over logical laws of this degree of complexity. The computor might then, unknown to us, break down and endorse the denial of one or both of the distributive laws. Given the evidence that the computor is not likely to err, we could then get empirical evidence against the distributive laws by discovering that the computor has rejected them.

The above kind of argument is hardly novel, and yet the result will strike many philosophers as paradoxical; for there appears to be good reasons for holding that if any of the laws of logic are empirically disconfirmable, then they are neither necessary nor a priori. I now want to look at some of these reasons.

III

First, it might be claimed that 'necessary' simply means 'not empirically confirmable or disconfirmable', (6) and, therefore, that it would be inconsistent to hold that logical laws are necessary and empirically disconfirmable. If that is how Putnam is using the term, however, then showing that none of the laws of logic is necessary in this sense does not, without additional argument, refute the first thesis of the classical view. In this thesis, 'necessary truth' means 'a truth which holds in all possible worlds'. In this explanation, nothing is said or implied about the possibility of getting empirical evidence for or against a necessary truth. A statement might be true in all possible worlds and yet on the basis of the empirical evidence available to us, we might be reasonable (although mistaken) in believing it to be false. Furthermore, showing that the laws of logic, one and all, fail to be 'necessary' in the sense of 'not empirically confirmable or disconfirmable' would not, without additional argument, refute (2) either. Laws of logic might be knowable a priori, as required by thesis (2), and yet be empirically confirmable or disconfirmable. If the intuitions of logicians can provide adequate evidence for accepting principles of logic, and if such principles can be known on the basis of such non-empirical evidence, then they are knowable a priori even if additional evidence obtained from empirical observations might outweigh such intuitive evidence and serve to disconfirm some of these principles.

A second reason for saying that if laws of logic are testable (i.e., either empirically confirmable or disconfirmable) they must be contingent, concerns their alleged analyticity ;and this reason would be endorsed by at least some holders of the traditional view of logic. It is widely held that if the laws of

(6) Gardner, for example, claims that there is a sense of 'necessary' according to which a sentence is a necessary truth just in case any evidence whatever would confirm it, i.e., just in case it could never be rational to abandon belief in its truth. See: Michael R. Gardner, op. cit., p. 508.

logic are necessary, they are also analytic and if they are analytic, they are empty. To cite a standard example «It will rain or not rain», it is alleged, does not tell us anything about the weather, nor does it tell us anything about our future experiences. No matter what we experience and no matter what the weather conditions, the statement will remain true. Even if this is so, however, that does not show that such statements are not testable. If a statement is empty, its truth or falsity is not determined by what occurs in the world; but that does not show that what happens in the world is incapable of affecting our reasons for believing the statement to be true. Some complicated statement of mathematics which I accept may be empty in the relevant sense, but the discovery that a seemingly infallible computor has rejected the statement might give me justification for believing it to be false. If it does, and if I discover that to be so through observation, I will have empirically disconfirmed the statement even if it is empty.

To put the matter slightly differently: a statement might fail to be about anything in the world and, hence, nothing in the world might be capable of affecting its truth value. The statement of the distributive laws may be like this. Yet what occurs in the world might affect our reasons for accepting or rejecting the statement: we might obtain empirical evidence that our previous reasons for thinking the statement necessary were incorrect and that the statement is not true. That might happen even if the statement is empty.

A third argument against my view might run as follows.

1. If any statement of logic, S, is necessary it is a priori.
2. If S is a priori, it is not a posteriori.
3. If S is empirical, it is a posteriori.
4. Hence, if S is necessary it is not empirical.
5. If S is empirically testable, it is empirical.
6. Therefore, S is not both necessary and empirically testable.

The above argument assumes that if statements of logic are necessary they are also a priori, and that may be reasonably doubted. (7) However, of more importance is the fifth assumption that if S is empirically testable, it is empirical. There is an established philosophic usage according to which a statement is empirical if it has empirical content, and it has empirical content if it is empirically testable (i.e., is either empirically confirmable or disconfirmable). If we use «empirical» in this sense, it does not follow from a statement's being empirical that it is a posteriori. «Empirical», in this sense, does not mean «a posteriori», and it cannot be assumed, without further argument, that the two terms are extensionally equivalent. Once again, it may be that we can know some statement of mathematics to be true in an a priori fashion, e.g., by reflecting on what the statement says: yet, that does not preclude the possibility of empirically confirming or disconfirming the statement. If we can be justified in believing the statement to be true by reflecting on it, we do not need to get supporting empirical evidence, but we can: for example, by checking the output of a computor or consulting expert mathematicians. Furthermore, our being justified on intuitive grounds does not rule out the possibility of discovering empirical data which would overrule our intuitions and which would disconfirm the statement. Thus, if «empirical» means «testable», premise 3 is either dubious or false.

Is there also a sense of «empirical» in which it means «a posteriori». It may be that philosophers are simply making a mistake when they equate the empirical and a posteriori, just as it is a mistake to equate the necessary and the a priori, but if not, then «empirical» is used in two distinct senses: to mean «testable» and to mean «a posteriori». In that case, statements of logic may be *empirical* in the first sense, the sense in which Putnam is using the term, but not the second. In any event, demonstrating that logic is empirical in the first sense does not

(7) For an argument against that assumption, see: Saul Kripke «Naming and Necessity», *in Semantics of Natural Languages,* ed. D. Davidson and G. Harman (Dordrecht Holland, 1971) and Edward Erwin, «Are the Notions 'Necessary Truth' and 'A Priori Truth' Extensionally Equivalent ?», *Canadian Journal of Philosophy* (1974).

show that it is empirical in the second sense, the sense in which it means «a posteriori» (if there is such a sense). Hence, even if all necessary truths are a priori, and no a priori truth is a posteriori, we cannot prove that truths of logic are contingent merely by proving that they are «empirical» in the sense of *being testable.* The above argument, therefore, commits the fallacy of ambiguity if it employs «empirical» in different senses in premises 3 and 5: and if «empirical» is employed in a single sense, either step 3 or 5 is dubious or false. Hence, the above argument fails.

A fourth argument which might be used against my view is due to Quine, and has recently been stated succinctly by Sumner and Woods. Referring to Quine's «Two Dogmas of Empiricism», they write:

> «The main force of his arguments is located in the last two sections of his article. There he says explicitly that no statement is immune to revision. If so, then there is no true statement whose falsity is impossible; but this is to say that no true statement is necessary, that there are no necessary truths. [8]

Quine, of course, does not merely *say* that no statement is immune to revision. He presents a very plausible view of confirmation which, in turn, supports the idea that any statement (or almost any statement) can be empirically disconfirmed, or, more precisely, any statement can be rationally abandoned in the face of certain recalcitrant experiences. Nevertheless, the above argument is fallacious. Even if no statement is immune to revision, it hardly follows that there is no true statement whose falsity is impossible. Principles of logic, in particular, can be empirically disconfirmed and we can, therefore, rationally reject them and yet, at the same time,, some or all of these principles may be necessarily true.

We might try to save the above argument by simply inserting the extra premise: that a statement which is subject to

[8] L.W. Sumner and John Woods, *Necessary Truth,* op. cit. p. 9.

revision is not necessarily true. However, we would then be assuming, instead of proving, that a statement which is empirically disconfirmable, i.e., subject to revision, is not necessary. The argument will then be explicitly question-begging.

IV

So far, I have been arguing against assumption (A) (that if the distributive laws are empirically disconfirmable, they are a posteriori and contingent). But why not dispense with (A) and simply argue, as Putnam in fact does, that evidence from quantum mechanics shows that the distributive laws *are* false (not merely that they could be false). If they are false, they are neither a priori nor necessary; and if it is likely that all or most principles of logic have the same epistemological and ontological status as the distributive laws, then the traditional view of logic is incorrect.

The above argument is unconvincing. If the distributive laws are false, then of course they are not necessarily true; but without assumption (A) we cannot infer that they are contingent: they might be necessarily false, and their negations might be knowable a priori. But if the explanation of why the distributive laws are neither necessary nor a priori is simply that they are false, and not because they are contingently false, then we cannot assume, without further argument, that it is likely that correct principles of logic will also be neither necessary nor a priori; for correct principles of logic are true.

It might be contended, however, that quantum theory shows that the distributive laws are *contingently* false; (9) and if that is so, then their negations are probably not knowable a priori. To support this contention, one might reason in the following manner. Quantum theory, or some approximation of it, is quite likely to be the true theory of the quantum phenomena of our world. Since the paradoxes of quantum theory can be resolved

(9) That this might be Putman's contention has been suggested to me by Steve Leeds.

only by giving up some principle of logic, and since the distributive laws are the principles we can most easily surrender here, we should give up these principles and admit they do not hold in our world. But just as Euclidean geometry is false in our world but true in some possible world where General Relativity theory is false, the distributive laws would be true in some possible world in which quantum theory does not hold. If this is so, the distributive laws are false in our world and true in some other possible world; consequently, they are contingently false.

The above reasoning is unsound. To see the mistake, consider the following principle of logic:

$$(P) : — [((p \vee q) . (p \supset q)) \supset (p \vee q)]$$

(P) is not a correct principle of logic; its negation is true and (let us assume) is true in all possible worlds. (P), then, is false in all possible worlds.

Suppose that (P) appears in some new textbook of logic. Suppose, further, that no one discovers that (P) is false, and it subsequently appears in many textbooks and is thus thought by many to be a correct principle of logic. However, a physical theory, (T), is developed which has a great deal of explanatory power and no serious rival for the same domain. We then use (P) to derive from (T) predictions which are incompatible with other very firmly entrenched beliefs. Rather than abandon either these other beliefs or (T), we decide that it is more reasonable to abandon (P). It would be a mistake, however, to reason that because there is a possible world in which (T) does not hold, there is a possible world in which (P) is true. By hypothesis, there is no such world. We might infer that in a world in which (T) is false, we would not have evidence against (P); but that, of course, is no reason to think that in that possible world (P) is true.

The above case has been designed to be exactly like what is alleged to be true in the case of quantum theory. In the latter case, we cannot assume, without begging the question, that the distributive laws are false in all possible worlds; but neither can we assume that because quantum theory is false in some possible world, that the distributive laws would be true in that world.

It might be that there is some other way to demonstrate that the paradoxes of quantum theory show that the distributive laws are not only false but are contingently false. However, it is unclear how that could be done. I conclude, then, that *if* Putnam is trying to refute the traditional view of logic, it is not clear that he will succeed even if he does show that the paradoxes of quantum theory refute the distributive laws.

It may be, however, that Putnam is not rejecting what I have called the «traditional view.» (10) The following is also a traditional view: «Principles of logic are employed in testing scientific theories, but they are never, in turn, to be accepted or rejected on the basis of empirical tests. Use *modus tollens,* for example, in falsifying theories, but never reject *modus tollens* itself no matter what empirical discoveries are made». It may be that it is this view that Putnam and others who appeal to quantum mechanics are primarily trying to refute. If it is, however, then there is no need to show that quantum paradoxes refute the distributive laws: if the goal is to show that principles of logic are empirically disconfirmable, it is sufficient to show that it is *possible* to get evidence against these laws by looking at discoveries in the quantum domain. But, then, talking about quantum theory and quantum logic, and generating difficult questions about meaning-change and the correct resolution of the paradoxes, is unnecessary. It is far simpler to describe, as I did earlier, a situation in which a seemingly infallible computor declared the distributive laws to be false. It would also help to go beyond the offering of counter-examples and to explain why the view that principles of logic are not testable has appeared so attractive. I suspect that

(10) Although the evidence is unclear, he does say: «The whole category of 'necessary truth' is called into question» and «What the classical point of view overlooks is that the *a prioricity* of logic vanishes as soon as *alternative logics* and *alternative geometries* begin to have serious physical applications.» See, «Is Logic Empirical ?», pp. 218 and 232.

this would involve, in part, explaining why so many have assumed that 'the empirical' (in the sense of 'the empirically testable'), 'the a posteriori', and 'the contingent' refer to the same items. It is unclear, however, that talk of quantum theory or quantum logic would have any special role to play in such an explanation.

It might be objected that speaking of what a computor might do is of little help. No matter how much evidence we might have that it could not err, we would still be irrational to abandon the distributive laws because of what the computor does. Laws of logic, it might be said, possess certitude which makes them impervious to empirical disconfirmation. If this view is correct, (11) however, then talking about discoveries in the quantum domain will not help either.

Finally, it might be objected that talk of a seemingly infallible computor at best shows how we might get a very indirect kind of empirical disconfirmation of the distributive laws. The evidence from quantum theory is not like this: it is evidence of a very different kind.

To sustain the above objection, we would have to explain the difference between the different kinds of evidence. It cannot be simply that there now exists no such evidence from an infallible computor and that there does exist disconfirming evidence from quantum theory. We have already been through that: If we are trying to show that certain principles of logic are disconfirmable, and not that they have been disconfirmed, then talking about evidence we might obtain is sufficient; it is not necessary to talk about evidence we already possess. The difference also cannot be that quantum theory entails the falsity of the distributive laws and that assumptions about computors do not. Quantum theory entails nothing about principles of logic. What is true is that postulates of quantum theory plus a certain complex set of auxillary assumptions entails the falsity of the distributive laws. But, then, the discon-

(11) For an opposing argument, see Edward Erwin, «The Confirmation Machine», *Boston Studies in the Philosophy of Science, Vol. VIII,* ed. by R. Buck and R. Cohen (Dordrecht, Holland: D. Reidel, 1971).

firming evidence that we get or might get from the quantum domain is indirect in the very same way as is the evidence gleaned from looking at the print out of a computor. In the second situation, we might have well-confirmed assumptions about what the computor will do, and from these assumptions plus auxillary assumptions, we can derive the negations of the distributive laws. We might even have a theory about how the computor works — the theory might even consist of quantum theory plus additional assumptions — and from this theory, plus auxillary assumptions, we might derive the conclusion we want. There is no warrant, then, given what has been said so far, for thinking that the evidence from a computor would be different in any way which matters from the evidence from the quantum domain. It is still open to someone who thinks that quantum theory and quantum logic have implications for our philosophic views about logic to try to explain the relevant differences between the two kinds of evidence. If that cannot be done, however, then there is reason to question whether there are such implications. It is, no doubt, of some interest to learn that the distributive laws, or any other widely accepted principles of logic, are false. But there is reason to think that Putnam and others who have appealed to quantum theory are also trying to make a point about the nature of logic. If they are, I think we need a clearer explanation then we have been given so far as to how the appeal to quantum theory is supposed to help (12).

University of Miami at Coral Gables Edward Erwin

(12) An earlier version of this paper was read at the 1974 meeting of the American Philosophical Association, Pacific Coast Division. I would like to thank E. Zemach and Jill Buroker for their helpful comments.