PERHAPS (?), NEW LOGICAL FOUNDATIONS ARE NEEDED FOR QUANTUM MECHANICS (1)

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In this paper we investigate the logical foundations of Quantum mechanics. In section 1 the concept of 'empirical logic' is discussed. We impose certain empirical constraints ony any logic which is supposed to serve as a logical foundations for Quantum mechanics. In section 2 we examine the nature of existing logics of Quantum mechanics. It is observed that they do not capture the INDETERMINATNESS, INEXACTNESS, UNDEFINDNESS and FUZZINESS which are exemplified in Quantum propositions. Hence we claim that one who constructs a Quantum logic must transfer the emphasis from the field of investigation of the nature of Quantum connectives, semantic models for quantum logic etc., to a much more fundamental problem: the construction of a theory of truth to Quantum atomic propositions and the investigation of the magnitude of the cardinality of the set of truth values of such a truth theory i.e. is it a two (three) value logic or a special type of a many-valued-fuzzy logic ?. In section 3 we try to supply physical evidence to our logical thesis. We observe that the IMPRECISE-UNDEFINED-FUZZY nature of Quantum propositions appears in the works of Heisenberg, Bohr, Born, Dirac, Margenau, Weizsacker, Kompaneeyets, and von Neumann. These physical evidences help us in proving the accusations we raised in section 2 against existing Quantum logics. Finally in section 4 we suggest new foundations for a Quantum logic. It is shown that even a classical fuzzy logic will not do. The conclusion directs us to a much more complicated FUZZY FUZZY LOGIC. i.e.a. fuzzy logic of a second degree of vagueness.

Such a logic captures the famous indeterminancy of Quantum proposition. It doesnt' assign to them exact real numbers in the interval (0,1) (as classical many-valued-fuzzy logics do) but rather assigns to each proposition a fuzzy set of points on the real line (0,1). Our logic is connected with Scott's work on

(1) 'To the memory of Y. Bar-Hillel'.

continuous lattices and type free calculi in which one has *degrees of undefindness* and *incomplete information* in objects which serve as values of Scott's partial functions. The investigation of the connection between Scott's continuous lattices and Quantum fuzzy lattices is left for a forthcoming paper. Finally we discuss some open problems concerned with our new logical foundations for Quantum mechanics. The philosophical conclusions are not drawn in this paper. They will be attacked in a future work.

1.0 EMPIRICAL LOGICS

The concept of 'empirical logic' seems to be contradictory. It is expected that if one will join the set of properties of a certain two valued logical system with the set of properties of an empirical system certain incompatibilities will arise. So perhaps instead of taking EMPIRICAL LOGIC to be equivalent to the union of the sets of properties of the logical and empirical systems we should claim that it is equivalent to their intersection. This move can be considered as a progress. Taking the nature of an empirical logic (EL) to be equivalent to the intersection of the properties of the corresponding systems will prevent the existence of contradicting features in the set of properties which characterizes the EL. But a standard intersection will not do. In forming the intersection of two sets we are assigning equal weights of importance to both sets and hence in the resulting intersection the importance of one type of properties will equal the importance of the second, to wit, if EL has the properties which are the intersection of the properties of the logical system and the empirical field in question it would not be an EMPIRICAL logic such that the logical force of the system is restricted and governed by the nature of the empirical field in discussion. Hence what we need is a logic whose nature is determined by the specific field of which it is a logic.

When we try to set some formation rules (in the syntax) and some interpretation rules (in the formal semantics) of EL we can adopt two methodological points of view:

1. The basic type of logic to be used as EL is a two valued classical logic when n-valued logics ($n > 2$) are regarded as special mechanisms which are ought to be used in order to give an account of DEVIANT and NON-NORMAL phenomena.

2. EL is normally a many valued or fuzzy logic. Reduction to two values is needed only in special cases.

It is interesting to see that the cardinality of the set of truth values has a metaphysical background. Thus if one adopts (1) he seems to hold that empirical propositions are normally two valued and cases where fuzzy or many valued propositions occur are special cases. On the other side, one who holds (2) seems to believe that the basic character of empirical propositions is not two valued and that some propositions in the empirical sciences are sometimes requiring indeterminate, inexact truth values. The one who holds (2) seems to hold that the function of logic in science must follow that of geometry. He equates the transformation from Euclidean geometry to a Rimenian one (via Minkowski's works) with the Reichenbachian transformation from two valued logic into a three valued or the von-Neumann-Birkhoff preference of non-distributive lattices over classical ones. Since we are especially interested in the logic of Quantum mechanics (LQM) let us remark tha the one who holds (2) must hold the following super-constraint on the nature of such a logic i.e., (LQM):

SUPER-CONSTRAINT ON ANY LQM: THE MATHEMATICAL STRUCTURE OF LQM IS A A FUNCTION OF THE EMPIRICAL PROPERTIES OF QM. THE CARDINALITY OF THE SET OF TRUTH VALUES OF LQM, THE NATURE OF QUANTUM CONNECTIVES AND THE STRUCTURE OF QUANTUM LATTICES, ALL SHOULD BE A DIRECT RESULT OF THE EMPIRICAL RESULTS GOT BY THE PHYSICST.

The line we shall follow in this paper will be based on (2). Our 'metaphysical' suggestion is to adopt (1) when one constructs a logic within a deductive system. On the other hand when one deals with empirical fields he is suggested to follow (2). (consult the works of Albrecht-Buchler (to appear), Goguen (1975) Hersch and Caramazza (1976) and Zadeh (1975, 1975a) for an adoption of (2) in other fields in the empirical sciences: Biology and theoretical genetics, artificial intelligence, experimental psychology and finally for Zadeh, Formal linguistics, Electrical enginnering, decision making processes and cybernetics. Consult the bibliography in Zadeh (1975a) for other applications in atomic physics, Biochemistry, Optics of the point of view of (2)). Finally let us end our metaphysical part by claiming that we are aware of the supposed dichotomy ANALYTIC-SYNTHETIC we are presupposing in giving advices of the term given above. It is not meant to be a critician of the Quine-Tarski view. It is an assertion saying that if this is the price we have to pay in order that our EL will be really empirically based and physically motivated then we are ready to pay it.

2.0 EXISTING LOGICS OF QM IN LIGHT OF OUR SUPER-CONSTRAINT

D. Scott in his (1973) claimed that: 'Formal methods should only be applied when the subject is ready for them, when conceptual clarification is sufficently advanced'. Then he attacked modal logic saying that 'colorful axioms have been strung up all over but few couples are dancing' and then Scott said: 'May be Quantum logic is another example, BUT AT LEAST THE MATHEMATICS BEING SERVED AT THAT PARTY IS VASTLY MORE SOPHISTICATED THAN THE COCA-COLA OF MODAL LOGICIANS'. Leaving aside modal logic, I shall claim that unfortunately this mathematical sophictication is the only *raison d'être* of existing LQM. I shall argue that the 'at least' mentioned by Scott has no 'at most' counterpart. Indeed, I shall claim that LQM (The ones known to me), have no physical reality, are not constrained by the existing theorems of QM, do not reflect the probabilistic nature of Quantum propositions, do not emphasize the VAGUENESS, INDETER-

MINANCY and GROWING ENTROPY that Quantum measurments cause. (For the connection between entropy and Quantum measurments consult von-Neumann (1955)).

Take for example the classical paper of von Neumann and Birkhoff (1936). The remarks made by these authors are concerned with the failure of the distributive law of classical logic. They changed it with a weaker modular law. (later this law was replaced by a weaker law of orthomodularity). With this remark they concluded i.e. apart of this failure, classical logic seem to them adequate for being a truth theory for Quantum propositions. Putnam (1970), Finkelstein (1970, 1972) continued this line. They tried to connect the failure of distributivity with a possible construction of a three-valued logic for QM, but again didnt' suggest that any other changes must be made in LQM in comparision with classical logic. The same is true of more sceptical approaches towards LQM like the one adopted by Jauch and Piron (1970). Others have been concerned with the nature of Quantum connectives, especially the implication connective (see Hardegree 1974, 1975 and Finch (1970)). Reichenbach suggested a three valued logic for quantum mechanics. In his (1944) he discussed various negation connectives and some implication connectives. Another field of investigations has been proposed lately by the use of semantic methods in LQM. The first steps were taken by Specker and Kochen (1965) while suggesting a model theoretic account of LQM. Presently, some research is done in this area especially by van Fraassen, Goldbatt (1974) and recently by Stachow (1976) who introduced game theoretical methods to LQM.

What is strange with these works is that normally a system deals with these topics after a theory of truth (satisfaction) was embedded in it. Thus a discussion of connectives, semantic models etc, occurs only after a discussion of the properties of atomic propositions, at least when one's system is constrained by a meta-constraint (like our SUPER-CONSTRAINT) although I am not sure that even if one deals with DEDUCTIVE systems, FREE OF EMPIRICAL CONSIDERATIONS, he should reject the primacy of the discussion of a truth theory for atomic propositions. Thus, in LQM discussions it was assumed that classical two valued logic suggests an adquate theory of truth for Quantum atomic propositions. This seems to be quite strange in light of the works of physicists in QM (Both of the Copenhaguen school and their rivals). As it will be shown in section 3, Physicts had repeatedly emphasize the indeterminate nature of statements describing Quantum measurements. Physics had altered us of the growing vagueness in a physical system as a result of measurements done in the system, Heisenberg and Bohr emphasized the probabilistic nature of a statement which assigns a property to a physical system (indeed they claim that such statements have the underlying structure of a relational statement, the second argument of the predicate being the measuring set-up). Von Neumann was aware of the connection between Quantum measurments and loss of information and hence inexact and indeterminate descriptions of the physical system. Some physicsts interpreted the Heisenberg formulae as representing the amount of VAGUENESS and INEXACTNESS needed to assert the dual character of particles and waves. Indeed the one who read De Broglie's dissertation in the early 20's can understand how strange it is to find no logical sensitivity to these developments toward the statistical nature of physical statements. The same is true of Schrödinger's wave mechanics and Born's statistical interpretation of the relation between photons and wave lights as occuring also in wave mechanics. The unawareness to these consequences of these statistical interpretations continues as far as logicians are concerned. How is it possible that logicians do not raise questions concerning the presupposed non-fuzzy nature of the truth theory so far found in LQM, when they encounter statements of the following form: (Landè 1976): 'A periodic (wave-like) probabilistic amplitude connecting p and q and E with t ... p-states and q-states are connected by Probability relations'.

The answer seems to me quite simple: Existing LQM try to maximize mathematical sophistication and elegance and thus neglect indeterminatness, inexactness, vagueness and FUZZINESS in physical descriptions of physical systems. Let us have an analogy. Formal semantics of natural languages tend to

neglect the fuzziness of many statements and concepts in natural language. In the so-called Montague formal grammar, fuzzy statements are supressed into a two valued framework of the classical Tarskian theory of truth and satisfaction. Indeed Montague semantics doesnt admit their fuzziness at all, because of heuristic reasons and quest for rigorous and formal representation. Now, back to LQM, it seems to me that we have here the same phenomenon. Hence, I think that we may raise the following accusations against existing LQM:
CHARGE 1:

> EXISTING LQM IGNORE THE FUZZY NATURE OF SOME OF THE ATOMIC QUANTUM PROPOSITIONS.

CHARGE 2:

> THE PROBABILISTIC NATURE OF PHYSICAL DESCRIPTIONS OF PHYSICAL SYSTEMS AND THE STATISTICAL NATURE OF QUANTUM 'LAWS' AS IT OCCURS IN ATOMIC QUANTUM PROPOSITIONS DESCRIBING THESE SYSTEMS AND LAWS, IS NOT REFLECTED IN A THEORY OF TRUTH FOR ATOMIC QUANTUM PROPOSITIONS.

CHARGE 3:

> THE CONCEPTS OF 'INDETERMINATNESS', 'VAGUENESS', 'INEXACTNESS' AND 'FUZZINESS' ARE NOT EMBEDDED IN THE THEORY OF TRUTH OF LQM BECAUSE OF THEIR TWO VALUED NATURE, WHICH MUST BE REPLACED BY A LOGIC OF INDETERMINATNESS, MANY VALUED LOGICS OR FUZZY LOGICS OF A TYPE WHICH DEPICTS QUANTUM FUZZY REALITY.

In the light of these charges one can raise the following general charge:

GENERAL (META-) ACCUSATION: EXISTING QUANTUM LOGICS DONT OBEY OUR SUPER CONSTRAINT ON ANY LQM.

It seems to me that with respect to the three first charges, existing LQM must plea 'guilty'. From this I derive the truth of my meta-accusation. Their punishment must be a cessation from functioning as adequate logical foundations of quantum mechanics. However since we accept liberalism in logic we let existing LQM appeal against our verdict. In the next section we shall try to show the fuzzy nature of atomic quantum propositions and by that we hope to show that the problems dealt by existing LQM are secondary in their importance. The reader who accepts our evidence is invited to consider our new logical foundations for quantum mechanics, to be found in section 4.

3.0 THE FUZZY NATURE OF QUANTUM PROPOSITIONS.

L. Zadeh (1973) describes in the following quotation what is the fate systems undergo in certain cases: 'As the complexity of the system increases our ability to make precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance (and relevance) become almost mutually exclusive characteristics' (p. 28).

This description seems very similar to various assertions made by physicsts which will be considered below. It would be quite amazing to see how Zadeh's analysis of fuzzy systems fits the quantum systems not only in the global analysis but even in particular details. In the late thirties Hempel suggested some measures of vagueness and fuzziness of a system. He suggested that given a term T and an object s, the consistency of the application of T to s, C(T, s), can be defined as:

$C(T, s) = \lim \frac{M}{M + N}$, when M stands for the number of judgements which verfied that $s \in T$ and N for the number of times in which $s \notin T$ was verfied. Thus the more the term is vague (fuzzy) the more are the objects of which it is true that C(T, s) approaches $\frac{1}{2}$. Hempel suggested to characterize VAGUENESS in the following way:

VAGUENESS = 1 — PRECISION

when PRECISION is defined to be:

$$\text{PRECISION} = \frac{4}{N} \sum_{K=1}^{N} \left(C(T, s_k) - \frac{1}{2}\right)^2$$

The relation between precision and vagueness is very important because the precision of measurements in QM is one of the causes of the indeterminancy. Indeed the Heisenberg formulae (a) and (b) received an interpretation connecting them to problems of precision and vagueness (imprecision):

(a) $\Delta E \ \Delta t \geqq h$

(b) $\Delta p_x \ \Delta q_x \geqq h$

This interpretation claimed that these formulae set the upper limit of the precision of our measurements (and thus the lower limits to their vagueness (imprecision)). Another variant was that these formulae represent the vagueness needed in order to assert consistently the dual character of particles and waves. Thus, in a way, these formulae may formulate the necessary growing fuzziness in a system as a function of the number of experiments involving measurements which were carried out.

This interpretation fits both Zadeh's remarks on the growing imprecision in complex systems (and that is for the logician side) and it fits von Neumann observation concerning the growing entropy in QM systems as measurements continue (von Neumann (1955)). Now, let us see how Zadeh's observations concerning fuzziness in systems of description fit the work in QM done by Dirac, Margenau, Weizsacker, Kompaneyets and von Neumann.

Let us consider some of the assertions made by Dirac, Margenau, and Weizsacker: (italics are mine) 'At the Quantum level, only EXCEPTIONAL circumstances enable us to assert meaningfully that a physical system HAS A GIVEN PROPERTY, IN CONTRAST TO MOST FREQUENT situations where only a PROBABILITY OF MANIFESTING THIS PROPERTY can be attributed to the system (Dirac 1935). Margenau differentiated between POSSESSED PROPERTIES OF CLASSICAL SYSTEMS and LATENT PROPERTIES OF QUANTAL SYSTEMS. Finally Weizsacker contrasted OBJECTIFIABILITY of properties at the macro level with UNOBEJECTIFIABILITY of properties at the quantal level. (see Margenau (1961), Weizsacker (1952)).

Thus if we let a, b, c ... range over physical systems and F, G, H over one place predicates for their corresponding properties then it is expected that for most of them a will have F probably rather than absolutely. Thus we shall need degrees of truth for sentences expressing Quantual propositions of the form 'Fa'. Suppose that pr($a \in F$) that is pr(Fa) is i. Then to say that '$a \in F$' or that 'Fa' is to say something true to the degree i. It is clear that both the sentential form "Fa' is true to i' and the set theoretical form "$a \in F$' is true at i' require non classical predicate calculus and non cantorian set theory. If we take Dirac, Margenau and Weizsacker assertions seriously (and also Heisenberg, Bohr and Born strong conviction in statistical properties) it seems that for any class S of Quantum propositions, given that T is a class of probability statements of the form 'pr(Fa)' and that T' is a class of absolute statements i.e. 'Fa' then:

$$\text{pr}\ (T \in S) > \text{pr}\ (T' \in S)$$

in a significant way such that the cardinality of the set of T-type statements is much greater in S than that of the set of T'-type statements. Now remember that our super constraint on any LQM asserted that the nature of LQM should be determined by the physical data of the empirical research done in QM. Hence if we judge in light of this super constraint the question of a formation of LQM truth theory for QM atomic propositions, we should prefer fuzzy-many valued logics in which the schema (A) holds, on two ot three valued logics in which it fails:

(A) (V = truth value) $\text{pr}(P) = i \rightarrow V(P) = i$

Thus we have a function from the real interval (0,1) into itself such that *there is a one-to-one correspondence between what the physicst assigns to a statements and what the logician assigns to it.*

Another interesting example can be derived from Kompaneyets's work on the law of space quantization, (in his (1961)).

Kompaneyets remarked there that when a silver atom, in an appropriate Quantum state, is placed in a horizontal non-homogenous magnetic field, the law of space quantization has two consequences:

a. If we check whether the atom is horizontally oriented, we shall certainly get a positive result.
b. If we check, instead, whether the atom is vertically oriented, we can expect a positive result with a probability

$$i > \frac{1}{2}.$$

Now let P stand for 'atom x is vertically oriented' (atom x is the silver atom discussed). It seems that any existing two valued theory of truth cannot give 'P' a natural truth value (it can assign 'P' an artificial truth value by saying: 'for all values above $\frac{1}{2}$, if 'P' has such a value it is TRUE. Such a procedure is arbitrary. The other defect it brings in, is that if 0.51 is as true as 1, indeterminate propositions (almost false, if 0.49 is already 'FALSE') are true as are necessary truths (like truths of mathematics)). A fuzzy logic of some sort will assign 'P' the value i such that it is a natural transformation of its probability (i.e. a one-to-one correspondence between probability and truth values). Note that fuzzy valuations are required not only for non modal quantum propositions. Consider the introduction of a TENSE OPERATOR, F which stands for 'it will be the case in the future'. Now any proposition 'F(P)' is true at t if there is a time t', $t < t'$ such that 'P' holds at t'. We saw already that given a moment of time 'P' may take a value i in the real interval (0,1). Thus 'P' may be at t' true to i and hence 'F(P)' is true at t to i. Fuzzy valuations are also required if the Quantum system is about to predict what is going to happen in all the moments of time in the future of i.e. this time 'P' stands for a certain lawlike statement. Let us introduce the operator G and read it 'it will ALWAYS be the case that'. Then 'G(P)' may be true to a degree at t. Actually the value of 'G(P)' at t will be the minimum of the values of 'P' in all t' $t < t'$ (in all the moments of time in the future) [1].

Finally consider von Neumann's remarks on the results of Quantum measurements: 'Quantum measurements always lead to an increase of dispersion and hence of the entropy of ensembles of Quantum systems' (1955). Now assume S is a Quantum system. Assume we check S after m measurements had been done on it and after n measurements had been performed in it such that $m < n$. Now if t is the time when m measurements had been carried out and t' is the time after n measurements, it seems that the sentence 'the entropy in S

[1] This observation concerning the fuzziness of Quantum propositions in the scope of tense operators is independent of the concept of time that we choose (dense or discrete).

grew comparing to the intial state of S' will be MORE TRUE at t' than at t (because the growth of entropy is a function of time and number of measurements carried out). If we will not use some sort of fuzzy-many-valued logic we will find ourselves in the following situation: Suppose t_1 is the time when one measurement had been performed on S . t_n stands for the time when n measurments had been carried out. I stands for the amount of information and degree of organization of S before any measurement. I_1 stands for the degree of organization and information content in t_1 and correspondingly for I_n and t_n. It is clear that if our truth theory is restricted to two values, we shall have the following anomaly: the more k is large $|I - I_k|$ grows with respect to $|I - I_1|$ but the truth value of the sentence 'the entropy of S grew with respect to I' doesnt become higher in t_k comparing to t_1. On the other hand if we use a many valued truth theory, we get a natural correspondence between growth of entropy and growth of the truth values in the interval (0,1). In such a theory the truth value of Q = 'the entropy of S grow in respect to I' in t_1 and t_2 will not be drastically different but there will be a drastic difference between Qs truth value at t_1 and at t_{23}, as a function of the sum of changes which S passed after each measurment between t_1 and t_{23} (thus in a way we have a strictly monotonic function connecting the magnitude of $|I - I_k|$ with the truth values Q takes in the real interval (0,1)).

I hope that we presented some evidence supporting our view concerning the importance of FUZZINESS, VAGUENESS, and INDETERMINANCY and their role in a theory of truth for Quantum propositions. If this argument is sound it leads us to the need to change the logical foundations of QM. Indeed it might be viewed also as a change in some of the philosophical foundations of QM. These changes (in the philosophical foundations) seem to be the result of the require changes in the logical foundations. We shall describe in section 4 some attempts to give QM new logical foundations. We shall leave the required changes in the philosophical basis for further work.

4.0 NEW FOUNDATIONS FOR THE LOGIC OF QUANTUM PROPOSITIONS

Reichenbach (1944) suggested that two-valued logic doesnt capture the nature of QM. In the final chapters of his book be introduced a system of three-valued logic which was viewed by him as an adequate logic for QM. Reichenbach suggested the following table:

Let m_u stand for 'a measurment of u', u stands for 'The measurements shows u':

OBJECT LANGUAGE		QM language
m_u	u	
T	T	T
T	F	F
F	T	M
F	F	M

Then T = true, F = false, M = meaningless (when this value is an adoption of the Heisenberg-Bohr approach). Reichenbach suggested that in his truth tables I= INDETERMINATE, will replace. M. This was supposed to account logically for his theorem 1, p. 143:

IF TWO STATEMENTS ARE COMPLEMENTARY, AT MOST ONE OF THEM IS MEANINGFUL AND ONE MEANINGLESS. (see his interesting remark on the use of 'at most'.

It is strange for the reader of Reichenbach (1938) to find no use of probability logic, especially in the light of the probabilistic character of Quantum predictions and the statistical nature of Quantum 'laws' (for this term in QM consult Landè (1976). But Reichenbach answered the doubts that might have been raised: (1944 p. 147) «Continuous logic corresponds more to classical physics than to Quantum mechanics since in it every proposition has a determinate probability *it has no room for a truth value of indeterminancy*. A probability of $\frac{1}{2}$ is not what is meant by the category indeterminated in Quantum mechanics. The use of sharp categories, TRUE, FALSE, must be considered in both cases as *an idealization applicable only* in the sense of approximation'.

It seems to me that Reichenbach has a very good point here, namely, that if Quantum propositions are imprecise and indeterminate they should not be mapped into exact points in the continuum (0,1) for if we know that P is indeterminate how is it that it can be mapped into 0.76 but not to 1 or 0 ? On the other hand Reichenbach's own solution seems to me to contradict his methodological fine remarks. His three valued logic suggested in (1944) or the reduction of probability logic he suggested in his (1938 and I mean both methods of reduction he suggested there pp. 326 — 7 and 334) are all violations of his methodological advice. The reasons will be discussed below. For the meantime let us see what we require from a non classical Quantum logic.

1. It should have a strong connection with the empirical findings of the physicist. (a derivation from our super-con straint)
2. It should not map indeterminate, vague (fuzzy) Quantum propositions into exact and determined truth values in the real interval (0,1).
3. It should be based on a non-distributive lattice.
4. (almost a variant of (2)) It should represent the vagueness and indeterminancy of QM propositions without a regimentation i.e. in a natural way.

Reichenbach's three valued logic doesnt fulfil all the requirments. It violates (1) (2) (4). It is quite strange that Reichenbach who interpreted the values of probability in terms of truth values didnt use such a many-valued logic to capture the fuzzy nature of Quantum propositions [2].

Let us consider some many valued logics as candidates for being the logic of QM. We shall not deal with Bochvarian, Lukasiewiczian, Kleene strong and weak systems. They are all restricted in the sense of fixing the cardinality of the set of truth values to three. In section 3 we argued hat QM exemplifies a fuzzy character and hence requires degrees of truth. The natural candidates seem to be fuzzy logics those of Scott, Zadeh, and Kamp.

Zadeh: (1975, 1975a) POSITIVE FEATURES: It permits a one-to-one mapping between probability values (assigned by the physicst) and truth values since the set of truth values is the real interval (0,1). Hence it depicts the fuzzy character of quantal propositions better than two or three values logics. (see however negative feature, no. 2). NEGATIVE FEATURES:

1. Zadeh's fuzzy logic is based on a complemented distributive lattice.
2. It assigns an *exact* number in the interval (0,1) to quantal propositions. (it is true that there is an extension of Zadeh's logic which uses linguistic truth values instead of numerical ones but it is designed for formal analyses in linguistics, not in physics).

Scott (1973): His fuzzy logic is based on the idea of the degree of error of a proposition, contra Zadeh who is interested in degrees of truth (however TRUTH = 1 — ERROR). POSITIVE FEATURES: The same as those of Zadeh. It is not a radical departure from two valued logic because of the special techniques used by Scott. It appears in a general program of giving a uniform treatment to modal and many valued logics and hence the close relation between LQM and the modal system B will be vindicated such that LQM may receive a Kripke-semantics type like the one B has. NEGATIVE RESULTS:

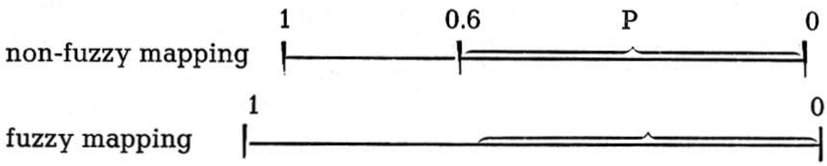
(2) We use the term 'MANY VALUED' to refer only to systems in which the cardinality of the set of truth values is not restricted to two, three or four values.

1. it assumes that there is an absolute truth in the sense of O degree of error, a state which doesnt exist in quantal systems.
2. Again, it assigns an exact truth value to fuzzy-indeterminate propositions.

Kamp (1974): Actually it is not exactly a logic but rather an application of intensional logic to fuzzy concepts and relative adjectives. POSITIVE FEATURES: The usual Zadeh-Scott sensitivity to degrees of truth 'and to' a rich model theory which, if adopted could serve as a model theory for Quantum logic. Existing model theories for continuously valued logics suffer from two main shortcomings: a. There seem to be no connection between them and the fuzzy logics discussed above. (b. They were not built in order to account for fuzziness and indeteminancy. They are more an exercise in constructing non classical logical basis for set theory and they seem to have no physical or even linguistic significance, (I am referring to Chang and Keisler (1965)). NEGATIVE FEATURES:

1. It was basically constructed in order to deal with problems in linguistics i.e. the analysis of the pragmatics of natural language and their relation to fuzzy adjectives.
2. Again, it maps sentences into exact points in the continuum (0,1).

Conclusion: In order to satisfy conditions (1) - (4), we need a logic which has more than two or three values but which is not a standard of fuzzy logic in the sense of assigning numerical values in the interval (0,1). Our Quantum propositions are indeterminate and vague and hence (going back to Reichenbach's quotation) we must not assign them exact values in the continuum. Hence what is needed is a kind of FUZZY MAPPING of sentences to truth values. This means that if a proposition (sentence) (I use these terms interchangeably but only in this paper) P is indeterminate it might be partly (or vaguely) true that it is true to 0.6. This is a very important point. In the begining we moved from two (three) values to degrees of truth. Now we make another move. We fuzzify even our assertion: 'P IS TRUE TO i' because if P is vague even our assertion which says that it is true, say to, 0.6 may be true to a degree. Hence if P's probability is partly determined we should map it fuzzilly to the continuum. What is a fuzzy mapping ?. A non fuzzy mapping is a function which assigns to a proposition true to 0.6 a set of points in the real interval i.e. the set of points from 0 to 0.6 such that in each, point in the set the proposition is true (since if $V(P) = 0.6 \rightarrow 0.6 \geqslant i$ it is true that $V(P) = i$).



A fuzzy mapping is a function which assigns to P a fuzzy set of points in the interval. This is an intuitive formal counterpart to our observation that assertions of the following type may be true to a degree as a result of P's indeterminancy: 'P IS TRUE TO 0.6'. Thus if the assignement of truth values is represented by a mapping of a proposition to a set of points in the interval, when the proposition is indeterminate some points in the interval may partly belong to the set of points in which the proposition is true. This is supposed to be the representation of the idea that the proposition is indeterminate and so is its truth value. Thus Quantum propositions are not assigned Cantorian sets but rather FUZZY SETS which are characterized by a membership function for each fuzzy set. Thus if the boundaries between the points on the line (0,1) in which P holds are vague it might be the case that we shall have' 'P is true to 0.6' is true to 0.8' or in order to avoid double quotations we may use a a nominalized form like 'that P is true to 0.6 is true to 0.8' when 'is true to 0.8' is a predicate. In a fuzzy logic calculus it is to be represented in the following form: Let T stand for the set of truth sentences, let

μ_T be a membership function for this fuzzy set and let Q = that P is true to 0.6 and then read $_T$(Q) = 0.8 as 'Q belongs to the truth set to 0.8.

We must note that such 'second order valuations' are not new. (I call a valuation a second order one, if it is a valuation of another assignement of valuation). Kit Fine and D. Sanford have suggested (independently) an operator of DEFINDNESS or DETERMINANCY such that D(P) comments on the truth of sentences like 'The value of P is i' i.e. this operator evaluates degrees of determinancy of P if P's truth value is i. Amore interesting and far reaching programme was raised by D. Scott in the context of information measures in the semantics of programming languages and the theory of lattices and data types. Scott uses there a certain relation of partial ordering to show how one can scale sentences according to the information content they have. He says in his (1973a) that each element in this ordering is assined its position as a function of its *degree of undefindness* (it is an interpretation of mine to Scott's paper. However the stressed term is his own idea which serves him to explain the scaling he does to sentences by his connective '$\subseteq$' which might be read '$x \subseteq y$' as 'x approximates y' or 'x's information is included in that of y but not vice versa' I hind his work fascinating and I am trying to work out a possible connection between it and the UNDEFINDNESS of Quantum propositions see Almog (forthcoming)) (3).

We shall use a completely different method in order to avoid the dubious modal character of Sanford 's and Fine's operators. We shall start by assigning to each quantum propositions a triple consisting of a TRUTH-value, FALSITY-value and INDETERMINANCY-value. Thus each proposition P will be assigned a triple (i, j, k) where i, j, k stand for the three types of valuations. In each coordinate, values may range over

(3) Actually Scott gave an axiomatization of what should be a space of partial objects which is a function algebra structured by the reciprocal operations of functional application and functiona abstraction. What is very important is that his functional application is type -free and CONTINUOUS.

the real interval (0,1). Now in correspondence to this triple we shall have a HYPER-triple which consists, as expected, of three coordinates. The valuations in this hyper-triple comment on the valuations in the basic triple i.e. each valuation in the hyper triple is a function which takes a proposition and a number in the basic triple to yield a new number in the real interval (0,1) which represents the degree of trutf of assigning the number from the basic triple to the, proposition. In this way the fuzziness of many valued valuations like 'P is true to 0.6' is captured (when P is QM proposition). In the basic triple the valuations stand in the following relations:

i (TRUTH) = 1 — i (FALSITY)
(INDETERMINANCY) = 1 —

$$\left[\frac{((i - 0.5) . 2) + ((i - 0.5 . 2)}{2} \right]$$

Such definitions lead to the following tables: (4)

P	T(P)	F(P)	I(P)
0	0	1	0
.1	.1	.9	.2
.2	.2	.8	.4
.3	.3	.6	.6
.4	.4	.4	.8
.5	.5	.7	1
.6	.6	.5	.8
.7	.7	.3	.6
.8	.8	.2	.4
.9	.9	.1	.2
1	1	0	0

(4) It is clear that my intention is to mark cases near 0.5 as borderline cases (to use Sanford's term) and hence indeterminate to high degrees. It is clear that the use of '| |' for 'absolute value» is necessitaited by our

Similarily for the relations ni the hyper-triple i.e. if the proposition Q = 'that P is true to 0.6' is evaluated at the basic triple and then the SECOND ORDER TRUTH VALUATION in the hyper-triple will say that Q itself is true to 0.8 then the SECOND ORDER FALSITY VALUATION will assign Q, 0.2 and the SECOND ORDER INDETERMINANCY VALUATION will assign to Q the value 0.4. Thus the relations between truth falsity and indeterminancy are the same in the basic and in the hyper-triples. (consult footnote 4 for the explanation of the relation between truth and indeterminancy).

This mechanism enables us to represent the fuzziness and indeterminatness of Quantum propositions. Some may doubt the almost synonymous use I am doing with FUZZINESS and INDETERMINANCY. I follow Scott in this matter. Scott suggested to, interpret Lukasiewicz fuzzy material implication as a tool to represent INEXACTNESS while Lukasiewicz suggested the term INDETERMINATNESS. I speculate that Scott supposed that the latter is included in the former. However if it isnt Scott's intention, I hold that given a system and given that we have the two following predicators F = INEXACT G = INDETERMINATE the following conditional holds:

$$(\forall X)\ (G(X) \rightarrow F(X))$$

Actually I implicitly hold that this conditional can be strengthened to a bi-conditional i.e. equivalence, at least in the context of QM propositions about measurements.

5.0 SOME OPEN PROBLEMS

The LQM we have presented is not a classical fuzzy logics. Its relation to fuzzy logic (of the Zadeh-Scott type) is just as the relation between classical fuzzy logic and classical two valued logic i.e. LQM suggested here FUZZIFIES classical

intention to evaluate values which are in the same distance from 0.5 by the same indeterminancy value.

fuzzy logic in allowing fuzzy mappings and hyper-triples which assign degrees of truth, falsity and indeterminancy, to the degrees of truth falsity, and indeterminancy which classical fuzzy logic assign to a proposition in the basic triple. Hence our LQM is a FUZZY FUZZY LOGIC or a fuzzy logic of a second degree of vagueness. This type of logic raises the following questions which we regard as open for further research:

1. Can we give a model theory to such a logic ? A first answer may be that with some modifications Kamp's model theory for intensional logic might do. The modification required is that the probability measure (one of the four elements of which consist Kamp's VAGUE MODELS (see his 1974)) will assign to vague propositions fuzzy sets of points in the real interval (0,1) instead of mapping them to an exact point i.e. since the probability measure assigns sets which in turn are associated with a real number, the probability measure should assign fuzzy sets. That will mean that the set of worlds in which a proposition holds is a fuzzy set. In some worlds it is just indeterminate whether it holds or not. However this is an ad hoc solution. Kamp's model theory is designed for vague predicates of natural languages and it deals with the pragmatics of these languages. We should, following the general ideas of Kamp, develop a similar model theory for QM propositions.

2. Since, as far as I know, no one dealt with a fuzzified fuzzy logic so far, we do not have any idea what kind of lattices can be used if one adopts that logic. One point is sure: they must be non distributive. For the rest, it seems that an adequate lattice theory for our LQM can be constructed on the basis of Scott's work on continuous lattices and their relations to data types which are a part of a general inquiry of a kind of a 'general function theory' (see Scott (1972, 1973a, 1975, 1975a). Scott's scaling ranging from the completely undefined object '$\perp$' to the over defined 'T', has interesting applications to QM because it captures the notion of degrees of defindeness such that, to use Scott's words (1975), the 'good' cases are somewhere below T but not far from it. This seem to have a great significance for the one who tries to set the limits of possible precision and definedness with the help of the above

mentioned Heisenberg formulae. Scott's investigation of partial functions having as their values INCOMPLETE objects can serve as a mathematical base for formalizing indeterminancy and vagueness of Quantum propositions. I hope to apply his methods successfully in my forthcoming paper (5).

3. This is really the most important problem which seems to have also a great philosophical import. We moved from a fuzzy logic of a first degree of vagueness to a fuzzy logic of second degree of vagueness with sentences of the form "P IS TRUE TO 0.6' IS TRUE TO 0.8' or "Q IS INDETERMINATE TO 0.9' IS FALSE TO 0.4'. But we stopped here. Why? Is there any justification not to continue to the n^{th} degree with sentences like ""P IS TRUE TO 0.6' IS TRUE TO Q.7' IS TRUE TO 0.4' IS TRUE TO 0.9'...... ?

My present answer is that the degree of vagueness of the fuzzy logic to be employed as the logical foundation of QM is strictly determined by the results and the requirements of the physicist. Thus in choosing the degree of vagueness of our logic we follow our SUPER CONSTRAINT advice. At the moment as far as I am competent to interpret empirical research in QM, a logic of a second degree of vagueness is exactly what is needed. However if one falsifies my assertion it should be clear that if his proof is based on empirical grounds no considerations of elegance and minimal complexity will be

(5) I must place the present paper in the right perspective. It is the first element of an ordered pair of papers, whose second element is Almog (1978). Almog (1978 develops the present paper in several directions. It discusses methodological, physical and purely mathematical aspects of quantum theory to support the need to have a fuzzy LQM. As such it contains a very large and detailed discussion of physical facts supporting my logical thesis. Aside of this it contains an answer to our first open problem (p. 48). Indeed I construct there a model theory for LQM based on Scott's work on real and continuous lattices. This gives my logical suggestions an elegant model theoretical counterpart.

Finally I must mention Almog (1978a, b). The first develops a fuzzy modal logic (aplicable to LQM) with two place operators of probability normally used in QM. The second generalizes to an infinite indenumerable valued modal logic for probabilistic sentences. These logics may depict the quantum notion of 'law of nature'.

authorized to be used as weapons against him. The only criterion has to be physical reality. We used it to prove that much more elegant systems of two or three valued logic or even classical fuzzy logic are not adequate. In case physical reality (empirical findings) will require a higher degree of vagueness for LQM, we shall accept the verdict.

6.0 CONCLUSION

Grammatici certant et adhuc sub iudice lis est (Horace)

REFERENCES

DIRAC P. (1935) The principles of Quantum mechanics, Clarendon press Oxford.

FINCH P. (1970) Quantum logic as an implication algebra, Bulletin of the Australian mathematical society 2.

FINKELSTEIN (1970) Matter, space and logic, Boston studies V, Reidel.

FINKELSTEIN D. (1972) The physics of logic, in Colodony (ed.) Paradigms and paradoxes.

GOGUEN (1975) Fuzzy robot planning, IN Zadeh et al. (eds) Fuzzy sets and their applications to cognitive and decision processes, Academic press 1975.

GOLDBATT (1974) Semantic analysis of orthologic, J, Of phi. logic 3.

HARDEGREE (1974) The conditional in Quantum logic, Synthese 29.

HARDEGREE (1975) Stalnaker conditionals and Quantum logic, J. of phil. logic 4.

HERSCH and CARAMAZZA (1976) A fuzzy set approach to modifiers and vagueness in natural language, J. of Experimental psychology.

JAUCH and PIRON (1970) What is Quantum logic; in Freuend et al. (eds) Quanta, Chicago U.P. 1970.

KAMP J.A.W. (1974) Two theories about adjectives, in Keenan (ed.) Formal semantics of natural language, Cambridge U.P.

KOMPANEEYETS (1961) Theoretical physics, Dower, N.Y.

LANDE (1976) The laws beyond Quantum laws, Phil. of science (March).

MARGENAU (1961) Open vistas; Philosophical perspectives of modern science, Yale U.P.

PUTNAM (1970) Is logic empirical ?, Boston studies V, Reidel.

REICHENBACH (1938) Experience and prediction, Chicago U.P.

REICHENBACH (1944) Philosophical foundations of Quantum mechanics, Berkely U.P., Los Angles.

SCOTT D, (1972) Continuous lattices, Lecture notes an mathematics, vol. 74 Springer, Berlin, 1972 pp. 97-136.

SCOTT (1973) Background to formalization, in Leblanc (ed) Truth, syntax and modality, North Holland.

SCOTT (1973a) Models for various type-free calculi, in Suppes et al. (eds.) Logic methodology and philosophy of sciences, North Holland.

SCOTT (1975) Lattice theory, data types and semantics, in Rustin (ed.) Semantics of programming languages.

SCOTT (1975a) Lambada calculus and recursion theory, in Kanger (ed.) Proceedings of the third Scandinavian logic symposium, North Holland.

STACHOW (1976) Completness of Quantum logic, J. of phil. logic 5.

VON NEUMANN (1955) Mathematical foundations of Quantum mechanics, Princeton.

WEIZSACKER (1952) The world view of physics, Chicago U.P.

ZADEH (1973) Outline of a new approach to the analysis of complex systems and decision processes, IEEE tans. of systems man and cybernetics vol. SMC-3.

ZADEH (1975) The concept of linguistic variable and its application to approximate reasoning, Information sciences (Parts I, II, III).

ZADEH (1975a) Calculus of fuzzy restrictions, in Zadeh et al. (eds.) Fuzzy sets and their application to cognitive and decision processes, Academic press.

BIRKHOFF and VON NEUMANN (1936) The logic of Quantum mechanics, Annals of mathematics 37 pp. 823-43.

SPECKER and KOCHEN (1965) Logical structures arising in Quantum mechanics, In Addison (ed) The theory of models, North-Holland.

CHANG C. and KEISLER J (1965) Continuous model theory, in Addison et al(eds). The theory of models, North Holland.

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