A FAVORABLE MARK FOR STRICT IMPLICATION

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The purpose of this note is to call attention to how failure of a deduction theorem for strict implication makes strict implication: $\prec$, a better representative for 'if—then—' than either material implication: $\supset$, or entailment: $\rightarrow$. For several years some major proponents of entailment systems have argued that disjunctive syllogism is an invalid argument form despite the fact its conclusion cannot be false when its premisses are true. See A.R. Anderson [3] and [4], and Routley [8]. A motive for trying to develop a basis for saying that disjunctive syllogism is invalid has been that its invalidity would provde a way to block C.I. Lewis' derivation of an arbitarily selected B from A & $\sim$ A. It is not my aim here to reconstruct and appraise the sophisticated efforts to show that in some sense disjunctive syllogism is invalid. Here I will assume that the impossibility of its premisses being true while its conclusion is false, where this impossibility does not depend upon the premisses being inconsistent or the conclusion being necessarily true, provides a strong presumption that disjunctive syllogism is a valid argument form. My aim is to observe that there is an invalid argument form whose invalidity some have mistakenly taken as showing the invalidity of disjunctive syllogism and then to point out that a system of logic which can discriminate between these two argument forms is superior to those which cannot.

By disjunctive syllogism I mean, of course, the argument form:
A or B, not-A therefore B, where «or» is the inclusive truth-functional «or». So, in symbols (A v B), $\sim$ A $\vdash$ B. The invalid form which bears a superficial resemblance, when spoken, to disjunctive syllogism is: (A v B) $\vdash$ If $\sim$ A, then B. I do not symbolize the 'if—then—' of the conclusion because a question of this note is: Should we symbolize it with $\supset$, $\prec$, or $\rightarrow$. Let

me call this, to me, obviously invalid argument form «or-equivocation» because those who accept an intensional sense of «or» would allow the inference from A or B to If not-A then B, if «or» were used in its intensional sense. See Anderson [3] as well as Hockney and Wilson [6]. It is obvious, isn't it, that we cannot infer from the true «Mr. Nixon is the U.S. President or Mrs. Nixon is the U.S. President,» that if Mr. Nixon is not the U.S. President then Mrs. Nixon is the U.S. President, where the «or» is truth-functional ?

In [6], Hockney and Wilson use an instance of an argument form very much like or-equivocation to try to show that disjunctive syllogism is invalid. They write on p. 217.

> «It is true that either Churchill was Prime Minister of England or that Fords are built in London (with the truth-functional 'or'): but it does not *follow* that if Churchill should not have been Prime Minister of England that Fords *would* be built in London. The truth-functional 'or' is simply not strong enough to warrant the plausibility of rule 4.» Here rule 4 is disjunctive syllogism.

I say that their argument is very much like an or-equivocation because they use «should» and «would» in their conclusion whereas if they had adhered strictly to the or-equivocation pattern their argument would be I below.

I) *Churchill was Prime Minister of England* or Fords are built in London. So, If Churchill was not Prime Minister of England Fords are built in London.

But I submit that argument (I) can be accepted as their alleged counterexemple to disjunctive syllogism. They want to give a counterexemple to disjunctive syllogism. But a disjunctive syllogism version of (I) would not contain «should» or «would». It would be (II).

II) Churchill was Prime Minister of England or Fords are built in London. *Churchill was not Prime Minister of England.*

So, Fords are built in London.

Also, unless one specifies that the form: (If not-A, then B) is simply a version of: (A or B) with truth-functional «or», then the falsity of the indicative conclusion is almost as obvious as the subjunctive version of Hockney and Wilson. (My Mr. and

Mrs. Nixon example also shows this). I suspect Hockney and Wilson give the subjunctive version simply to make it easier for us to recognize the falsity of the 'If—then—' conclusion despite the truth of the premiss.

Certainly pointing out the invalidity of or-equivocation does not give a counterexample to disjunctive syllogism because they are not the same argument form. But my main goal is not to criticize Hockney and Wilson for giving an inadequate counterexample due to a confusion of argument forms. I want to criticize systems for blurring the distinction between disjunctive syllogism and or-equivocation by failing to have disjunctive syllogism valid and or-equivocation invalid. My main point is: Disjunctive syllogism has a strong *prima facie* claim to validity, or-equivocation a strong *prima facie* claim to invalidity, and hence a system that marks the former as valid and the later as invalid has a mark in favor of it being the right logic.

Classical propositional logic blurs the distinction by certifying both $(A \vee B), \sim A \vdash B$ and $(A \vee B) \vdash (\sim A \supset B)$ as valid. Geach's elegant little entailment system ES in [5] blurs the distinction by having both $(A \vee B) \,\&\, \sim A \rightarrow B$ and $(A \vee B) \rightarrow (\sim A \supset B)$ as true entailments. Anderson and Belnap's system E blurs the distinction by having neither $(A \vee B), \sim A \vdash B$ nor $(A \vee B) \vdash (\sim A \rightarrow B)$. Similarly, Ackermann's Π' of [1] blurs the distinction by having neither. (Ackermann does not have $(A \vee B), \sim A \vdash B$. If he had it, he would by his rule H3 have $\vdash ((A \vee B) \,\&\, \sim A) \rightarrow B$, which Anderson notes in [2] is not a theorem for Ackermann. For Ackermann you have to assert $(A \vee B)$ and $\sim A$ as theorems to infer B from them).

In the Lewis strict implications systems; $(A \vee B), \sim A \vdash B$. But they do not have $(A \vee B) \vdash (A \prec B)$. We have here a special case of the *failure* of the following deduction theorem for the S-systems:

If $A_1, \ldots, A_{n-1}, A_n \vdash B$, then $A_1, \ldots, A_{n-1} \vdash A_n \prec B$. This failure was noted by R. Marcus in [7]. This failure of a deduction theorem enables the S-systems to mark the *prima facie* important distinction between the validity of disjunctive syllogism and the invalidity of or-equivocation. And strangely this failure gives the S-systems a favorable mark in the competition for being the right logic.

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