DAWSON MODELLING, DEONTIC LOGIC AND MORAL NECESSITY

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I

Before stating the purpose of this paper, we begin first with a few important definitions and some preliminary discussion.

An alethic system, A, of modal logic will be said to be 'normal' if it meets the following conditions:

1) All theses of the classical statement logic are theses of A.
2) The Rule of the Replacement of Equivalents holds.
3) Where 'L' and 'M' are altehic modal operators corresponding to 'necessity' and 'possibility' respectively, the following formulae are theses of A:
   a) $p \supset Mp$
   b) $M(pVq) \equiv (MpVMq)$
   c) $Lp \equiv \sim M \sim p$
   d) $L(p \supset p)$
4) The following formula is not a thesis of A:
   a) $Mp \supset p$

A 'modality' is defined as any unbroken sequence of zero or more of the monadic operators '$\sim$', 'L', and 'M'. A 'distinct modality' (sometimes called an 'irreducible modality') is defined thus: A modality, α, is distinct in a system, S, iff α is not provably equivalent to any other modality in S.

A deontic system, D, of modal logic will be said to be 'normal' if it meets the following conditions:

1) All theses of classical statement logic are theses of D.
2) The Rule of Replacement of Equivalents holds.
3) Where 'O' and 'P' are deontic modal operators corresponding to 'it ought to be the case that' and 'it is permissable that' respectively, the following formulae are

theses of D:

a) $Op \supset Pp$

b) $P(p \vee q) \equiv (Pp \vee Pq)$

c) $Op \equiv \sim P \sim p$

4) The following formulae are not theses of D:

a) $Pp \supset p$

b) $p \supset Pp$

5) If D is an extension of a normal alethic system, then the following formula is not a thesis of D:

a) $Mp \supset Pp$

We shall say that an alethic system of modal logic, A, provides a Dawson modelling of a deontic system of modal logic, D, iff the following conditions are met:

a) A is a normal system of alethic logic.

b) Where α and β are certain distinct modalities of A, the sequence of modal operators in α is abbreviated by 'O', and the sequence of modal operators in β is abbreviated by 'P'.

c) The resulting deontic system, D, is normal.

We follow both Lennart Aqvist in [5] and Charles F. Kielkopf in [11] in calling the above kind of modelling a 'Dawson modelling'. The label actually stems from the fact that E. E. Dawson, in [7], shows that if we abbreviate $ML\alpha$ as $O\alpha$ and $LM\alpha$ as $P\alpha$, modal system S4.2 contains a normal deontic logic. In effect, Dawson shows how to reduce deontic logic to alethic logic. S4.2 is not the only normal alethic system which admits of Dawson modelling; in [5], Aqvist shows that S4 with $LML\alpha$ abbreviated as $O\alpha$ and $MLM\alpha$ abbreviated as $P\alpha$ also contains a normal deontic system. In fact, Aqvist also shows that if we abbreviate $LLML\alpha$ as $O\alpha$ and $MMLM\alpha$ as $P\alpha$, S3 will also provide a Dawson modelling for a normal deontic system.

Of course, Dawson's method is not the only method available to us for reducing deontic logics to alethic logics. For example, A. R. Anderson shows how reductions of this sort may be accomplished by means of the following definitions:

1) $Op =_{df} L(\sim p \supset S)$
$Pp =_{df} \sim O \sim p$
2) $Pp =_{df} M(p . \sim S)$
$Op =_{df} \sim P \sim p$

In these definitions, 'S' represents a contingent statement saying that a sanction has been incurred. Anderson discusses these definitions in [1], pp. 170-171 and pp. 200-205, and also in [2], [3], and [4]. Again in [1], p. 171, Anderson shows that the addition of the single axiom, 'M ∼ S', to a normal alethic logic yields a normal deontic system. Later on, in the same paper, he shows how 'S' may be defined as 'M ∼ B · B' where the statement constant, 'B', replaces 'M ∼ S' in axiomatizing the system. Given this revised axiomatization, 'M ∼ S' is derived as a theorem. On the basis of this axiomatization, Anderson concludes that deontic logic, as least from a syntactical point of view, is merely a branch of alethic logic. However, in [11], Charles F. Kielkopf contends that Anderson's

> ... claim is not strictly speaking correct because he needs to supplement alethic systems with the constant B to develop deontic systems. I grant that Anderson is correct in noting that for formal manipulations we do not need to pay attention to the intended interpretation of B. Still, we can see B when we do deontic logic but not when we do plain alethic logic. And from a formal point of view what we can and cannot see is extremely significant. So, as I see it, the problem of reducing deontic to alethic logic in an Anderson development of deontic logic is the problem of eliminating the constant B.

In order to eliminate the constant 'B', Kielkopf makes use of Dawson modelling. He calls it a «Dawson modelling of Anderson's sense of 'ought.'» His justification for calling it this is due to the following line of reasoning: Using the second definition, Anderson defines 'Pp' as 'M[p · ∼ (M ∼ B · B)]' where 'M ∼ B · B' is the unabbreviated form of 'S'. Kielkopf notes that if Anderson did not have B, the defining formula for 'Pp' would be: 'M[p · ∼ (M ∼ q · q)]' where both 'p' and 'q' are statement variables. However, Kielkopf observes that this definition of 'Pp' would have the undesirable effect of not allowing the assignment of a value to 'Pp' to be a function of an assignment of a value to 'p'. Consequently, in an effort to keep 'Pp' unary, Kielkopf requires that the variable 'p' be used where 'q' is used in '$M[p \cdot \sim (M \sim q \cdot q)]$'. Thus he arrives at the following Andersonian definition of 'permissability' without the constant B:

$$Pp =df \quad M[p \cdot \sim (M \sim p . p)]$$

Kielkopf then proceeds to show that in any system of alethic logic strong enough to capture all of the requirements of alethic normalcy set forth at the beginning of this paper, the definiens of the above definition is equivalent to 'MLp'. Thus Anderson's definition of 'permissability' can be reduced to:

$$Pp =df \quad MLp$$

Defining 'Op' as '$\sim P \sim p$' we now arrive at the following Andersonian definition of 'ought':

$$Op =df \quad LMp$$

Now if we can find a normal system of alethic logic which permits abbreviating 'LMP' as 'Op' and 'MLp' as 'Pp', where 'LMp' and 'MLp' are distinct modalities, and the resulting deontic system is normal, then we shall have succeeded in providing a Dawson modelling of Anderson's sense of 'ought'. Kielkopf shows that the non-Lewis system of alethic logic, Kl, is the required system. Now we have, thanks to Kielkopf, a complete reduction of deontic logic to alethic logic even in an Andersonian development of deontic logic. Of course, this claim must be stated with some reservation. For, even Kielkopf confesses that his sole reason

> ... for calling the resulting Dawson modellings «Dawson modellings for Anderson's sense of 'ought'» is that they arise from the preceding alterations in Anderson's definition of P(p). Even if such alterations are a serious distortion of Anderson's development of deontic logic, I think

> that the sequel will show that the resulting deontic logics are interesting for their own sake. (Cf. [11], p. 405)

I agree with Kielkopf's claim that even if the considerations sketched above do represent a distortion of Anderson's intentions, the fact of the matter is that the deontic system resulting from employing Kl as a Dawson modelling is interesting for its own sake. In fact, the only reason why we have included Kielkopf's discussion of Anderson's sense of 'ought' is to show how Kl has been motivated as a candidate for Dawson modellling.

In any event, some of the interesting features of Kl as a Dawson modelling for deontic logic that Kielkopf observes concerns the iteration of deontic operators and the juxtaposition of alethic and deontic operators. For example, he notes that the following equivalences hold in Kl:

$$Op \equiv OOp \equiv OPp \equiv LOp \equiv LPp \equiv OLp \equiv OMp$$

$$Pp \equiv POp \equiv PPp \equiv MOp \equiv MPp \equiv PLp \equiv PMp$$

He also observes that any deontic operators in Kl lying in the scope of other deontic operators are equivalent to formulae having deontic operators not occurring in the scope of other deontic operators. (Cf. [11], pp. 407-408).

Finally, Kielkopf addresses himself to the philosophical problem of reducing deontic logic to alethic logic. For example, in [6], Casteneda argues that we are clear enough about the naturalistic fallacy to realize that if we reduce moral statements to claims of logical necessity, then we have, in effect, reduced morality to something which is not morality. He suggests that Anderson is guilty of doing this when he identifies ought-statements with strict implication-statements by means of his definitions. However, Kielkopf attempts to defend Anderson's reduction in the following fashion:

> ...if this Dawson modelling is not a total distortion of Anderson's sense of «ought», it provides a threefold defense against a charge that he reduced moral statements to logical ones. First the fact that O(p), viz., LM(p), is not

> equivalent to L(p) shows that ought-statements are not assertions of logical necessity if that is what L( ) is to be used for. The ought-operator contains L( ) but it is not L( ). The fact that LM is an irreducible modality in Kl can be construed as showing that O(p) is what it is and nothing else as G. E. Moore required at the beginning of his *Principia Ethica*. Secondly, the fact that O(p), viz. LM(p), is not a Kl-theorem shows that not all ought-statements are logical truths in the sense of being provable formulae. Thirdly, and of most significance for showing that there is no reduction of moral claims to logical claims, is the fact that any Dawson modelling for Anderson's sense of «ought» will have: $LM(p) \cdot LM(q) \supset LM(p \cdot q)$, as a theorem. To be sure, this is not the elementary modal fallacy: $M(p) \cdot M(q) \supset M(p \cdot q)$. But it is close to it! It is not at all clear that any natural, i.e., used, sense of «necessity» and «possibility» could fit into it. I am not saying that what is alethically bad is deontically good. I am saying only that if a reduction of deontic to alethic logic requires an alethic logic with no natural interpretation for L( ) and M( ) as necessity and possibility, the claim that such a reduction is a reduction of the moral to the non-moral is weakened. ([11], pp. 408-409).

We might view Kielkopf's threefold defense of Anderson's reduction as a defense resting primarily upon syntactical considerations. In fact, even those interesting features of Kl to which he directs our attention are of a syntactical nature. The purpose of this paper will be to press the defense a little further by complementing Kielkopf's syntactical defense with a semantical one. Moreover, this paper will attempt to point out additional interesting features of Kl as a Dawson modelling when viewed from a semantical point of view. One such interesting feature that even Kielkopf's syntactical arguments suggest is that Kl involves a sense of «necessity» with a moral tinge». This claim can be substantiated only from a semantical point of view. It is hoped that this paper will provide some of the substantiation.

II

Modal System Kl of Sobocinski and McKinsey (Cf. [14] and [12] respectively) is axiomatized by simply appending 'LMp ⊃ MLp' to some axiomatic basis for S4 containing the Unrestricted Rule of Necessitation as primitive. (We shall assume for the sake of the subsequent discussion that our axiomatization of Kl employs '∼', '⊃' and 'L' as its only primitive operators). Relying heavily on the terminology of Hughes and Cresswell in [10], it is well-known that a semantic model for S4 is defined as an ordered triple $<W, R, V>$ where W is a set of possible worlds, R is a reflexive and transitive accesibility relation defined over the members of W, and V is a value assignment satisfying the following conditions:

1. For any statement variable, p_k, and for any $w_i \varepsilon W$, either $V(p_k, w_i) = T$ or $V(p_k, w_i) = F$
2. For any wff, α, and for any $w_i \varepsilon W$, $V(\sim\alpha, w_i) = T$ iff $V(\alpha, w_i) = F$; otherwise $V(\sim\alpha, w_i) = F$.
3. For any wffs, α and β, and for any $w_i \varepsilon W$, $V((\alpha \supset \beta), w_i) = T$ iff either $V(\alpha, w_i) = F$ or $V(\beta, w_i) = T$; otherwise $V((\alpha \supset \beta), w_i) = F$.
4. For any wff, α, and for any $w_i \varepsilon W$, $V(L\alpha, w_i) = T$ iff for every $w_j \varepsilon W$ such that $w_i R w_j$, $V(\alpha, w_j) = T$; otherwise $V(L\alpha, w_i) = F$.

Given this model for S4, we may say that a wff, α, is S4-logically true iff for every S4-model $<W, R, V>$ and for every $w_i \varepsilon W$, $V(\alpha, w_i) = T$. Quite obviously, we can also derive the truth conditions for the other non-primitive operators of S4 without much difficulty. However, since they are well-known, we shall merely state them without proof:

5. For any wffs, α and β, and for any $w_i \varepsilon W$, $V((\alpha \vee \beta), w_i) = T$ iff either $V(\alpha, w_i) = T$ or $V(\beta, w_i) = T$; otherwise $V((\alpha \vee \beta), w_i) = F$.
6. For any wffs, α and β, and for any $w_i \varepsilon W$, $V((\alpha \cdot \beta), w_i) = T$ iff both $V(\alpha, w_i) = T$ and $V(\beta, w_i) = T$; otherwise $V((\alpha \cdot \beta), w_i) = F$.

7. For any wff, α, and for any $w_i \varepsilon W$, $V(M\alpha, w_i) = T$ iff for at least one $w_j \varepsilon W$ such that $w_i R w_j$, $V(\alpha, w_j) = T$; otherwise $V(M\alpha, w_i) = F$.

Now in [8], a semantic model for Kl is constructed by simply introducing «abnormal worlds» into an S4-model structure. Abnormal worlds possess two characteristic features: (1) They are accessible from any other world in the model. More specifically, every Kl-model possesses at least one of these worlds accessible from any other world. (2) Modal distinctions among statements within abnormal worlds collapse; in other words, these kinds of worlds do not recognize differences among actual truths, possible truths and necessary truths. Informally then, a Kl-model structure propounds the view that no matter what states of affairs within which we find ourselves, we are always able to conceive at least one other possible state of affairs where it would be to no avail to elaborate modal distinctions among statements. (1)

From a formal point of view, we say that $\langle W, R, V \rangle$ is a Kl-model iff (a) it is an S4-model; (b) there exists at least one *abnormal* $w_j \varepsilon W$ such that for every $w_i \varepsilon W$, $w_i R w_j$; and (c) V is a value assignment not only satisfying the conditions stated above, but also the following additional conditions concerning the evaluation of wffs in abnormal worlds:

8. For any abnormal $w_j \varepsilon W$,
   a) there exists a wff, $L\alpha$, such that $V(L\alpha, w_j) = T$ if for any wff α, $V(\alpha, w_j) = T$;
   b) there exists a wff, $L\alpha$, such that $V(L\alpha, w_j) = F$ if for any wff, α, $V(\alpha, w_j) = F$;
   c) there exists a wff, α, such that $V(\alpha, w_j) = T$ if for any wff, $L\alpha$, $V(L\alpha, w_j) = T$; and
   d) there exists a wff, α, such that $V(\alpha, w_j) = F$ if for any wff, $L\alpha$, $V(L\alpha, w_j) = F$.

(1) It is perhaps worth mentioning that an alternative interpretation for K1 to the one discussed here has been provided by Krister Segerberg in [13].

These additional conditions which the value assignment in a Kl-model must satisfy will guarantee that modal distinctions among statements will break down in abnormal worlds. Given this model for Kl, we can now say that a wff, α, is Kl-logically true iff in every Kl-model $\langle W, R, V \rangle$ and for every normal $w_i \varepsilon W$, $V(\alpha, w_i) = T$. The soundness and completeness of Kl on this interpretation are demonstrated in [8].

The truth conditions stated in 8 concerning the evaluation of wffs in abnormal worlds are only stated for the necessity operator since it is primitive in our axiomatization of Kl; however, the truth conditions for the possibility operator are, quite obviously, easily derivable. We state them below:

9. For any abnormal $w_j \varepsilon W$,
   a) there exists a wff, $M\alpha$, such that $V(M\alpha, w_j) = T$ if for any wff, α, $V(\alpha, w_j) = T$;
   b) there exists a wff, $M\alpha$, such that $V(M\alpha, w_j) = F$ if for any wff, α, $V(\alpha, w_j) = F$;
   c) there exists a wff, α, such that $V(\alpha, w_j) = T$ if for any wff $M\alpha$, $V(M\alpha, w_j) = T$; and
   d) there exists a wff, α, such that $V(\alpha, w_j) = F$ if for any wff $M\alpha$, $V(M\alpha, w_j) = F$.

An interesting feature of Kl is that it is not contained in S5. Moreover, it is incompatible with S5; incompatible in the sense that if the characteristic axiom of either system is appended to the axiomatic basis of the other system, they collapse into the bi-valued statement logic. For example,

$$\{S5: LMp \supset MLp\} = \{Kl; MLp \supset Lp\} = PC.$$

This is easily demonstrated by showing that '$p \supset Lp$' is a thesis of both $\{S5; LMp \supset MLp\}$ and $\{Kl; MLp \supset Lp\}$ (Cf. [12], p. 93). Now since an S5-model is pretty much the same as an S4-model except for the consideration that an S5-model possesses the additional stipulation that the accessibility relation is symmetrical, we should expect that the model resulting from appending the additional requirement of symmetricality to a Kl-model would validate '$p \supset Lp$'. That it does indeed is demonstrated below.

Assume for the sake of reductio that $V((p \supset Lp), w_i) = F$. It follows that

(1) $V(p, w_i) = T$

and

(2) $V(Lp, w_j) = F.$

Clearly then, it follows from (2) that there exists at least one w_j such that

(3) $V(p, w_j) = F.$

Now w_j is abnormal since it is accessible from every world in the above model; viz., from w_i and from itself. Hence, in view of truth condition 9b, we have from (3) that there exists a wff 'Mp' such that

(4) $V(Mp, w_j) = F.$

But R is symmetrical, therefore from (4)

(5) $V(p, w_i) = F$

which is inconsistent with (1). Thus, $V((p \supset Lp), w_i) = T.$

Having discussed some of the distinctive features of a Kl-model, we now prove, for the sake of the subsequent discussion, a few methatheorems concerning a Kl-model.

MT1: For every $w_i \varepsilon W$, there exists at least one abnormal $w_j \varepsilon W$ such that $w_i R w_j$.

We prove this metatheorem informally. A characteristic feature of a Kl-model is that there exists an abnormal $w_j \varepsilon W$ such that for every $w_j \varepsilon W$, $w_i R w_j$; i.e., there exists an abnormal world accessible from every world in the model. But if this is true, then quite obviously it follows that for every world in the model there exists an abnormal world such that the latter is accessible from every one of the former; i.e., for every $w_i \varepsilon W$, there exists at least one abnormal $w_j \varepsilon W$ such that $w_i R w_j$.

MT2: For any wff, α, and for any $w_i \varepsilon W$, $V(LM\alpha, w_i) = T$ iff for every *abnormal* $w_j \varepsilon W$, such that $w_i R w_j$, $V(\alpha, w_j) = T$.

Assume that for any wff, α, and for any $w_i \varepsilon W$,

(1) $V(LM\alpha, w_i) = T$.

Further suppose that w_iRw_j where w_j is any abnormal world in W. Clearly, in view of truth condition 4, it follows from (1) that

(2) $V(M\alpha, w_j) = T$.

But then, in view of truth condition 9c, it follows from (2) that there exists a wff, α, such that

(3) $V(\alpha, w_j) = T$.

Now for the converse. Suppose that for every abnormal $w_j \varepsilon W$, such that w_iRw_j,

(1') $V(\alpha, w_j) = T$.

Then, in view of truth condition 9a, there exists a wff 'Mα' such that

(2') $V(M\alpha, w_j) = T$.

It also follows from (1), in view of truth condition 7, that

(3') $V(M\alpha, w_i) = T$.

Consequently, in view of truth condition 4, we have from both (2') and (3') that

(4') $V(LM\alpha, w_i) = T$.

MT3: For any wff, α, and for any $w_i \varepsilon W$, $V(ML\alpha, w_i) = T$ iff for at least one *abnormal* $w_j \varepsilon W$ such that w_iRw_j, $V(\alpha, w_j) = T$.

Assume that for any wff, α, and for any $w_i \varepsilon W$,

(1) $V(ML\alpha, w_i) = T$.

Then clearly, in view of truth condition 7, it follows that there exists at least one $w_j \varepsilon W$ such that w_iRw_j, and

(2) $V(L\alpha, w_j) = T$.

But w_j is abnormal since it is accessible from every world in the model. Hence, in view of truth condition 8c, there exists a wff, α, such that

(3) $V(\alpha, w_j) = T$.

For the converse we assume that there exists at least one abnormal $w_j \varepsilon W$ such that $w_i R w_j$, and

(1′) $V(\alpha, w_j) = T$.

It follows, in view of truth condition 8a, that there exists a wff, $L\alpha$, such

(2′) $V(L\alpha, w_j) = T$.

Consequently, because of truth condition 7, we have

(3′) $V(ML\alpha, w_i) = T$.

Metatheorems 2 and 3 indicate that the distinct modalities 'LM' and 'ML' achieve special status in a Kl-model structure. They are not simply iterated modalities; rather they are iterated modalities gaining their unique significance by virtue of the consideration that they are realizable in *different kinds of worlds*. Our model permits us to characterize the status of each of the ten distinct modalities of system Kl in the following way:

1. α	α is true in the initial world of the model; i.e., the actual world.
2. $L\alpha$	α is true in all accessible possible worlds whether normal or abnormal.
3. $M\alpha$	α is true in at least one accessible possible world whether normal or abnormal.
4. $LM\alpha$	α is true in all accessible *abnormal* possible worlds.
5. $ML\alpha$	α is true in at least one accessible *abnormal* possible world.
6. $\sim \alpha$	α is false in the initial world of the model; i.e., the actual world.
7. $\sim L\alpha$	α is false in at least one accessible possible world whether normal or abnormal.
8. $\sim M\alpha$	α is false in all accessible possible worlds whether normal or abnormal.

9. $\sim$ LMα — α is false in at least one accessible *abnormal* possible world.

10. $\sim$ MLα — α is false in all accessible *abnormal* possible worlds.

That Kl has no more than ten distinct modalities is easily verified. It is well-known that S4 possesses fourteen distinct modalities; those of Kl plus LMLα and MLMα and their negations. Now the reader can readily satisfy that both

$$LMLp \supset LMp$$

and

$$MLp \supset MLMp$$

are provable in S4. Consequently, in order to demonstrate that there are no more than ten distinct modalities in Kl, we must show that their converses, viz., both

$$LMp \supset LMLp$$

and

$$MLMp \supset MLp$$

are Kl-logically true. (2)

Assume for the sake of reductio that $(V((LMp \supset LMLp), w_i) = F$. It then follows that both

(1) $V(LMp, w_i) = T.$

and

(2) $V(LMLp, w_i) = F.$

In view of MT 2, we have from (2) that there exists at least one abnormal $w_j \varepsilon W$ such that

(3) $V(Lp, w_j) = F.$

However, because of truth condition 8d, it follows from (3) that there exists a wff, 'p', such that

(2) That Modal System K1 has at most ten distinct modalities is indicated by McKinsey in [12], p. 93.

(4) $V(p, w_j) = F$.

Again MT 2 permits us to infer from (1) that

(5) $V(p, w_j) = T$.

But (4) and (5) are mutually inconsistent; therefore $V((LMp \supset LMLp), w_i) = T$.

Now let's suppose that $V((MLMp \supset MLp), w_i) = F$. Clearly then,

(1) $V(MLMp, w_i) = T$

and

(2) $V(MLp, w_i) = F$.

In view of MT 3, it follows from (1) that there exists at least one abnormal $w_j \varepsilon W$ such that

(3) $V(Mp, w_j) = T$.

From (3), because of truth condition 9c, there exists a wff, 'p' such that

(4) $V(p, w_j) = T$.

Again, in view of MT 3, it follows from (2) that

(5) $V(p, w_j) = F$

which contradicts (4). Thus $V((MLMp \supset MLp), w_i) = T$.

Quite obviously, if, along with Kielkopf, we identify $O\alpha$ with $LM\alpha$ and $P\alpha$ with $ML\alpha$, then we can undoubtedly understand, at least from a semantical point of view, the difference, on the one hand, between necessity and obligation and, on the other hand, between possibility and permission. If by necessity in KI we are to understand logical necessity, then clearly there is a radical difference between necessity and obligation. Whatever is necessary is realizable in all possible worlds whether normal or abnormal, whereas whatever is obligatory is realizable in only all of the abnormal worlds. Clearly, the contention that Anderson's development of deontic logic involves reducing moral statements to claims of logical necessity has surely been weakened. After all, it might prove convenient for certain purposes to view abnormal worlds as possible moral situations, and normal worlds as possible states of affairs or situations of a non-moral character. On this view, necessity would be applicable to all possible situations, whereas obligation to only moral situations. It would hardly seem worthwhile now to claim that morality has been reduced to something which is not morality.

Of course, when viewed syntactically, it certainly does appear as though moral statements have been reduced to statements of logical necessity. After all, if by necessity in Kl we are to understand logical necessity, then, since possibility is defined in terms of logical necessity, the concept of possibility we are dealing with must be logical possibility. Hence the operators governing α in $LM\alpha$ are of a logical character rather than of a moral character.

Consequently, in identifying $O\alpha$ with $LM\alpha$, we have in effect reduced morality to something which is not morality. Viewing it in this way, it couldn't possibly be the case that $O\alpha$ represents any sense of obligation compatible with our moral intuitions. But this, as mentioned above, overlooks the fact that the truth conditions of $LM\alpha$ rely upon different kinds of considerations than the truth conditions of eiher $L\alpha$ or $M\alpha$. The concept of an abnormal world in our version of a Kl-model is not a derived concept, it's primitive. From a semantical point of view, the claim that morality has been reduced to something which is not morality would involve showing that abnormal worlds are reducible to normal ones.

III

We have already mentioned that Kielkopf's investigations suggest, as he himself notes, that Kl involves a sense of «necessity with a moral tinge.» If this is correct then of course the issue of reducing moral claims to claims of logical necessity doesn't arise; not even from a syntactical point of view. For now it could be argued that the operators governing α in $LM\alpha$ posses, as it were, a «moral flavor» and accordingly there isn't any difficulty in identifying $O\alpha$ with $LM\alpha$. This would simply be a case of defining a certain moral concept in terms of other moral concepts. Of course we would now have the problem of understanding exactly what kind of moral necessity and possibility 'L' and 'M' are supposed to express in a Kl-model. Ordinarily, obligation is also thought of as a kind of moral necessity. Now since both 'L' and 'LM' are different modalities in a Kl-model, it appears that a Kl-model discriminates between two kinds of moral necessity. Kielkopf suggests that the concept of necessary truth in Kl might be viewed as expressing what Kant might have possibly meant by a natural or universal law (Cf. [11], p. 409). On this interpretation whatever is necessary is a moral law.

Perhaps our version of Kl-model might lend some plausibility to this interpretation. Let us now construe the set W in a Kl-model as a set of morally possible worlds. Informally, we might view these worlds as the set of all moral situations. Following Hintikka (Cf. [9]), we might construe the abnormal worlds in W as deontologically perfect worlds; worlds in which all moral obligations are fulfilled. Hintikka views the way deontic alternatives are related to a given world in pretty much the same way as a Kantian «Kingdom of Ends» is related to the actual world ([9], p. 189). From the point of view of the actual world, deontologically perfect worlds are realizations of normative ideals (obligations) obtaining in the actual world. In Kantian language, they are mere ideals which can be realized only if all maxims based upon the categorical imperative are followed without exception. Of course, for Kant the *Reich der Zwecke* is a unique entity, whereas there are normally several deontic alternatives to a given morally possible world. In a sense then, as Hintikka himself observes, the concept of a deontologically perfect world is a relativization of the notion of a *Reich der Zwecke* (Cf. [9], p. 190).

In any event, if we identify the abnormal worlds in a Kl-model with Hintikka's deontologically perfect worlds, then whatever is obligatory (necessarily possible) is realizable in all accessible deontologically perfect worlds. On this interpretation then, whatever is obligatory is not realizable in just any morally possible situation; being normative ideals from the point of view of any given moral situation, obligations can be fulfilled only in those moral states of affairs where all maxims based upon the categorical imperative are followed without exception.

We have seen that in a Kl-model, abnormal worlds are characterized as worlds where all modal distinctions break down. This is hardly surprising in light of the above analysis. Deontologically perfect worlds are ideal moral situations, situations where what ought to be the case is the case.

Being ideal, we should also expect that what must be the case is the case, what can be the case is the case, and so on.

We have also seen that there is at least one abnormal world accessible from every world in a Kl-model. Informally, this means that, in terms of our re-interpretation, that a Kl-model propounds the view that we can always conceive at least one deonologically perfect world from any moral situation within which we find ourselves, at least one other moral situation where we can fulfill our obligations. In a sense then, what ought to be done can be done. Thus we should expect that our Kl-model would adopt the «sollen-Können» principle. To see that it does indeed, we demonstrate that

$$Op \supset Mp$$

is validated by a Kl-model. Assume that it isn't; i.e. $V((Op \supset Mp), w_i) = F$. Clearly, it then follows that

(1) $V(Op, w_i) = T$

and

(2) $V(Mp, w_i) = F.$

From (1) we have by definition that

(3) $V(LMp, w_i) = T.$

But, in view of MT 1, for every $w_i \varepsilon W$, there exists at least one abnormal $w_j \varepsilon W$ such that $w_i R w_j$. Hence it follows from (3), in view of MT 2, that

(4) $V(p, w_j) = T.$

Consequently, from (2) that

(5) $V(p, w_j) = F.$

and so we have a contradiction. Therefore, $V((Op \supset Mp), w_i) = T$. A Kl-model then commits us to the view that ought implies can.

A necessary statement is a statement which is true in all accessible possible moral situations whether deontologically perfect or not. Moral necessity is stronger than obligation; it is universal and applicable to all moral situaions. From the point of view of any given moral situation our obligations can only we fulfilled in certain worlds, those ideal worlds in which the categorical imperative is followed without exception. However, morally necessary truths hold in all worlds accessible from any given one indepently of the categorical imperative — they are moral laws. The categorical imperative, on this account, doesn't tell us what the moral laws are, it tells us what we ought to do. What the moral laws are, must be determined on other gorunds — reason, intuition, etc. A Kl-model does not provide the means for resolving this issue.

A Kantian view of ethics places a high premium on rationality. Whatever action we do in fact perform should be consistent with the moral laws holding in the situations within which we find ourselves. Thus, whatever action I can perform morally, i.e., whatever is morally possible for me to do, must be consistent with all those moral principles holding in a given situation. Now in determining what we ought to do, Kant asks us to «Act only according to that maxim by which you can at the same time will that it should become a universal law.» But we are obliged to do only what we *can* do, and we can do only what reason tells us is consistent with the moral laws. Hence any maxim in accordance with which we shall act is a statement of the moral possibility of an action. Accordingly, the statement of a maxim in accordance with which we can act has the logical form, 'Mp' where 'p' is a description of an action. But ought I to do what 'p' states ? The categorical imperative says that we ought to do 'p' if and only if we can will that the maxim 'Mp' should become a universal law; that is, if and only if we can will 'LMp'. On this interpretation, we undoubtedly see why Kl can serve as a Dawson modelling for a Kantian version of deontic logic when $O\alpha$ is defined in terms of $LM\alpha$.

Of course I am not insisting that the interpretation sketched above is what Kant either says or should have said. Not at all. All that I have hoped to show is that a Kl-model provides a way of gaining an intuitive understanding along Kantian lines of the concepts of moral necessity (moral law), obligation, permission and moral possibility. From a purely formal point of view, it is enough to know that these concepts differ from one another because of the truth conditions governing them with respect to the two different kinds of possible worlds in a Kl-model.

Hopefully it is now clear that if by necessity in Kl we are to understand «logical necessity», then there is a semantical justification for claiming that Andersonian developments of deontic logic or Dawson modellings in general do not necessarily involve, contrary to what syntactical considerations might suggest, the reduction of morality to something which is not morality. On the other hand, given a certain intuitive understanding of Kl-model, we can now recognize why it is possible to view modal system Kl as a logic of moral necessity and possibility. (8)

(8) This paper has profited from discussion with Robin Smith, James R. Hamilton and Michael P. O'Neil all of whom are members of the philosophy department of Kansas State University.

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