TENSE LOGIC AND STANDARD LOGIC

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1. *Introduction*

It is a matter of debate whether the logical «theory of time» should constitute an independent discipline (consisting of ordinary predicate logic with added tense operators), or be viewed as an applied predicate calculus (in which quantification occurs over moments in time). The former point of view was advocated by A. Prior (cf. his influential [13]) — and seems to be widely accepted as part of the package deal involving tenses, modalities, deontic notions, etc., called *intensional logic* —, whereas the latter opinion is voiced by sporadic dissidents (cf. Massey [11] and Needham [12]). In this paper we try to shed some light upon this controversy (or even: a clash of Kuhnian paradigms, according to Massey) by comparing the two approaches from a technical point of view. The relevant material is arranged as follows. Section 2 is concerned with the general argumentation pro or contra the two approaches, which is partly philosophical, partly linguistical, etc. It is argued that the burden of justification lies with the Prior approach, which is only partially successful in this respect. In section 3 several publications of «Prioreans» are discussed, as far as relevant to the present theme. These are Prior [13], Kamp [6], Kamp [7], Vlach [18], Åqvist [1], Åqvist & Guenthner [2] and Gabbay [5], in that order. It will be seen that, as the complexity of the respective tense-logical languages increases, they «converge» towards predicate logic (although some authors do not seem to be aware of it). This section extends the analogous discussion in Needham [12], chapter V. 1, 2, 3. Section 4 is devoted to the latter book, which is one of the clearest expositions of the non-Priorean approach. Notwithstanding this clarity, it leaves something to be desired from the point of view of technical precision. A paper of Quine ([14]) showing how to formulate predicate logic without using individual variables (using permutation and substitution operators — for argument places — instead), is explained in section 5. Since such operators occur in recent tense-logical languages (cf. section 3), section 5 supports the «convergence» claim made above. Finally, section 6 contains our main conclusions, viz. (1) the two approaches are not mutually exclusive, in fact (2) Priorean tense logics may be profitably regarded as (often interesting) *subsystems* of the predicate logic with time variables, but (3) as the complexity of Priorean tense logics increases, they reach a point where an open conversion to the second approach may be preferable.

2. *The general case*

Before proceeding to the battle field, let us sketch the two positions. On Prior's view, a sentence like

(1) A child is born (B)

expresses a proposition which may be true or false, depending on the time of its utterance. (We abstract from other context factors for the moment). Its past tense formulation

(2) A child was born

may be viewed as the result of a *past tense operator* P (to be read as «it was the case that») transforming B into PB. Similarly, the *future tense operator* F (to be read as «it will be the case that») would transform B into FB, i.e.,

(3) A child will be born.

A typical Priorean procedure is the *iteration* of such operators, to arrive at combinations like PPB: «It was the case that (it was the case that B)», or, in plain English:

(4) A child had been born,

or PFB: «It was the case that (it will [= would, in idiomatic English] be the case that B)», i.e.,

(5) A child would be born.

This is all very elegant, and the opposing view (let us call it the *classical* one) seems rather cumbersome by comparison. It uses a two-sorted predicate calculus with variables x, y, z, ... for individuals and t, t', t'', ..., t_0, t_1, ... for moments («points in time»), to bring out graphically the dependence on time. E.g., Bt will stand for «A child is born at time t». Paradoxically, this policy may also be described as *de-tensing* (cf. Massey [11]), because Bt, if true, will be timelessly true. (Many combatants feel an abysmal divergence here between the two approaches — whereas we can only see a notational difference). The next step is more momentous. Let E denote the relation of precedence among points in time (*earlier than*). The above sentences will then be expressed by means of *quantifiers* as follows.

(1)' Bt (t always denotes the «point of evaluation»),

(2)' $\exists t'(Et't \wedge Bt')$ («A child is born at some time t' earlier than t»),

(3)' $\exists t'(Ett' \wedge Bt')$ («A child is born at some time t' later than t»),

(4)' $\exists t'(Et't \wedge \exists t''(Et''t' \wedge Bt''))$,

(5)' $\exists t'(Et't \wedge \exists t''(Et't'' \wedge Bt''))$.

The classical approach seems at a clear disadvantage from the point of view of mere complexity. But, let us not judge too early. A finer analysis of (1) might read

(6) $\exists x(Cx \wedge Bx)$ («Some individual is a child which is born»), which would have the «classical» counterpart (Ctx: «x is a child at t», Btx: «x is born at t»)

(6)' $\exists x(Ctx \wedge Btx)$.

A sentence like

(7) A child was born which would be king

(an example taken from Kamp [5]) will get the Priorean reading

(8) $P\,\exists x(Cx \wedge Bx \wedge FKx)$.

Its classical counterpart is

(8)' $\exists t'(Et't \wedge \exists x(Ct'x \wedge Bt'x \wedge \exists t''(Et't'' \wedge Kt''x)))$,

which is surely less perspicuous. But, now consider the following sentence — also due to Kamp —

(9) A child was born which will be king.

A minor modification of (8)' will yield its classical transcription

(9)' $\exists t'(Et't \wedge \exists x(Ct'x \wedge Bt'x \wedge \exists t''(Et(!)t'' \wedge Kt''x)))$.

(9) cannot be expressed in terms of the above operators, however, whence a third operator N (for *now*) has to be added. (9) then gets the Priorean reading

(10) $P\,\exists x(Cx \wedge Bx \wedge NFKx)$.

It seems the classical approach has scored a point: a simple alteration («simple» from *its* point of view, that is) had to be accounted for by the introduction of a whole new operator in the rival theory.

The above will suffice to show in what way the two approaches differ, but also which obvious parallels exist. Such parallels, i.e., classical transcriptions of Priorean formulas, can always be found (cf. section 3), at least, as long as Priorean tense logicians keep using operators which can be expressed in Kripke semantics (and they show every intention of doing so). Now, predicate logic is a venerable system which is well-understood from a technical point of view. From this point of view, then, the burden of justification would seem to lie with the Prior approach: why not use existing logical systems ?

The argument of the preceding paragraph applies not only to tense logic, but also to other variants of intensional logic, like modal logic. (E.g., the theory of *counterfactuals*, in which authors manage to come up with legions of operators in one paper, would be an obvious next candidate — and, indeed, Needham (written communication) is planning the campaign already). The first line of defence then often consists in an appeal to authority. D. Scott writes in [16]:

«One often hears that modal (or some other) logic is pointless because it can be translated into some simpler language in a first-order way. Take no notice of such arguments. There is no weight to the claim that the original system must therefore be replaced by the new one. What is essential is to single out important concepts and to investigate their properties. The fact that the real numbers can be defined in terms of sets is no argument for being interested in *arbitrary* sets. One must look among the sets for the significant ones and cannot be censured if one finds the intrinsic properties of the reals more interesting than any of their formulations in set theory».

The analogy seems to fail, however. It is, admittedly, ridiculous to reduce real analysis in practice to set theory. This would require such an amount of «unpacking» of definitions that simple results would fill whole volumes. But the distance between intensional logic and its classical counterpart is not all that great, and the «reductionist» approach is practically viable as well as theoretically enlightening (we will argue for this below). And, anyway, even Scott admits that the intensional operator approach should justify itself: by yielding results (the tree is to be known by its fruits). Fruitfulness is a rather subjective notion, however, so we turn to other arguments advanced in favour of the Priorean approach.

According to Massey, arguments are of no importance here, because the two approaches are different paradigms (in the sense of Kuhn [8]), between which no rational decision is possible. This view is exaggerated, since both parties often seem to differ not so much in what they are doing (technically), as in their *interpretation* of what they are doing. But, if the picture of two competing theories is correct, then, again, the matter of the «burden of proof» arises. Which of the two is the revolutionary theory, having to display a better performance in order to dislodge the established one ? Nowadays, the Prior approach is so widely accepted, that Needham pictures himself as a revolutionary David advancing against Goliath. But, only as far back as 1969, Massey could describe Prior and his «dedicated band of revolutionaries» as subversive elements turning against the main logical tradition of «Russell, Keynes and Quine» (cf. [11], p. 18). The latter view seems historically more correct. (This does not mean that Massey's account of Priorean tense logic is correct. E.g., some of his attacks concern irrelevant details, like Prior's Polish preoccupations: «languages which boast theses like 'CpNFNPp' and 'CNFNCpqCFp Fq'» ([11], p. 19). Moreover, he is too extravagant both in his criticism and in his praise:

«In this paper I have argued that Prior's tense-logic programme is unviable, because it purports to lead to logics of systems of tokens, and ill-advised, because grounded in bad physics and indefensible metaphysics (...). Yet few if any viable and well-advised programmes have been more interesting, yielded more insights, or abounded more in ingenious innovations than has the programme I have argued against». ([11], p. 31/2.)

Arguments advanced in favour of the Priorean approach fall into three categories: philosophical, linguistical and technical (i.e., purely logical). Some of the *philosophical* arguments are peculiar to *predicate* tense logic, like those involving the (in) validity of the *Barcan formula*

$$F\,\exists x Ax \rightarrow \exists x\, F\, Ax.$$

It may be doubted if these arguments are anywhere as cogent here as they are in the case of *modal logic*. Moreover, we want the discussion to apply to propositional tense logic as well; which leaves us with enough arguments anyway.

The main philosophical argument relies on Quine's doctrine of ontological commitment. The classical approach uses quantification over moments, thereby committing itself to the existence of moments. Much of the discourse involving time, however, — it is said — is not committed to this, so it should not be described in this fashion. The Priorean languages keep our hands free, so to speak, allowing different interpretations, of which the «time axis» is only one. In a sense, this argument — which also applies to, e.g., modal logic — is correct. Intensional notations are not inextricably tied up with Kripke semantics; which leaves room for possible new associations. Still, one wonders how much of this is mere ideology, especially in tense logic. Why are Kripke type semantics always introduced in the same breath with the Priorean language ?

Some will reply that this is for technical convenience only. This then often turns out to amount to a completeness proof. But completeness results refer to the semantics in their formulation, and are even usually interpreted as giving priority to semantics: the axiomatic system succeeds in capturing the richness of the semantic notion of truth. The reply, therefore, only has some force coming from people who use the semantics as nothing more than a convenient tool for proving purely syntactic results, like e.g., Hamblin's «fifteen tenses theorem». But these are rare. One gets the impression that there is some truth in Needham's accusation that tense-logicians want to have their cake and eat it. By banishing moments to the metalanguage (of semantical interpretation), they hope to enjoy all the practical benefits of operating with moments, without being ontologically committed to them. But one does not escape from Quine so easily !

It is difficult for a logician to assess the merits of *linguistical* arguments. Nevertheless, we mention a few heard in recent years. (1) «Tenses *are* operators: this is just common sense about language.» This type of argument should be thoroughly distrusted. Is it not one of the tasks of logic to attack so-called «linguistically evident» facts ? And, to put it more bluntly, what *is* linguistic evidence but traditional school grammar raised to the level of «intuitive insight» ? (2) «The classical approach gives verbs a higher number of arguments than they actually have. E.g., 'is born' is unary, not binary (as in (1)' above). 'Is born at' is binary, but that is a different verb.» This argument points backwards at the user as well: indexical semantics also raises the arity, but is more clever at hiding this fact notationally. (3) «The classical approach uses a language that is *too rich*. Maybe a Priorean procedure of adding enough operators, no matter how ad hoc, will turn out to suffice for the description of natural language». This argument is dangerous (as well as being unflattering to natural language). A similar line of thought might show that no general theory of quantification is required, because natural language contains no more than, say, five types of iterated quantifier (which could be described ad hoc by means of some kind of «suppositio» theory). Frege's general theory, however, turned out to be both simpler and stronger than the ad hoc one !

From a *technical* point of view, it may be advanced that Priorean tense logics have a more elegant and efficient notation than the classical one. This is true, but only to a certain extent (cf. the above discussion and also section 3). Also, these logics are often interesting *as such* (true again) and have turned out to be natural in the sense of being connected with other mathematical subjects (like algebra and topology). Nevertheless, these considerations, if true, only establish that Priorean tense logic is interesting, not that it is — in any deep sense of the word — better. Moreover, exclusive concentration on tense logic as such may create problems as well. E.g., one sometimes finds superfluous proofs of pendants of classical results, or wrong estimates of the value of results obtained about tense logics (cf. section 3). And, even where genuinely novel results are obtained (like in Segerberg [17]), one often wonders if tense logic means tying one's hands behind one's back and then trying to prove something which used to be easy. (Compare Segerberg's proof of the completeness of Lin DA with respect to the rationals, [17] theorem 2.1., with the classical proof of $\aleph_0$-categoricity (and, therefore, completeness) of the theory of dense linear orderings.) To be fair, however, it should be added that almost all interesting questions result from *specialization* with respect to a very general question.

Reviewing the three kinds of arguments advanced in favour of the Priorean approach, we see that the philosophical ones could carry some weight if tense-logicians were to live in accordance with them, the linguistical ones are outside our range of competence, and the technical ones merely establish that tense logics are interesting objects of study — if only as nice subsystems of an applied predicate calculus. As the complexity of tense logics increases, however, it may become preferable to use that predicate calculus in its full strength (cf. section 3).

To conclude this section, we recall some arguments employed by Massey *against* Priorean tense logic. The above quotation contained the following charges: it is *unviable* (leading to logics of systems of tokens), and *ill-advised* (being grounded in bad physics and indefensible metaphysics). Two of these will not be considered here. Tense logic is viable, for it exists; and most activities in life are based upon indefensible metaphysics, without, thereby, becoming uninteresting. But an accusation of «bad physics» hurts. E.g., Massey shows that the formula

CKPpPqAAPKpqPKpPqPKqPp, i.e.,
$(Pp \wedge Pq) \rightarrow (P(p \wedge q) \vee P(p \wedge Pq) \vee P(q \wedge Pp))$,

which is accepted by most tense logicians (it holds on a linear time axis), is falsified by the Standard Theory of Relativity. Two lines of defense are open here. First, by attacking in this way, Massey accepts an *intra-theoretical* question, about the validity of some Priorean formula; which almost amounts to admitting that the Priorean framework *is* interesting. The more common objection is that, at least for the description of natural language, «common sense physics» (i.e., physics before it became too difficult) suffices. On this view, tense logic should not describe how the world is, but how language users imagine it to be. This objection has a limited force, however, because (at least) philosophers should require more than this. Moreover, language is not as static as is presupposed here: it may be influenced by science. Nevertheless, tense logic need not be bothered (yet ?) with fine technical points about physical time, because natural language cannot be expected to be definite about each of these. (Its wide applicability and adaptability are based upon a certain indeterminacy). E.g., it will be hard to find linguistical arguments to decide between the rationals and the reals as the «correct» time axis. Take this one for example: «Dying is a continuous process. Yet we talk about a 'moment of death'. This means we presuppose the Upper Bound Theorem, hence Dedekind Completeness, hence the reals.» Similar arguments exist with respect to the marriage ceremony. But all this is asking too much from language. E.g., should we say that the marriage takes place when both say 'Yes' ? This takes time, so should not one be more precise ? At the moment, then, when both have said 'Yes' ? But how is it defined ? When the last longitudinal vibrations of the 'Yes' have died out to amplitude ... ? Surely, this is nonsensical: some *interval* determines the marriage; all further decisions will be theoretical stipulations.

3. *Some Priorean tense logics*

In this section several systems of tense logic are discussed, as far as possible in increasing order of strength. Most of the discussion will be devoted to *propositional* tense logic, where the tense operators may be studied without interference of the individual quantifiers (which creates problems of its own).

3.1 *A. Prior*

In [13] Prior considers various systems of tense logic, of which we mention only the simplest one. (We will use our own notation). Let the language contain proposition letters p, q, r, ..., a unary connective $\urcorner$ (negation), a binary connective $\rightarrow$ (material implication) and unary tense operators F («it will be the case that») and P («it was the case that»). The other Boolean connectives $\wedge$ (and), $\vee$ (and/or), $\leftrightarrow$ (if and only if) may then be introduced in the standard fashion. Two additional tense operators G («it will always — from now on — be the case that») and H («it has always been — up to now — the case that») are defined by $G\varphi \models \neg F \neg \varphi$ and $H\varphi = \neg P \neg \varphi$. Formulas are then defined in the obvious way.

In this language, one may formulate tense-logical principles that are thought to be intuitively valid, like $FFp \rightarrow Fp$ or $P \rightarrow HFp$. This results in various axiomatic systems of which we mention two (cf. Segerberg [17]). The *minimal tense logic* K_t consists of the axioms of some calculus complete (with respect to modus ponens as its sole rule of inference) for propositional logic together with the above definitions, the additional axioms

(1) $G(\varphi \rightarrow \psi) \rightarrow (G\varphi \rightarrow G\psi)$
(2) $H(\varphi \rightarrow \psi) \rightarrow (H\varphi \rightarrow H\psi)$
(3) $\varphi \rightarrow GP\varphi$
(4) $\varphi \rightarrow HF\varphi$

and the additional rules of inference

$$\frac{\varphi}{G\varphi}, \qquad \frac{\varphi}{H\varphi}.$$

Intuitively, K_t imposes no restriction whatsoever upon the structure of the time axis. The next system imposes several, however. $K_tD4.3$ adds the following axioms to K_t:

(5)	$G\varphi \rightarrow GG\varphi$	(«transitivity»)
(6)	$GG\varphi \rightarrow G\varphi$	(«pseudo-density»)
(7)	$F(\varphi \rightarrow \varphi)$	(«succession to the right»)
(8)	$P(\varphi \rightarrow \varphi)$	(«succession to the left»)
(9)	$F\varphi \rightarrow G(F\varphi \vee \varphi \vee P\varphi)$	(«pseudo-connectedness to the right»)
(10)	$P\varphi \rightarrow H(P\varphi \vee \varphi \vee F\varphi)$	(«pseudo-connectedness to the left»)

The relational properties on the right-hand side will be explained below. To avoid misunderstandings, we remark that the *dual* forms (i.e., G replaced by H) of (5) and (6) are not needed: they are derivable in $K_tD4.3$.

Very often, axioms just represent a haphazard choice of «first intuitions». A more systematic view of the matter requires a semantics. Let us define a *structure* as a triple $M = \langle T, <, V \rangle$, where T is a non-empty set (of «moments»), $<$ a binary relation on T («precedence») and V a *valuation* yielding, for each proposition letter p, a subset V(p) of T (the set of «moments when p holds»). The truth definition is then as follows. $M \models \varphi [t]$ («φ holds at t in M»), where $t \in T$, is defined by a recursion whose only non-trivial clauses are

(11) $M \models F\varphi [t]$ iff, for some $t' \in T$ such that $t < t'$, $M \models \varphi [t']$.

(12) $M \models P\varphi [t]$ iff, for some $t' \in T$ such that $t' < t$, $M \models \varphi [t']$.

The most basic result of tense logic is the completeness of K_t:

3.1.1 *Theorem*

For any tense-logical formula φ, φ is provable in K_t if and only if φ holds at t in M for all M and t.

In other words, provability in the minimal tense logic and universal validity coincide. Once we add restrictions on $<$, however, — and there are some obvious candidates — K_t is not sufficient any more. We then need additional axioms «corresponding» to these restrictions. One particular such correspondence is the following. A couple $F = \langle T, < \rangle$ as above (a «bare time structure») will be called a *frame*, and the tense-logical formula φ is said to *hold in* F if, for all $t \in T$ and all valuations V, $\langle T, <, V \rangle \models \varphi [t]$. (Notation: $F \models \varphi$.) Now φ is said to *express* the relational property R if and only if φ holds on exactly the frames satisfying R. Some examples are (cf. Van Benthem [3]):

(5)' $Gp \rightarrow GGp$ expresses $\forall x \forall y (x < y \rightarrow \forall z (y < z \rightarrow x < z))$

(6)' $GGp \rightarrow Gp$ expresses $\forall x \forall y (x < y \rightarrow \exists z(x < z \wedge z < y))$

(7)' $F(p \rightarrow p)$ expresses $\forall x \exists y\, x < y$

(8)' $P(p \rightarrow p)$ expresses $\forall x \exists y\, y < x$

(9)' $Fp \rightarrow G(Fp \vee p \vee Pp)$ expresses $\forall x \forall y (x < y \rightarrow \forall z(x < z \rightarrow (y < z \vee y = z \vee z < y)))$

(10)' $Pp \rightarrow H(Pp \vee p \vee Fp)$ expresses $\forall x \forall y (y < x \rightarrow \forall z(z < x \rightarrow (y < z \vee y = z \vee z < y)))$

Now let R be the conjunction of the properties mentioned in (5)', ..., (10)'. It can be proved that

3.1.2 *Theorem*

For any tense-logical formula φ, φ is provable in $K_tD4.3$ if and only if φ holds in all frames satisfying R.

In fact, one can do better than this (cf. Segerberg [17]). The frame $<\mathbb{Q}, <>$ ($\mathbb{Q}$ is the set of rational numbers, $<$ the «smaller than» ordering) satisfies R, and $K_tD4.3$ «characterizes» this frame:

3.1.3 *Theorem*

For any tense-logical formula φ, φ is provable in $K_tD4.3$ if and only if φ holds in $<\mathbb{Q}, <>$.

A similar theorem holds with respect to $<\mathbb{R}, <>$ (where $\mathbb{R}$ is the set of real numbers), if one adds a certain «completeness» axiom (cf. [17]) to $K_tD4.3$. But, as was argued at the end of section 2, the decision whether or not to accept this additional axiom seems to be of a more philosophical than linguistical nature.

The above will have given some impression of the interest of tense logic. We now proceed to the connection with a two-sorted predicate calculus, i.e., its «classical» rival. There are variables x, y, z, ... for individuals, and $t, t', t'', \ldots, t_0, t_1, t_2, \ldots$ for moments. As long as we are concerned with propositional logic, the only predicate constants needed are unary P, Q, R,... (corresponding to the proposition letters p, q, r,...) taking one (moment) variable, and a binary E («earlier than») taking two (moment) variables. Now define a «translation function» $^{-}$ taking each tense-logical formula φ to a formula $\overline{\varphi}$ of this predicate calculus containing one free (moment) variable (say a fixed variable t):

3.1.4 *Definition*

$\overline{p} = Pt$ for proposition letters p

$\overline{\neg\varphi} = \neg\overline{\varphi}$

$\overline{\varphi \rightarrow \psi} = \overline{\varphi} \rightarrow \overline{\psi}$

$\overline{F\varphi} = \exists t^i(Ett^i \wedge \overline{\varphi}(t^i))$, where $t^i = t'^{\ldots(\text{times})\ldots'}$ and i is the smallest number such that t^i does not occur in $\overline{\varphi}$, and $\overline{\varphi}(t^i)$ is the result of substituting t^i for t in $\overline{\varphi}$,

$\overline{P\varphi} = \exists t^i(Et^it \wedge \overline{\varphi}(t^i))$.

A tense-logical structure as described above may be regarded as a structure for this two-sorted language in an obvious way: in fact, it *is* such a structure already. For $M = \langle T, <, V \rangle$ and $t_0 \in T$, this yields the following basic equivalence, for any tense-logical formula φ:

(13) $M \models \varphi\,[t_0]$ if and only if $M \models \overline{\varphi}\,[t_0]$ (where t_0 is assigned to t).

Do not think of (13) as something to be proven: it is so utterly obvious!

As a subclass of all predicate-logical formulas, the $\overline{\varphi}$'s (for a tense-logical φ) are particularly interesting in that they contain only *restricted quantifiers* of the forms $\exists t'(Et't \wedge \ldots$ and $\exists t'(Ett' \wedge \ldots$ (where t is a variable *distinct from* t'). Such quantifiers play an important role in various branches of mathematics, e.g. in number theory ($\exists y \leqq x \wedge \ldots$), and set theory ($\exists y (y \in x \wedge \ldots$). For this reason, this class of formulas has been characterized (as a subclass of all formulas) model-theoretically by S. Feferman (cf. [4] and also Van Benthem [3]), in terms of invariance under certain model-theoretic constructions. We mention this to show the technical interest in these questions.

A stray remark made above about the *elegance* of Prior's notation will be clarified now. Call a formula of the form $\overline{\varphi}$ (for a tense-logical formula φ) a *P-formula*. The most natural corresponding class in our predicate logic are the *p-formulas*, constructed from atomic formulas of the form Qt (t may be any moment variable) using $\neg$, $\rightarrow$ and restricted quantifiers as described above. Not all p-formulas are P-formulas, e.g., $\exists t''(Ett'' \wedge Rt''')$ is not, but this is not essential:

3.1.5 *Lemma* (cf. Van Benthem [3], 1.3)

Any p-formula φ is logically equivalent to a Boolean combination of P-formulas whose (single) free variable is among that of φ.

3.1.6 *Corollary*

Any p-formula with one free variable is logically equivalent to a P-formula (and hence to a tense-logical formula).

The elegance of Prior's notation thus consists in its providing a *variable-free* notation for an important class of predicate-logical formulas. Note also that, once the relation $<$ in structures is required to be *connected* (i.e., $\forall x \forall y (x < y \vee x = y \vee y < x)$), *any* quantifier becomes definable in terms of restricted ones, by means of the equivalence (t' is any variable free for t in φ):

$$(14)\quad \exists t\, \varphi(t) \leftrightarrow \exists t(Et't \wedge \varphi(t)) \vee \varphi(t') \vee \exists t(Ett' \wedge \varphi(t)).$$

We will return to this below.

3.1.6 fails for the case of predicate tense logic. We do not explain this system here, trusting the reader to understand (or look up) the notation:

A Priorean formula like Px will now be translated into Ptx, $\forall x\, F\, Qx$ into $\forall x\, \exists t'(Ett' \wedge Qt'x)$, $P\, \exists x\, Qx$ into $\exists t'(Et't \wedge \exists x\, Qt'x)$, etc. Note that the translation will now depend on the particular Kripke semantics one uses for the structures. E.g., if not all points in time have the same domain of individuals, then a decision has to be made at the «quantifier clause»: $P\, \exists x\, Qx$ might have to be translated as

$$\exists t'(Et't \wedge \exists x(\in t'x \wedge Qt'x)),$$

where $\in t'x$ means: x *exists at* t'.
Now the following formulas are not equivalent to a Boolean combination of Priorean ones:

$\forall x(Qt_1x \rightarrow Qt_2x)$ («whatever is Q at t_1 is Q at t_2)
$\exists t_2(Et_1t_2 \wedge \forall x(Qt_1x \rightarrow Qt_2x))$.

(A formal proof of this is not even easy!)

Even if one does not accept the above translation as a *reduction* of tense logic to standard logic, it serves at least as a channel of information in the following sense: many known results about standard logic become applicable. E.g., *Löwenheim-Skolem* and *compactness* theorems about tense logic are immediate (and tense-logical proofs of these are, therefore, superfluous). (Cf. the remarks made in section 2.) A *completeness* result is also forthcoming: a tense-logical formula φ is universally valid iff $\forall t\, \overline{\varphi}(t)$ is provable in predicate logic. In other words, theorem 3.1.1. is not important as a completeness result (though many people think so), but as a technical result to the effect that K_t suffices as a complete system of deduction, rather than the more comprehensive standard axiom system of predicate logic. Accordingly, the proper way to view the Henkin technique of Makinson (cf. [10]) and Lemmon-Scott (cf. [11]) is as an interesting, but not sensational tool. (In fact, application of this technique has quickly become a routine procedure, often applied — and published — unthinkingly.) Similar remarks apply to *decidability*. The above equivalence implies that the class of universally valid tense-logical formulas is recursively enumerable. A purely semantical argument establishes that non-universally valid tense-logical formulas have *finite* counterexamples, which makes this class recursively enumerable as well. It follows, by Post's theorem, that universal validity is a decidable notion: again a result obtained by standard means.

Finally, we mention a purely syntactical result provable by means of semantic considerations. This serves as an example of the kind of benefit a «strict» tense-logician could accept from Kripke semantics without committing himself to the existence of moments. If $\neg\varphi$ holds at t in a structure M, then, for any $t' < t$, $\neg G\varphi$ holds at t' in M. It follows that, if all structures satisfy $\forall x\ \exists y\ y < x$, $G\varphi$ is universally valid only if φ is. This yields the corresponding syntactical result: for the logic $K_tD = K_t$ with the additional axiom (8), and for all tense-logical formulas φ, $G\varphi$ is provable in K_tD only if φ is. Once he knows this, the strict logician will try to prove it purely syntactically, of course — but he might still be grateful.

(By the way, the syntactical proof is:

(i)	$G\varphi$	(supposed to be proven)
(ii)	$HG\varphi$	(rule of inference of K_t)
(iii)	$HG\varphi \rightarrow PG\varphi$	(provable in K_tD, thanks to (8))
(iv)	$PG\varphi$	
(v)	$\neg\varphi \rightarrow HF\neg\varphi$	(axiom (4))
(vi)	$\neg HF\neg\varphi \rightarrow \neg\neg\varphi$	
(vii)	$PG\varphi \rightarrow \varphi$	(use equivalences provable in K_t)
(viii)	φ.	

Also, note that $GP(p \rightarrow p)$ is provable in K_t, but $P(p \rightarrow p)$ is not).

3.2 *H. Kamp*

In [6] Kamp introduces a «now»-operator N, to account for the difference between (6) and (7) of section 2:

(6) A child was born which would be king,
i.e., in Prior's notation,
$P\ \exists x(Cx \wedge Bx \wedge FKx)$ and
(7) A child was born which will be king,
which becomes
$P\ \exists x(Cx \wedge Bx \wedge NFKx)$.

Semantically, this means that *two* points in time will be needed in the evaluation of a tense-logical formula: t_0, the moment of speech («now») and a «running variable» t, the moment of evaluation. The truth definition may now be given for $M \models \varphi[t_0, t]$, with clauses [recall that $M = \langle T, <, V \rangle$)

(15) $M \models p\,[t_0, t]$ if and only if $t \in V(p)$
(16) $M \models F\varphi\,[t_0, t]$ if and only if, for some $t' \in T$ such that $t < t'$, $M \models \varphi\,[t_0, t']$, etc., and a new clause
(17) $M \models N\varphi\,[t_0, t]$ if and only if $M \models \varphi\,[t_0, t_0]$.

The above translation $^{-}$ (cf. 3.1.4) may be taken over as it stands, with one additional clause (let $\overline{\varphi} = \overline{\varphi}(t)$)

(18) $\overline{N\varphi} = subst_{t_0, t}\,\overline{\varphi}$, where $subst_{t_0, t}\,\overline{\varphi}$ is the result of substituting t_0 for t in $\overline{\varphi}$.

(An equivalence like (13) above will hold now, where t_0 may be regarded as a fixed free variable or an individual constant.) In other words, N functions as a *substitution operator*. This explains very neatly why «now» dominates all other contexts in a sentence. (Cf. Needham [12], chapter III.) Note that, if $\overline{\varphi}$ is to correspond to a sentence actually uttered, then the variable t must be set equal to t_0: the first moment of evaluation coincides with the moment of speech.

Kamp proves that, for any propositional formula φ in his language, there exists a φ' in the Priorean language of 3.1 such that, for all M and t_0,

(19) $M \models \varphi\,[t_0, t_0]$ if and only if $M \models \varphi'\,[t_0]$.

A similar result could be proved for our corresponding predicate logic, but, there, it would not be interesting. (19) implies that the full force of N only shows in predicate tense logic, like the examples (6) and (7) already suggested. The proof that N is not eliminable from the latter system is quite involved — although the result is intuitively obvious from a few examples.

A second paper by Kamp, his doctoral dissertation [7], is maybe the most (logically) sophisticated publication in the field yet. We mention a few of his results. Let two *binary* tense operators S («Since») and U («Until») be added to a propositional language. Their interpretation is as follows:

(20) $M \models S(\varphi, \psi)\,[t]$ if and only if, for some $t' < t$, $M \models \varphi[t']$ and, for all t'' with $t' < t'' < t$, $M \models \psi\,[t'']$

(i.e., «it has been the case that ψ since it was the case that φ») and a dual

(21) $M \models U(\varphi, \psi)\,[t]$ if and only if, for some $t' > t$, $M \models \psi\,[t']$ and, for all t'' with $t < t'' < t'$, $M \models \varphi\,[t'']$

(i.e., «it will be the case that φ until it will be the case that ψ»).

This language is the strongest one available for the case of a «single point index», in a sense to be explained below. First, note that P and F are definable now, by means of

$P\varphi =_{\text{def}} S(\varphi, \varphi \rightarrow \varphi)$ and

$F\varphi =_{\text{def}} U(\varphi \rightarrow \varphi, \varphi)$.

The same holds for the «present progressive tense» Pr φ, defined by

(22) $M \models Pr\,\varphi\,[t]$ if and only if, for some t', t'' with $t' < t < t''$, $M \models \varphi\,[t''']$ for all t''' such that $t' \leqq t''' \leqq t''$.

$\mathrm{Pr}\,\varphi =_{\mathrm{def}} S(\varphi, \varphi) \wedge \varphi \wedge U(\varphi, \varphi)$. It was shown by Kamp that Pr is not definable in terms of P and F alone. Kamp's main result states that his language is *functionally complete* in the following sense.

3.2.1 *Theorem*

For any formula φ (with one free variable) in the first-order language of 3.1 there exists a formula φ' of the above language whose translation $\overline{\varphi'}$ is logically equivalent to φ, provided that attention is restricted to structures in which $<$ is a *complete linear* ordering.

($<$ is *complete* if any non-empty subset S of T with a lower (upper) bound also has a greatest lower (smallest upper) bound.)

Examples of such orderings are the integers with «smaller than», or the reals with «smaller than». This ordering is not complete on the rationals, however. The question if a similar result can be proved for the class of all linear orderings is still open. Kamp shows that S and U do not suffice in this general case. The following formula φ is not definable (on the class of linear orderings) in terms of S and U. (Cf. [7], chapter IV, theorem 5).

(23) $\exists t_1(t_1 < t_0 \wedge Pt_1 \wedge \forall t_2(t_1 < t_2 < t_0 \rightarrow$
$(Pt_2 \wedge \exists t_3(t_2 < t_3 < t_0 \wedge \forall t_4(t_2 < t_4 < t_3 \rightarrow Pt_4))) \vee$
$(\neg Pt_2 \wedge \exists t_3(\underline{t_1 < t_3} < t_2 \wedge \forall t_4(t_3 < t_4 < t_2 \rightarrow \neg Pt_4)))))$.

Leaving out the underlined atomic subformula yields a formula equivalent to (23) on the class of *dense* linear orderings:

(24) $\exists t_1(t_1 < t_0 \wedge Pt_1 \wedge \forall t_2(t_1 < t_2 < t_0 \rightarrow$
$(Pt_2 \wedge \exists t_3(t_2 < t_3 < t_0 \wedge \forall t_4(t_2 < t_4 < t_3 \rightarrow Pt_4))) \vee$
$(\neg Pt_2 \wedge \exists t_3(t_3 < t_2 \wedge \forall t_4(t_3 < t_4 < t_2 \rightarrow \neg Pt_4)))))$.

We will return to (24) in 3.4 below.

Kamps notes (p. 29) that there are tense operators, like «mostly» or «usually», which are not definable in our first-order language, and, therefore, outside the scope of theorem 3.2.1. Being *second-order* in nature, they are outside the scope of the present discussion as well. In fact, even (23) and (24) have a second-order flavour. They do not hold on complete orderings for any interpretation of t_0 and P (which is why they are, trivially, definable in the sense of 3.2.1). But, this fact itself comes close to defining the (second-order) property of completeness. On p. 38, Kamp remarks «We do not believe that such tenses can be expressed in English without explicit reference to moments», which points at a detached stand in the «operator/quantifier controversy».

Finally, note that from the classical point of view, 3.2.1 amounts to a *normal form theorem* for predicate-logical formulas, as Kamp himself states on p. 39. (Its proof is a technical tour de force which cannot even be sketched here). The relevance of the theorem depends, of course, on the acceptability of the completeness restriction and the naturalness of the tenses Since and Until as explained above. What does the theorem contribute to our controversy, however ? It shows that, under certain assumptions, the Priorean operator approach is as rich in expressive power as the classical one (for the case of «single point indices», that is). In this case, a technical choice between the two approaches would be dictated by considerations of perspicuity (how complicated are the φ''s of 3.2.1, when compared to the original φ's ?).

3.3 *F. Vlach*

There are simple tensed sentences not expressible in the symbolism of Kamp [6], as was noted by F. Vlach in [18]. Consider the sentence:

(25) One day, all persons alive now will be dead,

or, in Kamp's notation,

(26) $F\ \forall x(NAx \rightarrow Dx)$,

and in our transcription:

(27) $\exists t'(Ett' \wedge \forall x(At_0x \rightarrow Dt'x))$.

According to most people, its past tense formulation would be

(28) One day, all persons alive then would be dead.

The Priorean recipe would consist in prefixing (26) by P to get a reading for (28):

(29) $PF\ \forall x(NAx \rightarrow Dx)$.

(29) does not capture the intended meaning, however, since N still refers back to the *present* moment. This is brought out by the translation of (29):

(30) $\exists t''(Et''t \wedge \exists t'(Et''t' \wedge \forall x(At_0(!)x \rightarrow Dt'x)))$.

This is not what we want for a reading of (28) — which is rather

(31) $\exists t''(Et''t \wedge \exists t'(Et''t' \wedge \forall x(At''(!)x \rightarrow Dt'x)))$.

Note how easy this whole matter is from the point of view of predicate logic: the nature of a certain occurrence of a free moment variable is to be specified. As a true Priorean, Vlach introduces a new operator K, with the semantical stipulation:

(32) $M \models K\varphi\ [t, t_0]$ if and only if $M \models \varphi[t, t]$.

In other words, a second *substitution operator* is introduced, this time

(33) $\overline{K\varphi} = \text{subst}_{t,t_0}\ \overline{\varphi}$.

(28) may, them, be read as:
(34) PKF $\forall x(NAx \rightarrow Dx)$.

An easy calculation shows that, indeed $(\overline{34})$ is the desired (31). In many cases, K may be given the natural language reading «then».

As the reader will have expected by now, this addition does not solve all problems. Needham [12] contains an example allegedly not expressible in Vlach's language (p. 73/74):

(35) Everyone who has come will be going to meet those who play after the concert.

His reading of (35) is

(36) $\exists t'''(Et_0t''' \wedge \forall x(\exists t''(Et''t_0 \wedge At''x) \rightarrow \exists t'(Et'''t' \wedge \forall y(Bt'''y \rightarrow Ct'xy))))$.

In spite of his claim to the contrary, (36) *is* expressible in Vlach's language, however, viz. by

(37) NF $\forall x(NPAx \rightarrow KF \forall y(NBy \rightarrow Cxy))$.

But the following sentence is really a counter-example:

(38) There will always jokes be told that were told at one time in the past.

Its predicate-logical reading (or, at least, one of its readings) is:

(39) $\exists t''(Et''t_0 \wedge \forall t'(Et_0t' \rightarrow \exists x(At'x \wedge At''x)))$.

No formula in Vlach's language can describe this: all suitable candidates fail. E.g., NPKNG $\exists x(Ax \wedge NAx)$ becomes

(40) $\exists t''(Et''t_0 \wedge \forall t'(Et''t' \rightarrow \exists x(At'x \wedge At''x)))$

and NPNKG $\exists x(Ax \wedge NAx)$ becomes

(41) $\exists t''(Et''t_0 \wedge \forall t'(Et_0t' \rightarrow \exists x(At'x \wedge At_0x)))$.

(NPGK $\exists x (Ax \wedge NAx)$ and NPG $\exists x(Ax \wedge KNAx)$ fail in a similar manner).
This is no formal *proof* of course, but we hope the reader will grant us that.

In an appendix, Vlach mentions a safety valve which blocks this counterexample and similar ones. It consists in adding operators $N_1, N_2, \ldots$ and corresponding $K_1, K_2, \ldots$ in any quantity. This will take care of all cases of cross-reference ,but — as Needham rightly observes — such a move degenerates into using a typographical variant of predicate logic (with subscripts instead of variables), merely without *calling* it predicate logic.

3.4 *L. Åqvist*

In [1], a paper devoted to absorbing H. Reichenbach's ideas (cf. [15], § 51) into Priorean tense logic, L. Åqvist introduces the following operator in addition to Kamp's N: $\Box_x$, «for which no independent reading is codified» (p. 4). Its semantical interpretation is as follows:

(42) $M \models \Box_x \varphi\, [t, t_0]$ if and only if $M \models \varphi\, [t_0, t]$.

(This clause is our, simplified, version of Åqvist's elaborate semantical construction.) Note that a momentous step has been taken: $\Box_x$ is a purely technical operator for which no natural language reading exists. (It has only been added for technical purposes.) $\Box_x$ may be viewed as a *permutation operator:*

(43) $\overline{\Box_x\varphi} = \mathrm{perm}_{t,t_0}\overline{\varphi}$,

where $\text{perm}_{t,t_0}\,\overline{\varphi} = [t/t_0, t_0/t]\,\overline{\varphi}$, i.e., the result of simultaneously replacing t by t_0 and t_0 by t in $\overline{\varphi}$.

Åqvist uses in fact *two* symbols $\Box_x$ and $\Diamond_x$ to denote the same permutation operator. Although there is indeed some formal similarity with modalities (in that $\Diamond_x\varphi$ is equivalent to $\neg\Box_x\neg\varphi$), nothing is gained, of course, by treating permutation in this way. This unnecessary obedience to the ritual of the Priorean approach even carries the false suggestion that $\Box_x$ is some sort of quantifier. But, a simple notational detail like this shows very vividly how strong is the hold of the Priorean approach over its adherents.

A second salient feature of Åqvist's system is its introduction of the *propositional constants* bf, id and af, which are interpreted as follows:

(44) $M \models \text{bf}\,[t, t_0]$ if and only if $t < t_0$
$M \models \text{id}\,[t, t_0]$ if and only if $t = t_0$
$M \models \text{af}\,[t, t_0]$ if and only if $t_0 < t$.

In other words:

(45) $\overline{\text{bf}} = Ett_0$

$\overline{\text{id}} = t{=}t_0$

$\overline{\text{af}} = Et_0t$:

we have added new atomic formulas (besides the already existing Pt's and Pt_0's).

The addition of id alone already yields bf and af, however. bf may be defined as Fid, and af as Pid. Moreover, Åqvist assumes $<$ to be a *linear dense* ordering, which allows us to define $N\varphi$ as $H(\text{id} \rightarrow \varphi) \wedge (\text{id} \rightarrow \varphi) \wedge G(\text{id} \rightarrow \varphi)$. So, let us restrict attention to the Priorean language of 3.1 with an added propositional constant id, on the class of linear dense orderings. This language is quite strong, as may be seen from the following examples:

$S(\varphi, \psi) =_{def} NP(\varphi \wedge G(bf \rightarrow \psi))$
$U(\varphi, \psi) =_{def} NF(\psi \wedge H(af \rightarrow \varphi))$.

Åqvist himself shows how to define *positional quantifiers* $\exists t \in (t', t'')$ in his language, where

$$(46) \quad \exists t \in (t', t'')\varphi =_{def} (t' < t'' \wedge \exists t(t' < t \wedge t < t'' \wedge \varphi)) \vee (t' = t'' \wedge [t'/t]\, \varphi) \vee (t'' < t' \wedge \exists t(t'' < t \wedge t < t' \wedge \varphi)).$$

(His notation is $\Diamond_{\rightarrow} \varphi$ where

$$(47) \quad \Diamond_{\rightarrow} \overline{\varphi} = \exists t' \in (t, t_0)\overline{\varphi}.$$

Next, readmitting the operator $\Box_x$ yields a very strong language. E.g., a laborious calculation shows that the formula (24) of 3.2 (which was not definable by means of Since and Until) has the following definition in Åqvist's language:

$$(48) \quad P(p \wedge G(bf \rightarrow (p \wedge \Box_x P(af \wedge H(af \rightarrow p))) \vee (\neg p \wedge \Box_x NPG(bf \rightarrow \neg p))).$$

The reader is invited to check this by himself.

In view of this and similar examples we think it would be interesting to find out if Åqvist's language is *functionally complete* with respect to first-order formulas with at most two free variables, always assuming the orderings to be dense and linear. One might try to adapt Kamp's proof in [7].

The «conversion» operator $\Box_x$ has a peculiar behaviour, witness universally valid principles like

(49) $\Box_x p \leftrightarrow p$ for proposition letters p
(50) $\Box_x \neg \varphi \leftrightarrow \neg \Box_x \varphi$
(51) $\Box_x(\varphi \wedge \psi) \leftrightarrow \Box_x \wedge \Box_x \psi$ (similarly for $\vee$)
(52) $\Box_x \Box_x \varphi \leftrightarrow \varphi$.

$\Box_x$ cannot be defined in terms of the other primitives, however. This may be shown by considering the formula

(53) $\exists t'(Et't_0 \wedge t' \neq t \wedge Qt')$,

definable by $\Box_x P(\neg id \wedge q)$, but not definable without using $\Box_x$.

Finally, we want to comment upon Åqvist's question about a completeness proof for an axiomatic system presented in [1]. Universal validity in his semantics is axiomatizable in standard logic, by means of our translation. Therefore, no very deep interest attaches to finding a completeness result with respect to the «purely Åqvistian» language. (Cf. our comments in 2 and 3.1.) Moreover, Reichenbach himself was quite definitely in the non-Priorean tradition (he presented his analysis of tenses as an *application* of standard logic), so it does not seem appropriate to attach his name to a purely Priorean problem.

After this paper had been finished, K. Segerberg's paper «Two-dimensional modal logic» (The Journal of Philosophical Logic 2 (1973), 77-96) came to our attention. Segerberg proves a completeness theorem for a tense logic with six operators $\Box_1$, $\Box_2$, $\Box_3$, $\Box_4$, $\Box_5$ and $\Box_6$. These get the following semantic interpretation (translated into the terminology of this paper):

(i) $M \models \Box_1\varphi\, [t, t_0]$ iff *for all* $t, t_0 \in T$, $M \models \varphi\, [t, t_0]$
(ii) $M \models \Box_2\varphi\, [t, t_0]$ iff *for all* $t_0 \in T$, $M \models \varphi\, [t, t_0]$
(iii) $M \models \Box_3\varphi\, [t, t_0]$ iff *for all* $t \in T$, $M \models \varphi\, [t, t_0]$
(iv) $M \models \Box_4\varphi\, [t, t_0]$ iff $M \models \varphi\, [t, t]$
(v) $M \models \Box_5\varphi\, [t, t_0]$ iff $M \models \varphi\, [t_0, t_0]$
(vi) $M \models \Box_6\varphi\, [t, t_0]$ iff $M \models \varphi\, [t_0, t]$

As Segerberg notes himself, $\Box_4$, $\Box_5$ and $\Box_6$ are «very strong K-modalities of rather unusual kinds» (p. 82). Like Åqvist, he seems to regard them as some kind of quantifiers, withness his (deliberately ?) parallel clauses like
«$\models_u \Box_2 B$ iff for all $v \in U$ such that $Xu = Xv$, $\models_v B$»
(which was transcribed as (ii) above) and
«$\models_u \Box_4 B$ iff for that $v \in U$ such that $Xv = Yv = Xu$, $\models_v B$»
(which was transcribed as (iv) above). (X and Y are some kind of coordinate functions). Unlike Åqvist, Segerberg does not use both $\Box_x$ and $\Diamond_x$ for the same permutation operator.

From our point of view, there is nothing more to «twodimensionality» than the presence of two temporal parameters. Segerberg thinks it so important, however, that he discusses priority questions regarding its «discovery» (p. 79). We would rather look upon Segerberg's paper as providing an interesting complete axiomatization for a fragment of Quine's variable-free predicate logic (cf. section 5). One would like to have such an elegant complete axiomatization for that predicate logic as a whole.

3.5 *L. Åqvist & F. Günthner*

An extended version of Åqvist's system is found in [2]. Again the language is that of a propositional tense logic, enriched with tense operators. There are the, by now familiar, P, F, H, G, N (written as «NOW»), as well as technical operators (written in our notation as) $\Box_x$, $\Box'_+$, $\Box_d$, $\Box_w$, $\Box_s$, about three of which it is remarked on p. 7:
«For the time being, we refrain from offering any natural language renderings of these three operators». In fact, none of them receive a natural language reading, so we proceed to the semantics for their explication. (There are even more operators than these ten, but these will do for the time being).

In order to give a Reichenbachian account of tenses, *four* moments are now involved in the evaluation of a formula. In our notation, these are

t_0: the *point of speech*
t_1: the *designated point*
t_2: the *point from where*
t : the *point of evaluation.*

For our purposes, a structure may still be regarded as a triple $M = \langle T, <, V \rangle$, but the truth definition will now describe the notion

(54) $M \models \varphi\,[t, t_2, t_1, t_0]$.

We prefer this to the original cumbersome notation $\overset{M^{t'/t''}}{\underset{t}{\models}} \varphi$.

To aid the reader's memory, we give it in full, repeating former clauses:

(i) $M \models p\,[t, t_2, t_1, t_0]$ iff $t \in V(p)$
(ii) $M \models \neg\varphi[t, t_2, t_1, t_0]$ iff *not* $M \models \varphi\,[t, t_2, t_1, t_0]$
(iii) $M \models \varphi \rightarrow \psi\,[t, t_2, t_1, t_0]$ iff *if* $M \models \varphi\,[t, t_2, t_1, t_0]$ *then* $M \models \psi\,[t, t_2, t_1, t_0]$
(iv) $M \models P\varphi\,[t, t_2, t_1, t_0]$ iff, *for some* $t' \in T$ *with* $t' < t$, $M \models \varphi\,[t', t_2, t_1, t_0]$
(v) $M \models F\varphi\,[t, t_2, t_1, t_0]$ iff, *for some* $t' \in T$ *with* $t < t'$, $M \models \varphi\,[t', t_2, t_1, t_0]$
(vi) $M \models NOW\varphi\,[t, t_2, t_1, t_0]$ iff $M \models \varphi\,[t_0, t_0, t_0, t_0]$ (thus a kind of «total» now)
(vii) $M \models \Box_x\varphi\,[t, t_2, t_1, t_0]$ iff $M \models \varphi\,[t_2, t, t_1, t_0]$
(viii) $M \models \Box_+\varphi\,[t, t_2, t_1, t_0]$ iff $M \models \varphi\,[t_1, t_2, t, t_0]$
(ix) $M \models \Box_d\varphi\,[t, t_2, t_1, t_0]$ iff $M \models \varphi\,]t_1, t_2, t_1, t_0]$
(x) $M \models \Box_w\varphi\,[t, t_2, t_1, t_0]$ iff $M \models \varphi\,[t_2, t_2, t_1, t_0]$
(xi) $M \models \Box_s\varphi\,[t, t_2, t_1, t_0]$ iff $M \models \varphi\,[t_0, t_2, t_1, t_0]$

The clauses (vi), ..., (xi) will now be discussed from a mathematical point of view. If we translate into first-order logic, we will get translations $\bar{\varphi} = \bar{\varphi}\,[t, t_2, t_1, t_0]$ and the new operators will turn out to be *permutation* and *substitution operators.* Although Åqvist & Günthner do not motivate these operators, the following «rational reconstruction» may be given. We want to be able to permute t, t_2, t_1 in any order (t_0 is always to remain fixed, being the point of actual utterance). This means we have to describe $3! - 1$ (for the identity permutation) $= 5$ permutations. As is well-known, two inter-changes suffice for this. The authors have

$\Box_x = perm_{12}$ (permutation of the first and second member)
$\Box_+ = perm_{13}$ (permutation of the first and third member)

This indeed yields the remaining three permutations:

$perm_{23}$ = the composition $perm_{13}\, perm_{12}\, perm_{13}$
and

$$\binom{123}{312} = perm_{13}\, perm_{12}$$
$$\binom{123}{231} = perm_{12}\, perm_{13}.$$

(Note the order convention: e.g., $perm_{12}\ perm_{13}\,(t, t_2, t_1) = perm_{12}\,(t_1, t_2, t) = t_2, t_1, t$. This is the reverse of the order convention in the truth definition !)

Next, we want all mappings $\binom{t\ t_2\ t_1}{u\ v\ w}$, where u, v, w are chosen from t, t_2, t_1 such that at most two of them are different. (The case of three different u, v, w has been taken care of by the permutations). For this, it suffices to have the substitution operators:

$subst_{12}$ (substitution of the first member for the second)
$subst_{31}$ (substitution of the third member for the first),

etc. E.g., $\binom{t\ t_2\ t_1}{t_1\ t_1\ t}$ will then be obtained as the composition $subst_{21}\ subst_{13}\ subst_{32}$. The authors have two such operators:

$\Box_d = subst_{31}$ and
$\Box_w = subst_{21}$.

But, in fact, only one suffices, thanks to the permutations. These will yield all substitutions, given any one of them. E.g., $\Box_d$ may be defined in terms of $\Box_w$ as follows:
$subst_{31} = perm_{23}\ subst_{21}\ perm_{23}$, or — using the above definition of $perm_{23}$ — $\Box_d = \Box_+ \Box'_x \Box_+ \Box_w \Box_+ \Box'_x \Box_+$.
Conversely, $\Box_w$ may be defined as $perm_{23}\ subst_{31}\ perm_{23}$.

Finally, we want to be able to substitute t_0 anywhere in t, t_2, t_1. By the presence of the $subst_{ij}$ operators, it suffices to have any single such substitution. The authors have

$\Box_s$ = substitution of t_0 for the first member.
They themselves remark that NOW (= simultaneous substitution of t_0 in all three places) φ is definable as $\Box_s \Box'_x \Box_w \Box_+ \Box_s\varphi$. Another definition. which does not use $\Box_w$, is
NOWφ = $\Box_s \Box_x \Box_s \Box'_+ \Box_s\varphi$.

In other words, four operators (instead of the original six) allow for any combination of arguments to be obtained from t, t_2, t_1. This fact is used when positional quantifiers are introduced: it suffices to have only $\underset{\rightarrow}{\Diamond}$, defined by

(xii) $M \models \underset{\rightarrow}{\Diamond}\ \varphi\, [t, t_2, t_1, t_0]$ iff, *for some* $t' \in (t, t_2)$,

$M \models \varphi\, [t', t_2, t_1, t_0]$.

All other positional quantifiers will then be definable, and available for use in Åqvist & Günthner's theory of verb aspects.

Finally their system contains «comparative operators» («More», «Exactly as Much», etc.), which fall outside the scope of this discussion. The same holds for their interesting «theory of events».

The system presented here becomes comparable to that of Åqvist in 3.4, once we add a propositional constant id, to be interpreted as, e.g. (there are several equivalent choices):

(xiii) $M \models id\, [t, t_2, t_1, t_0]$ iff $t = t_2$.

A conjecture about its expressive power (now with respect to formulas with at most four free variables) may then be stated analogous to the one in 3.4.

3.6 *D. Gabbay*

The last author on our list has written a voluminous book ([5]) on modal and tense logics, of which we only mention the chapters 10 («Two dimensional tense logics»), 11 («A theory of proper names and conceptual change») and 12 («Tense logic and the tenses of English»). We will not discuss these, but only draw attention to a few passages in them illustrating our present theme. As we have seen up to now, the tendency exists to add ever more points in time at the *index* (of evaluation), which are then «manipulated» by operators without moment variables in the object language. The alternative, which should have been kept in mind throughout the discussion, is the use of predicate-logical formulas containing moment variables and overtly displaying these «manipulations». Clearly, if one is willing to increase the complexity of the index to any extent (while adding enough operators to take profit of it), there is no need to ever resort to predicate logic *technically*, but, in our opinion, it is a Pyrrhic victory. This tendency is evident in Gabbay's work, which is why it is mentioned here.

The most interesting chapter of the three is 12, where Gabbay says:

«In this chapter we outline the kind of modal logic and semantics that is suitable for the representation and analysis of a non-trivial body of tensed statements in English. (...)

To achieve our goal, we shall need to depart radically from traditional tense-logics, with regard to both semantic and syntactic concepts.» (p. 165)

He turns out, however, to be fighting a concept of «traditional tense-logic», which does not exist any more. It consists in, amongst others,

«(II) Choice of interpretation (i.e., the fact that each atomic proposition has a truth value at a point t of time and not e.g., an interval of time or a sequence of points, etc.)

(III) Choice of truth tables (i.e., the fact that we evaluate the truth value of A at a point and not something else).» (p. 167/8). But, as we have seen, Priorean tense-logicians are quite willing to complicate both their languages and their structures. Moreover, they often realize that *intervals* are needed in addition to moments (cf. Åqvist & Günthner [2]). Gabbay then presents a lot of examples showing the need for ever more complex systems of tense logic. Most of them are quite convincing, although some have a distinctly modal (or, at least, intensional) flavour (involving phrases like «he told me that», «he will hate you for», «I knew that», «he will realize that», etc.), which points to a possible confusion. Such sentences cannot be treated by tense-logic alone, and no tense-logician would pretend otherwise. On the other hand, Gabbay does seem to be aware of this point, because he says on p. 165:

«To be sure, what is known as the tense-system of English incorporates more than what is accounted for in our system, since it involves also the factors of aspect and mood. But at least we can claim that our system analyzes some of the complexities of the tense system that are the result of temporal references alone».

Now our main point is that Gabbay, despite his criticism of «traditional tense-logic», is well within the Priorean tradition, because he does not even *consider* the possibility of using ordinary predicate logic to resolve his difficulties. Several examples involving an ever increasing number of events to be mutually related (in terms of «earlier» and «later»), lead him to introduce ever more «points» corresponding to such events. These are then to be used for future reference. On p. 174 we read:

«In the meantime, the tentative conclusion is that we must gives tables for evaluating sentences $||A||_{(u,t_1,t_2,t_3,\ldots)}$ i.e., we must keep record of the entire sequence of points and not only that, but also keep track of the kind of operators used (i.e., whether t_3 was introduced because of an F or not, because if we have another F, the next point may have to be chosen in the future of t_3!)».

Such a passage is quite interesting from a historical point of view. The difficulty described here is *exactly* that which logicians had before Frege invented the theory of quantifiers with attached variables for cross-reference. But, even a competent logician like Gabbay does not recognize it for what it is.

In the earlier chapter 10, Gabbay defines an «n-place, m-dimensional truth table with parameters in T», where the *para-*

meters are the *points* referred to above. We quote the definition, not for its technical content, but for another illustration of how the author refuses the helping hand of predicate logic (p. 140):

«Given a flow of time $\tau = (T, R, o)$ we now give a general definition of the notion of an n-place, m-dimensional truth table Ψ with parameters $s_1, \ldots, s_k \in T$. Let $Q_1, \ldots, Q_n$ be n variables ranging over T^m, let $\Psi(t_1, \ldots, t_m, s_1, \ldots, s_k, Q_1, \ldots, Q_n)$ be a wff with the indicated free variables in a language with quantifiers $\forall$, $\exists$ ranging over elements of T, the usual classical connectives (...) and the predicates and constants 0, $s_1, \ldots, s_k$, R. Then Ψ is said to be an *n-place, m-dimensional truth table with parameters in* τ».

One cannot help wondering: why not use the truth table, and forget the corresponding operator?

4. *Temporal Perspective*

This section is devoted to P. Needham's dissertation «Temporal Perspective, a logical analysis of Temporal Reference in English» ([12]). Needham is an opponent of the Priorean approach, witness his preface:

«The book is (...) critical, in that it is opposed to the basic tenet in the writings of Arthur Prior — the founder of modern tense logic — that tenses behave essentially very much like *propositional* operators».

We will not review Needham's arguments against Prior, Kamp and Vlach: many of them have been mentioned already. Our aim is to give some impression of what a «classical» tense logic looks like:

«The constructive aspect lies in developing the expressive powers of a first-order logic and demonstrating the applicability of its greater flexibility over conventional tense logic». (Note, by the way, that Massey's «revolutionary tense logic» of 1969, was already «conventional» in the perspective of 1975, the year in which [12] appeared).

We will explain Needham's languages, and discuss some of his results.

«The important results are theorems 5 and 10. Theorem 5 embodies within the formal system what I take to be the crucial principle, that the point of view of the speaker dominates all subordinate contexts. The standard prenex normal form theorem, theorem 10, lays the foundation for the definition of a tense».

Needham presents the case against Priorean tense logic as follows. By means of examples taken from natural language, he shows that the tense logics of Prior, Kamp, and Vlach are inadequate for a description of all tense phenomena. Of course, new tense-logical operators may be added ad hoc to cope with any amount of counter-examples, but he contrasts this with the simplicity and perspicacity of two predicate-logical languages L* and L, which can handle all these counterexamples effortlessly.

Like Åqvist, Needham claims Reichenbach for his ancestor, whose *S, R, E-analysis* (i.e., a *point of speech*, a *point of reference* and a *point of event* are to account for verb tenses, (cf. [15], § 51)) is incorporated into his languages. As Reichenbach uses quantification over moments as a matter of course (cf. his account of time indications as descriptions in § 47 and his analysis of the sentence «the earth-quake was followed by the explosion of the factory» in § 48), Needham's claim seems to be the stronger one. (It must be admitted, however, that the key section 51 is ambiguous on this point). At least in Åqvist & Günthner [2], there is no exact parallel between their «four point index» and Reichenbach's $<S, R, E>$. But, the same holds for Needham, whose allegiance to Reichenbach is nominal in that he has no need for a distinction S/R (there are only reference points) and leaves out E altogether (it has to be accounted for by quantification). Moreover, Needham needs *three* reference points, one «now» ,a past «then» and a future «then». (Cf. his chapter II.4).

In a chapter called «A logic of tenses and dates», Needham introduces a language L with the following salient features. It is a two-sorted first-order language, having variables and constants for both individuals and moments. All predicate constants are either of type $<n, 1>$ for some n (i.e., they take n individual terms and one moment term) or one of E («earlier than») and = (identity, both for individuals and moments). The logical operators are as usual, but for the addition of three so-called indexical quantifiers Π, Δ and Γ. Semantical structures are four-tuples $<D, T, <p, n, f>, V>$, where D is a non-empty set (of individuals),

T is a non-empty set (of moments) disjoint from D,
p, n, f are elements of T such that $p < n < f$, where $<$ is the interpretation of E on T: a strict linear order relation, V is a valuation assigning, to each predicate constant of type $<n, 1>$, an $(n+1)$-ary predicate, whose first argument is to belong to T, and the other n to D.

The truth definition is obvious, except for the clauses for the indexical quantifiers. We will not give these, however, because — as Needham points out himself (p. 41) — these quantifiers may be regarded as *substitution operators*. $\Pi x\varphi$ (this is not Needham's notation) $= [p/x]\varphi$ (p is an individual constant for the reference point «then» in the past) and, similarly, $\Delta x\varphi = [n/x]\varphi$ («now») and $\Gamma x\varphi = [f/x]\varphi$ («then» in the future).

«The fundamental principle of our analysis of tenses, that the point of view of the speaker dominates all subordinate clauses» is expressed by theorem 5 of chapter III. This is a quite trivial technical result, which amounts to the realization that any number of substitutions $[p/x_1], \ldots, [p/x_k]$ in a formula φ may be replaced by one substitution $[p/y]$ in front of φ, for a suitable new variable y not occurring in φ which replaces all former $x_1, \ldots, x_k$. (Similarly for n and f). Tenses are, then, connected with *restricted* moment quantifiers like we did in section 3, and the phenomenon of *sequences of tenses* (cf. the Priorean iterations PP, FP, etc.) is investigated.

An analysis of sentences like «I am taller than I was», involving a comparison through time, leads to the introduction of an extended language L*, allowing predicate constants of any type $<n, k>$ (n individual argument places, k moment argument places). The relevant argument, though highly interesting, does not concern us here.

In chapter IV.5, Needham devotes a lot of attention to a rigorous definition of a *tense*. Tenses turn out to be certain *chains of restricted quantifiers* to be read off from a special kind of *prenex normal form* of the formula under consideration. Now the informal concept of a «tense» is so vague that it can hardly support such formal scrutiny. Moreover, prenex normal forms do not seem to provide very natural ways of defining complexity. What Needham needs for his definition of a «tense» is rather some modification of the concept of *nested* quantifiers. Prenex normal forms may introduce «spurious» complexity in this respect. E.g., $\exists x\, Ax \vee \neg \exists x\, Ax$, where only nestings of depth 1 occur, becomes (say) $\exists x\, \exists y\, (Ax \vee \neg Ay)$, with a nesting of depth 2. One will, therefore, have to use a measure like the *quantifier depth* (d), defined as the maximal depth of a nesting of quantifiers occurring in the formula, or — inductively —

$$
\begin{aligned}
d(\varphi) &= 0 \text{ if } \varphi \text{ is atomic} \\
d(\neg\varphi) &= d(\varphi) \\
d(\varphi \rightarrow \psi) &= \text{maximum } (d(\varphi),\, d(\psi)) \\
d(\exists x\varphi) &= d(\forall x\varphi) = d(\varphi) + 1, \text{ etc.}
\end{aligned}
$$

In chapter 5, Needham compares his languages with those of Prior, Kamp and Vlach. He shows how to translate tense-logical formulas into L*. But, his method of showing that certain formulas of L* cannot (conversely) be expressed by tense-logical formulas is clumsy; in fact, it is not a method at all. He takes, e.g., an L*-formula, tries a few ways in which it may be expressed as a tense-logical one, fails, and then concludes that it is not so expressible. (Cf. pp. 68, 69). The worst thing is that, like we have seen in 3.3, this procedure results in mistaken claims. Now, Needham is partly excused in that claims of the kind «formula φ is not logically equivalent to any formula of the kind ...» are often exceedingly hard to prove. (One needs the logical ingenuity of a Hans Kamp to prove them, cf. [7]). Moreover, exact proofs of these claims are often quite uninformative.

In this discussion of expressive power, Needham should have mentioned Kamp [7]. The tense logic with Since and Until is one of the strongest ones around, and it would be much harder to find natural examples of formulas outside its scope. Needham required $<$ to be a linear ordering without first or last element. Therefore, Kamp's result about functional completeness does not apply here (since $<$ need not be complete), but it may be that all «natural» tenses are still definable in terms of Since and Until. It is up to Needham to disprove Kamp's claim ([7], p. 38) that «there is good reason to assume that even if time is like the rational numbers those first-order tenses which can be expressed by means of English tense operators are expressible by means of NOT, AND, SINCE, and UNTIL». Moreover, if Kamp is right in suggesting that all counterexamples «must be intimately related to the differences between the reals and the rationals», i.e., with the *second-order* property of Dedekind Completeness (cf. the end of section 2), then Needham's *first-order* language would be unsuitable for expressing this difference as well. This omission is all the more amazing, since Needham mentions [7] in his list of references. But, maybe, he felt justified in omitting Kamp's technical results because they only apply to *propositional* tense logic, whereas Needham does not consider such a subsystem. (Kamp's completeness result probably does not extend to *predicate* tense logic).

Summing up, [12] is an interesting exposition and defense of the predicate-logical approach. The positive results obtained are rather simple, however (theorems 5 and 10).

One rather serious mistake has been signalled, pointing at a certain lack of precision. Finally, the attack upon the Priorean approach, although not without force, is marred by its omission of perhaps the most mature work in tense logic: Kamp [7].

5. *Predicate logic without variables*

In this section, a kind of appendix to the preceding discussion, we explain an idea, due to Quine (cf. [14]), for writing predicate-logical formulas without using variables. It will appear that this is feasible, once *permutation* and *substitution* operators are added to the language. Clearly, such a notational change will not alter predicate logic in any essential respect, particularly not in the respect of ontological commitment. This, then, throws a new light on the operators or Åqvist [1] and Åqvist & Günthner [2]. They constitute an important step on the road of getting the full expressive power of predicate logic, while restricting oneself to operators all the time. So, banishing moment variables is only a fight against *symptoms* of the predicate-logical «disease».

Many students of logic are struck by the fact that, at a certain simple level, predicate-logical notation is more involved than seems required by the sentence being analyzed. E.g., «Everyone agrees» becomes, say, $\forall x\, Ax$, which introduces two variables where none seem needed. Let us, then, drop these and write $\forall(A)$, to be interpreted as «the property of universal instantiation holds of the predicate A». But what about more complex sentences, involving cross-reference, like $\forall x(Ax \rightarrow Bx)$? Language needs no variables here either («Every A is B»), so let us write $\forall(\rightarrow(A, B))$, where A, B are predicates combined by $\rightarrow$ to form the complex predicate «being B if A». But, if dependences are expressed, like in «Everyone agrees with someone», then variables become more important. How, for instance, is $\forall x\, \exists y\, Axy$ to be written without variables ? A simple convention turns out to suffice. What is needed is an explanation of $\exists(A)$, where A may now be a *binary* predicate. Let us stipulate that $\exists$ «fills» the *last* argument place. E.g., $\exists(A)$ means here: «agreeing with someone». (A similar convention applies to $\forall$.) Now, $\exists(A)$ has only one argument place left, so that one is «filled» by $\forall$ in the reading $\forall(\exists(A))$ of the above sentence. Obviously, this will not do yet, however. How is $\forall x\, \exists y\, Ayx$ to be expressed ? $\exists$ now will have to «fill» the first argument place of A. The order of arguments in A will, therefore, have to be changed. E.g., let *conv*(A) denote the converse predicate of A. Then $\forall(\exists(\text{conv}(A)))$ will be a reading for our sentence. In fact, sentences like $\forall x\ \exists y\ \exists z\ Azxy$ show that *any* permutation of arguments will have to be allowed: for any i, j ($1 \leqq i$, $1 \leqq j$, $i \neq j$) $perm_{ij}$ (interchange of i'th and j'th argument place) is, therefore, added to the notation. (E.g., conv may be regarded as perm_{12}). Quine shows that two permutations suffice, instead of the infinitely many perm_{ij}:

major inversion (INV): interchange of *first* and *last* argument place
and

minor inversion (inv): interchange of *last* and *one but last* argument place.

Reviewing our notation, we now have $\forall$, $\exists$, $\rightarrow$, INV, **inv**. Clearly, we also need $\urcorner$ for taking the negation of predicates. Moreover, $\rightarrow$ is not satisfactory as it stands: how are we to write $\forall x\ \forall y(Ax \rightarrow By)$? $\rightarrow$ (A, B) will yield $\lambda x.(Ax \rightarrow Bx)$ instead of $\lambda x\, y\,.\,(Ax \rightarrow By)$. The remedy is as follows: if A is n-ary and B m-ary, then $\rightarrow$ (A, B) is to be the n + m-ary predicate which holds of objects $d_1,\ldots,d_n$, $e_1,\ldots,e_m$ if and only if (if $A(d_1,\ldots,d_n)$, then $B(e_1,\ldots,e_m)$). The above formula then becomes $\forall(\forall(\rightarrow(A, B)))$. But, this remedy seems to do more harm than good: what will become of $\forall x(Ax \rightarrow Bx)$ now? $\rightarrow$ (A, B) will yield $\lambda x\, y\,.\,(Ax \rightarrow By)$, whereas we want $\lambda x\,.\,(Ax \rightarrow Bx)$. It follows that an *identification* operator *id* is needed, which identifies argument places. $\forall(\text{id}(\rightarrow(A, B)))$ is, then, the required reading. The presence of arbitrary permutations spares us a multitude of identification operators: an *id* identifying the *last* and *one* but *last* argument place suffices. (Cf. the remarks about the substitution operator of Åqvist & Günthner in 3.5). Our discussion has yielded the following result.

5.1 *Theorem* (W.V.O. Quine)

The notation $\forall$, $\exists$, $\neg$, $\rightarrow$, INV, inv, id as introduced above suffices for writing any predicate-logical formula without occurrences of individual or functional constants.

One example will show that this reading need not be more perspicuous, however. Consider $\forall x(Ax \rightarrow \forall y(Rxy \rightarrow Ay))$ and its «translation», which is constructed as follows:

expression	predicate denoted
A	$\lambda y.Ay$
R	$\lambda x\, y.Rxy$
$\rightarrow(R, A)$	$\lambda x\, y\, z.(Rxy \rightarrow Az)$
$id(\rightarrow(R, A))$	$\lambda x\, y.(Rxy \rightarrow Ay)$
$\forall(id(\rightarrow(R, A)))$	$\lambda x.\forall y(Rxy \rightarrow Ay)$
$\rightarrow(A, \forall(id(\rightarrow(R, A))))$	$\lambda x\, z.(Ax \rightarrow \forall y(Rzy \rightarrow Ay))$
$id(\rightarrow(A, \forall(id(\rightarrow(R, A)))))$	$\lambda x.(Ax \rightarrow \forall y(Rxy \rightarrow Ay))$
$\forall(id(\rightarrow(A, \forall(id(\rightarrow(R, A))))))$	$\forall x(Ax \rightarrow \forall y(Rxy \rightarrow Ay))$.

On the other hand, this notation may have some explanatory power. E.g., consider the simple sentence «Everyone loves someone», symbolized as $\forall x\ \exists y\, Lxy$. The equally simple predicate-logical formula $\forall x\ \exists y\, Lyx$ corresponds, in sequential order, to the involved «For everyone there is someone who loves him». Much more natural, however, is the synonymous «Everyone is loved by someone», in which the *passive* corresponds to the above conversion operator. In other words, the Quinean reading $\forall(\exists(inv(L)))$ reflects a tendency of natural language.

Theorem 5.1 may be extended to cover the case of individual and function constants by adding a *substitution operator* subst, which turns a term t with n argument places and a term or predicate α with m argument places into a $(n+m\text{-}1)$-ary term or predicate subst(t, α), as in the following examples. Let a be an individual constant, f a unary and g a binary function constant.

subst(a, f) denotes the object fa
subst(a, g) denotes the function $\lambda x . gxa$
subst(f, g) denotes the function $\lambda xy . gxfy$
subst(a, R) denotes the predicate $\lambda x . Rxa$
subst(g, R) denotes the predicate $\lambda xyz . Rxgyz$

Note that a term like gfxfy will require *inv:* subst(f, inv(subst (f, g))).

6. *Conclusions*

The discussion of section 2 showed that the philosophical case for Priorean tense logic is not convincing. Linguistic evidence for the operator approach may be more telling, but, as we saw in section 3, tense-logical systems start appearing which contain operators without natural language readings. Moreover, since natural language seems averse to displaying *any* kind of quantifiers plus explicit variables, the case against *moment* variables may seem stronger than it actually is. (It should be stated, however, that our discussion left out possible arguments arising from the combination of tense phenomena with quantification over individuals). From a technical point of view, tense logics could be considered to be sublogics of predicate logic. Some of these are quite elegant (and worth studying for its own sake), like Prior's original system, or Kamp's system with Since and Until. But, as tense logics become stronger and stronger (containing ever more exotic operators), predicate logic itself becomes a serious rival as regards elegance and simplicity.

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