EXTENSIONAL AND INTENSIONAL LOGIC FOR CRITERIA OF IDENTITY

Leslie STEVENSON

In this paper I attempt to analyze some of the principles concerning identity and sortal concepts which have been explored by Geach, 1962 and by Wiggins, 1967. I start by investigating how much can be done in a purely extensional way (and my treatment here will differ in certain respects from what I have attempted elswhere — in the *Notre Dame Journal of Formal Logic 1975*. But I will suggest reasons for thinking that some of the conceptual analysis done by Geach and Wiggins can be expressed only in an intensional logic, so I will introduce modality at a certain stage. This may suggest some connections between criteria of identity and recent discussions of essentialism. But there are many questions about identity and sortals which I do not answer here. I am trying to explore the logic of these notions from the inside, so to speak, and even this project may involve a certain amount of idealization and lack of realism. This is just a preliminary exploration of a logical territory which so far as I know has not yet been mapped.

The claim has been made that if *a* is identical to *b*, then there must be some concept F such that *a* is the same F as *b*. (This is one thing which may be meant by the thesis that identity is relative). A connected claim is that for every proper name there must be a concept F which gives the criterion of identity of the thing named. One classic source for these claims is Frege's *Foundations of Arithmetic* (1884, § 62) where he says «If we are to use the symbol *a* to signify an object, we must have a criterion of identity for deciding in all cases whether *b* is the same as *a*». A criterion of identity is, presumably, expressed in a phrase of the form *the same F*.

But not all general terms can fill the bill here. Frege realised that not all concepts «isolate in a definite manner what falls under them» (op. cit. § 54), e.g. something red can be divided up in many ways without the parts ceasing to fall under the concept *red;* but the parts of a cat are not themselves cats. Following Strawson (1959, p. 168) I define a *count noun* or ***sortal concept*** as one which supplies a principle for distinguishing and counting the individual particulars to which it applies, and can therefore play the role of F in identity statements of the form *a is the same F as b.* Geach (op. cit. § 31) says that countability is not a necessary condition of a term's making sense in *the same F,* for we can talk of *the same gold* (and, indeed, of *the same weight).* But I would question whether *a is the same gold as b* or *a is the same weight as b* are *identity*-statements, for both seem to allow that something true of *a* is not true of *b.* These coins may be the same gold as the statue we melted down, but in such a case the statue ceased to exist before the coins came into existence. Wiggins says that it is not necessary for F to supply a *generally applicable* principle of counting in order for it to give an intelligible sense to a particular statment of identity. For example, we might be able to settle whether you saw the same oily wave as I saw, without having a way of counting waves in general. (Wiggins, op. cit., pp. 39-40). However, to settle the truth-value of an identity statement is to settle whether *one* or *two* individuals of the relevant sort have been referred to, so there must be at least the beginnings of a principle of counting which could be applied to more individuals of the same sort, given appropriately similar conditions. As far as the logic of it goes, if we have an identity relation is *the same F as,* we can define all statements of the form *There are n F's which ∅* (where n is an integer), by devices familiar in the ordinary logic of quantification with identity.

Identity is a logical notion which we use in talking about all sorts of things — numbers (integral, rational, and real), events limited companies, nations, etc.), persons, animals, and material objects (tables, cars, mountains, rivers, etc.). And the principle that every identity statement between individuals has a corresponding sortal concept which gives the relevant criterion of identity would seem to be of completely general application. (Frege in the places cited above was applying it to numbers, but his notion of an object was completely general — persons, planets, cities, numbers, classes, and truth-values all counted as objects for him, and each sort of object had its appropriate criterion of identity). So I intend that what I say, and the formal definitions and axioms I lay down, should be of completely general application. That is, I am trying to analyze the *logic* of identity, assuming that its logical features are the same no matter which sort or category of individual it is applied to.

Let us assume, then, that the fundamental form of an identity statement is x *is the same A as* y, where *A* represents a place to be filled by a count noun. We are thus making a *syntactic* distinction between one-place predicates and the kind of expression which can fill the place of *A* in identity statements. So we must add a new basic syntactic category of count nouns (C) to the standard basic categories of sentences (S) and (individual) names (N) (cf. Lewis, 1972). Our assumption can then be expressed in the claim that identity is a logical constant in the category S/CNN, so that it needs completion by a count noun and two names to make a sentence.

What about other kinds of occurrence of count nouns ? They occur of course in what are standardly treated as atomic sentences in which a one-place predicate *is an A,* of category S/N, is attached to a name, of category N, to form a sentence of the form x *is an A*. We will have to treat such one-place predicates as compound, formed from a count noun, and a copula of category S/CN. But we do not need to take this copula as primitive, for it can be defined in terms of our primitive expression for identity. As Geach suggested, (op. cit., §109) we can define *is an A* as *is the same A as something,* or, more simply, as *is the same A as itself* (cf. Geach, 1973, p. 291). So for our logic of identity let us lay down as our first definition:

$$\text{(D1)} \qquad \text{x is a A} =_{\text{df}} \quad x \underset{A}{=} x.$$

Count nouns also occur in quantifier phrases such as *some boy, every pig, most weddings, at least three integers,* etc... Such phrases can go with one-place predicates to make up sentences, so they are of category S/(S/N), and the words *some, every,* etc., must themselves be of category S/C(S/N). We shall take *every* as primitive, and represent *Every A is* ∅ in our logic by '$(\forall xA)\varnothing(x)$'.

What about the quantifier phrases *something, everything* and *nothing*? They are of category S/(S/N) of course, but if we are unwilling to admit that *thing* has the proper semantic qualifications (of supplying a criterion of identity) to be allowed into our intended category C, then we cannot split up these expressions in the above way. But there is another suggestion of Geach's which can be exploited at this point, namely that unrestricted quantification *something is* ∅ should be interpreted as *For Some A; some A is* ∅, where the second quantifier is restricted by a count noun (in our notation it could be written '$(\exists xA)\varnothing(x)$', but the first occurrence of *for some A* is a quantifier whose meaning is that there is some count noun such that what follows is true. (See Geach, 1962, § 93.) So if we are prepared to take quantification over count nouns as primitive in category S/(S/C), we can define unrestricted quantification in terms of the restricted quantifiers introduced above. Using S,S',S'', ... for count noun variables, the definition would be:

$$\text{(D2)} \qquad (x)\varnothing(x) =_{\text{df}} (S)\,(\forall xS)\varnothing(x).$$

Quantification over count nouns may seem a rather obscure and controversial notion to take as primitive. But if the analysis is to be justified, it must be in terms of its results; and my immediate purpose in this paper is to see what *can* be done by way of logical analysis of identity, and thus to press on with the logic for the moment. Full philosophical discussion can come later, when we have the logical results before us. One of the attractions of using quantification over count nouns is that we can also apply it to derive absolute or unrestricted identity from the relative or restricted identity that we have taken as primitive above. We can simply define that x is identical to y iff for some S, x is the same S as y:

$$\text{(D3)} \qquad x = y =_{df} (\exists S)\ (x \underset{S}{=} y).$$

We can now begin to lay down some axioms to express what we think are the logical truths about identity. With our quantification over count nouns, which are supposed to be the terms which express criteria of identity, we can symbolize Frege's insight that every object must have a criterion of identity by the axiom:

$$\text{(A1)} \qquad (x)(\exists S)\ (x \underset{S}{=} x),$$

which, by our definition (D1) of the copula *isa,* tells us that for every object x there is a count noun *A* such that x is an A.

Unrestricted identity is often described as the minimal reflexive relation, that relation which every object bears to itself and to no other object. But for *is the same A as* it is clear that only an A, not any object at all, can be the same A as itself. So the analogue of the reflexivity of identity which we want is:

$$\text{(A2)} \qquad (x)(y)(S)(x \underset{S}{=} y \supset x \underset{S}{=} x).$$

The other law of identity which is used as an axiom in standard first-order logic is Leibniz's law of the indiscernibility of identicals; for us, the analogue would be the axiom schema:

$$\text{(A3)} \qquad (x)(y)(S)(x \underset{S}{=} y \supset (\varnothing(x) \supset \varnothing(y))).$$

This is the point at which we part company from Geach, for as Wiggins has shown (op. cit., § 1.2) Leibniz's law quickly yields the impossibility of the situation which Geach (1962, § 94) thinks possible, that different A's could be one and the same

B. In our present notation, by putting $\S \underset{A}{=} x$ for $\emptyset$ (§) in (A3) we can get $x \underset{B}{=} y \supset (x \underset{A}{=} x \supset y \underset{A}{=} x)$, and so if x is an A at all (i.e. $x \underset{A}{=} x$) we have $x \underset{B}{=} y \supset y \underset{A}{=} x$. However, I have already (Stevenson, 1972) given my reasons for disagreeing with Geach on this point, so I will not repeat them here.

It is worth noting that an identity statement x *is the same A as* y can be false in different ways — because x is not an *A*, or y is not an A, or both or because they are A's but *different* A's. We can pick out the latter case by the following definition:

$$\text{(D4)} \qquad x \underset{A}{\neq} y =_{df} (x \underset{A}{=} x \;\&\; y \underset{A}{=} y \;\&\; —(x \underset{A}{=} y)).$$

We can read '$x \underset{A}{\neq} y$' as x *is a different A from* y, for this English sentence-form entails that x and y are both A's, unlike the form *it is not the case that* x *is the same A as* y, which means just '$—(x \underset{A}{=} y)$'.

From the above three axioms and our definition (D3) of unrestricted identity, it is easy to derive the standard laws of identity used in first order logic. Applying (D3) to (A1) gives us $(x)(x = x)$ immediately. And if $x = y$, then by (D3)$x \underset{S}{=} y$ for some S, and so by (A3) we can derive $\emptyset$ (x) $\emptyset(y)$, and so we have $(x)(y)(x = y \supset (\emptyset(x) \supset \emptyset(y)))$. (I am assuming the use of standard first-order logic for both individual variables and count noun variables.) We could do a similar derivation of unrestricted quantification from restricted quantifiers, exploiting (D2), if we assume appropriate axioms and rules for operating with the restricted quantifiers (see Stevenson 1975). But since the main concern of this paper is with identity, I will not go into this here. The economy would not be very genuine, anyway, since we would have to assume the standard laws of first order logic for the count noun variables in order to perform the derivation.

From what we have laid down so far, it follows that (S)(x)(y)($x \underset{S}{=} y \equiv (x = y$ & x isa S)) is a theorem. For if $x \underset{A}{=} y$, then ($\exists$S)($x \underset{S}{=} y$), i.e. x = y, and we also have $x \underset{A}{=} x$ by (A2), i.e. x isa A. Conversely, if x = y, then for some B, x = y, so putting x = ξ for ∅ (ξ) in A3) we can derive x = y from x isa A. Geach would regard this theorem as a *reductio* of our position to triviality, for according to him, (1962 emended edition, § 109), the crucial point about *is the same A as* is that it does *not* admit of the analysis *is an A and is the same as*. However, we can reply that our *analysis* is embodied not in the above theorem but in the definitions (D1) and (D3). If claims about conceptual priority are to be made, our candidate would of course be the mixed three-place relation x *is the same A as* y, of category S/CNN.

But Geach may reply (cf. 1973, p. 289) that the fact that one expression can be taken as primitive in one axiomatization proves nothing about conceptual priority, for a different axiomatization may yield the same result from different primitives. (Such a situation is very familiar from the propositional calculus.) So what if we started from standard first-order logic, with x = y as primitive, and defined '$x \underset{A}{=} y$' as 'x = y & x isa A' ? Clearly, if *isa A* is treated as an atomic one-place predicate, we would have no explanation of why not all such predicates can yield a term which can occur in $x \underset{A}{=} y$ in the position of A. Suppose then we graft on to standard first-order logic the syntactic distinction between count nouns and one-place predicates, and define $x \underset{A}{=} y$ as just suggested ? We would then have to assume the copula *isa* as a primitive (mixed) relation of category S/CN. Such a procedure *would* yield the same results as ours.

Does this give the game away to Geach, and show that all we have really done is to introduce a notation '$x \underset{A}{=} y$' for the standard «absolute» identity relation when restricted to

A's ? Certainly, we are denying that identity is relative in the way that Geach thinks it is, namely that it is possible that x could be the same A as y but a different B from y, for we have seen that our axiom schema (A3) entails the impossibility of this. So it would be appropriate to call the present theory one of restricted rather than relative identity. But is it anything more than a notational variant of the standard theory of identity in first-order logic ? It cannot be merely that, for we have just seen that in order to derive our results from the standard theory we need to add a new syntactic category C and a new primitive ««isa».

Given that we are going to have the new syntactic category C, is there anything to recommend our choice of '$=_S$' as primitive rather than *isa* ? Since the intuitions which motivate the whole theory are about criteria of identity, it looks as if the former expresses them more directly; but it might be said that since on the latter choice only those terms which can occur after *isa* can be used to form relations of the form '$=_S$', it simply expresses the same intuitions. And in any case, such talk can appeal only those already sympathetic with the intuitions. However, there is a stronger defence to be made, which is that if we are going to use quantifiers over count nouns whichever choice we make, (and we shall see in a moment that there is motivation for this), then the choice of '$=_S$' as primitive allows us to derive both *isa* and '=' (by (D1) and (D3)), whereas taking *isa* as primitive requires us *also* to introduce '=' as primitive. Such reduction in the number of primitives is an objective logical result which does not depend on intuitions.

Let us now begin to explore the relations between count nouns, and criteria of identity they supply. Very often, one such term is subordinate to another (a restriction of another, as Geach puts it). We can express the extensional part of such a relation by the definition:

$$\text{(D5)} \qquad \text{A sub B} =_{\text{df}} (x)(y)(x \underset{A}{=} y \supset x \underset{B}{=} y).$$

This yield as an immediate consequence that A sub B ⊃ (x) (x isa A ⊃ x isa B). There will be a corresponding relation of coextensionality:

$$\text{(D6)} \qquad \text{A equ B} =_{\text{df}} \text{A sub B \& B sub A}.$$

We can also define intersection, the relation which holds iff two count nouns apply to one object:

$$\text{(D7)} \qquad \text{A int B} =_{\text{df}} (\exists x)(x \underset{A}{=} x \;\&\; x \underset{B}{=} x).$$

These are obviously two-place relations between count nouns, of category S/CC.

If we think for a moment of what form of referential semantics can be given for the language we are building up, then it looks as if the extension of a count noun will be simply the set of objects to which it applies. But count nouns are the terms which supply criteria of identity, so the extension of a given count noun will be a set all of whose members have the same criterion of identity. We can call such sets *sorts,* since they are the extensions of what are called sortal terms, i.e. count nouns. All sorts are sets, but not all sets are sorts. What the last three definitions capture is just the set-theoretic relations of inclusion, identity, and non-emptiness of intersection, holding between sorts.

But what formal properties of sorts would reflect our intuitions about criteria of identity (CI's for short) ? Any subset of a sort will be a sort, for if all the A's share a C1, then the A's which are ∅ share that C1. So the intersection of a sort with any set will be a sort (provided, of course, that we accept that the empty set is a sort). But the union of two sorts will not always be a sort, although it will of course be a set. For if it is ever true to say that two objects, or two sorts of object, have different CI's, then any set containing those two objects, or objects of those two sorts, will not be a sort, since its members will not share a CI. For instance, dogs and weddings surely have different CI's, so there can be no sort containing some of both. But weddings and church services must presumably have the same CI, since some services *are* weddings. But not all services are weddings, nor do all weddings take place in church. So it is not necessary, for two sorts to share a CI, that one should include the other; it is sufficient that they overlap. But even this is not necessary, for girls and boys surely have the same CI, namely that for human beings, but the two sorts are disjoint. Two sorts' having the same CI would seem to amount to there being some sort which includes both; linguistically, a more general count noun which supplies the CI supplied by the two count nouns subordinate to it. (In the case of weddings and services, such a term is *ceremony*.) Using our quantifiers over count nouns, we can express this by the following definition of *being in the same family:*

$$\text{(D8)} \qquad \text{A fam B} =_{\text{df}} (\exists \text{S}) (\text{A sub S} \ \& \ \text{B sub S}).$$

This is our attempt to formalize the relation which holds between two count nouns when they supply the same CI; the relation which Wiggins calls "restricting the same sortal", (op. cit. p. 31), and claims to be an equivalence relation. But the attempt is not wholly successful, for (D8) allows an empty count noun to bear the *fam* relation to any count noun whatsoever, since if *A* applies to nothing then A sub B for any B. So *unicorn* bears *fam* to *wedding* and to *rational* number, whereas we do *not* want to say that these terms supply the same CI. The kind of thing we would like to say is that unicorns have the CI of mammals, not of ceremonies or numbers, because *if* there were any unicorns they would be mammals. But this is something we cannot express in a purely extensional logic.

One possible reaction is to refuse to admit empty count nouns, and put in an axiom (S) ($\exists$x) (x isa S). This is one of the features of the treatment I have given elsewhere (Stevenson 1975). But this choise is unrealistic, for there undoubtedly are in our language some perfectly meaningful count nouns which lack application, and as Geach says, (1962, §106), in order to use the term *dragon* one need only claim that one *would* be able to identify a dragon if confronted with one! The other possible reaction is to try to develop an intensional logic in which we can express such counterfactual talk. We will explore this option later in the present paper, but before we allow ourselves the luxuries and the problems of modality, let us see how much can be done in more austere extensional surroundings, in which we admit empty count nouns.

We have yet to lay down any axioms involving the S/CC relations we have been defining. But one intuitively valid principle which we mentioned above is that if two count nouns apply to one and the same object, then they must supply the same CI (for an object can have only one CI, on our understanding of *object* as opposed to Geach's). The nearest expression of this with our present resources is:

$$\text{(A4)} \qquad (S)\,(S')\,(S \text{ int } S' \supset S \text{ fam } S').$$

Such a principle is appealed to by Wiggins when he says "the cross-classifications which two sortals can impose on an object must be subordinate to some logically sound principle of classification under which the object falls. Whether named or unnamed there must then exist a corresponding sortal which both sortals restrict" (op. cit. p. 33.) It follows easily from (A4) that *fam* is an equivalence relation over *non-empty* count nouns, so we can collect them into equivalence classes which we can conveniently call *families*.

The non-empty common nouns in a family all share the same CI. And the intention behind the whole theory is that there are many different families, because there are many different CI's. For instance *horse, battle,* and *integer* must be in different families, since they supply different CI's. There is no such family as that of all objects or things.

It seems natural to suppose that each family must have a head, i.e. a common noun to which all the rest of the family are subordinate, which supplies the pure CI which the others supply together with extra requirements.
Dummett, (1973, pp. 75-6), says that among any class of nouns associated with the same CI there will always be one which is most general, and he calls such terms *categorical predicates.* Wiggins, (op. cit. p. 33 and footnote 40), commits himself to the principle that every sortal is a restriction of some 'ultimate' sortal; but his definition of *ultimate* is a disjunctive one, only the first disjunct of which - "restricts no other sortal' - corresponds to the idea in hand. We can try to express the idea in definition:

$$\text{(D9)} \qquad \text{cat (A)} =_{\text{df}} \text{(S) (A sub S} \supset \text{A equ S),}$$

where cat is of course a property of count nouns, in the syntactic category S/C. But nothing we have said so far commits us to the existence of category terms in the sense just defined. The appropriate axiom to add would be:

$$\text{(A5)} \qquad \text{(S) } (\exists \text{S}') \text{ (S sub S}' \text{ \& cat (S}'\text{)).}$$

If we call the sorts that are the extensions of category terms *categories* or *ultimate sorts,* then it is easy to show that on the assumptions we have made, every sort is included in one and only one category, and every object is a member of one and only one category. We could therefore introduce functions, in categories C/C and C/N, which take us from a count noun or individual term to the corresponding category term. In my treatment elsewhere (Stevenson 1975) I represented both these functions by a primitive constant *U* and did without variables for count nouns (sortals) by putting in axioms governing *U* to give the same effect. The development of a corresponding set-theoretic semantics, in which sorts are distinguished from sets in general, makes possible a completeness proof on standard lines.

What I would like to do in this paper is to make a tentative foray into modal logic, to see what light the introduction of modality may throw on the logic of restricted identity. Immediately the question arises of which modal principles we should permit ourselves. But remembering that we have both individual variables and count noun variables in our system already, we must realize that they may behave differently in modal contexts. As far as individuals are concerned, we would presumably want to allow the domain of objects in each possible world to vary arbitarily, so that possible worlds may contain objects which do not exist in the actual world, and may lack objects which do exist in the actual world. This entails that for individual variables, neither the Barcan formula $(x)\, L\, \varnothing\, (x) \supset L\, (x)\, \varnothing\, (x)$ nor its converse will be valid (cf. Hughes and Cresswell, 1968, Chapter 10, and Kripke, 1963). But as regards our count noun variables, it seems natural to suppose that we have the same count nouns available in every possible world, (even though some of them may be empty in some worlds). We would therefore expect the Barcan formulas for count noun variables. $(S)L\varnothing \supset L(S)\varnothing$, and its converse, to be valid.

What modal additions should we make, then, to our theory? The choice between systems of type T, S4, S5, and Brouwerian (at least) lies open to us. S5 is the simplest in the sense that all iterations of modalities reduce to a single modality, and it has been argued to give the best representation of *logical* necessity, so it might be the natural one to try for a start. But I can leave the choice open as far as this paper is concemed. However, it is notorious that it makes a substantial difference to *which* non-modal basis for first-order logic we add the modal axioms (cf. Hughes and Cresswell op. cit. Chapter 10). If we are to avoid making the Barcan formula and its converse provable we must either have a basis in which only closed formulas are theorems, as in Kripke (op. cit.), or else use a 'free logic' in which the usual axiom $(x)\varnothing(x) \supset \varnothing(y)$ is amended to $(x)\varnothing(x) \supset (Ey \supset \varnothing(y))$ where E is a predicate for existence, as in Hintikka (1959, 1963). I suspect that the latter choice may suit our purposes best, for we can amend the axiom as suggested for individual variables while leaving its analogue for count noun variables unchanged. But again I shall leave the choice open in this paper. It is worth noting, however, that whichever way it goes, the formula (x) (y) (x = y ⊃ L(x = y)), which is regarded as paradoxical by some, is not derivable as a theorem; for we can only get it with (x) L(x = x) as an antecedent, but the latter is not derivable from L(x) (x = x). Still less does the result apply to restricted identity, for L(x) $(x \underset{A}{=} x)$ is not even true, since not everything is an A.

With modal operators at hand, we can now strengthen some of our previous definitions and axioms, to capture more closely our intuitions about criteria of identity. The simplest way to do this is to put an appropriate modal operator at the front; thus (D5 - 7) would become:

(D5') $\text{A SUB B} =_{df} L(x)(y)(x \underset{A}{=} y \supset x \underset{B}{=} y)$,

(D6') $\text{A EQU B} =_{df} \text{A SUB B \& B SUB A}$,

and (D7') $\text{A INT B} =_{df} M(\exists x)(x \underset{A}{=} x \ \& \ x \underset{B}{=} x)$.

(D5') would seem to a better representation of the notion of subordination or restriction than (D5), for we can say that an empty count noun does not bear *SUB* to any count noun at all, but only to those whose extensions include its extension in all possible worlds. So *unicorn* will bear *SUB* to *mammal,* since in any possible world in which there are unicorns they would be equine mammals with one horn; but it will not bear *SUB* to any arbitrary count noun such as *ceremony* or *rational* number.

However it will follow from (D5') that any count noun which is necessarily empty will bear *SUB* to any count noun whatsoever. (This is the obvious analogue of one of the paradoxes of strict implication - the analogue of the other does not arise, since no count noun is necessarily true of everything.) For ⊢L - (∃x) (x isa A) ⊃ (S) (A sub S) by non-modal logic, hence ⊢L - (∃x) (x isa A) ⊃ L(S) (A sub S) by modal logic in system

T, but $\vdash$ L (S) (A sub S) $\supset$ (S)L(A sub S), i.e. $\vdash$ L(S) (A sub S) $\supset$ (S) (A SUB S), by the converse of the Barcan formula for count noun variables. So if *married bachelor* and *rational square root of two* are admitted as count nouns they will bear *SUB* to all count nouns. But I'm not sure whether we should be worried by this result. We could put in an axiom requiring every count noun to be capable of application, $(S)M(\exists x)(x \underset{S}{=} x)$, but there seems little harm in tolerating self-contradictory count nouns (and in mathematics we may have to!).

The natural way to strengthen our notion of *being in the same family*, which is supposed to represent the having of the same criterion of identity, is:

$$\text{(D8')} \quad \text{A FAM B} =_{df} (\exists S)\ (\text{A SUB S}\ \&\ \text{B SUB S}).$$

An empty count noun will not bear *FAM* to any count noun, although a self-contradictory one will. *Unicorn* will now go in the same family as *horse*, but not in the family of *ceremony* etc. . The natural strengthening of our axiom (A4) is:

$$\text{(A4')} \quad (S)\ (S')\ (\text{S INT S}' \supset \text{S FAM S}').$$

which says that if it is possible for two count nouns to apply to one and the same object, then they must be in the same family, i.e. supply the same criterion of identity. It will follow that *FAM* is an equivalence relation over non-contradictory count nouns.

The strongest way to modalize the notion of category term is

$$\text{(D9')} \quad \text{CAT (A)} =_{df} L(S)\ (\text{A sub S} \supset \text{A equ S}).$$

This entails, but is not entailed by (S) (A SUB S $\supset$ A EQU S). The obvious strenthening of the axiom for the existence of a category term to which each count noun is subordinate is:

$$\text{(A5')} \quad (S)\ (\exists S')\ (\text{S SUB S}'\ \&\ \text{CAT (S')}).$$

In this strengthened sense of category, there will be one and only one category for each object and each count noun.

But now that we have introduced modality, is there not a rather different notion which we can define, namely that of a count noun which is *essentially* true of whatever it is true of? Our idea has all along been that count nouns are the terms which supply criteria of identity (CI's), although some of them have extra requirements as part of their sense, restricting their application to only some of the objects having the relevant CI. Thus *tailor* supplies the CI of *human being,* but also says something about the occupation of the human beings it applies to. Now instead of looking for the category term to which a given count noun is subordinate, could we not look for an essential count noun? The notions may well be different ones, for *human being* is subordinate to *mammal* and is therefore not a category term, yet it might be argued that anything that is a human being cannot cease to be one without ceasing to exist, so that *human being* would be an essential term. The second disjunct of Wiggins' disjunctive definition of *ultimate sortal,* (op. cit. p. 32) - a sortal which "has a sense which both yields necessary and sufficient conditions of persistence for the kind it defines and is such that this sense can be clearly fixed and fully explained without reference to any other sortal which it restricts" - suggests some such notion of an essential term. For if a term "yields necessary conditions of persistence" then nothing to which the term applies can cease to fall under that term without ceasing to exist. Wiggins' distinction between "substance-sortals", which must apply to an object throughout its existence, and "phase-sortals" like *boy,* which need not (op. cit. p. 7) is also relevant here, for substance-sortals would seem to be essential terms.

How can we represent these essentialist notions in our logic? Let us not restrict ourselves to notions like *cease* which involve time, but just think of an essential property of something as a property which is necessarily true of that thing. A first shot at representing *a is necessarily F* would be 'LFa', but this will not do because it requires 'Fa' to be true in all possible worlds and hence requires *a* to exist in all possible worlds. We must use the existence predicate 'Ex' and say instead 'L(Ea ⊃ Fa)' -that *a* is F in all worlds in which it exists. We can now define a general notion of essential property as one which must be necessarily true of whatever it is true of - thus the formula will be L(x) (Fx ⊃ L(Ex ⊃ Fx)). Applying this to the case of count nouns, we can give the following definition of an essential count noun:

$$\text{(D10)} \quad \text{ESS (A)} =_{df} \text{L(x) (x} \underset{A}{=} \text{x} \supset \text{L(Ex} \supset \text{x} \underset{A}{=} \text{x)).}$$

To assert the intuitively plausible principle that every count noun must be subordinate to an essential count noun, we need the axiom:

$$\text{(A6)} \quad \text{(S) (}\exists\text{S') (S SUB S' \& ESS(S')).}$$

Such an axiom explicitly commits us to essentialism, for together with (A1), which says that every object has a criterion of identity, it entails that every object has at least one essential property, namely that it has the criterion of identity that it has. But my argument is that just such a conclusion follows from unpacking Frege's original insight that every object must have a criterion of identity. [1]

University of St. Andrews Leslie Stevenson

DUMMETT, M.A.E.

1973 *Frege: Philosophy of Language* (London, Duckworth).

FREGE, G.

1884 *Foundations of Arithmetic* trans. Austin, 1950 (Oxford, Blackwell).

GEACH, P.T.

1962 *Reference and Generality* (Ithaca, Cornell University Press).

1967 «Identity» *Review of Metaphysics*, 21, pp. 3-12.

(1) This paper was first presented at a conference at the University of York in 1973; it has improved substantially as a result of the discussion there.

1973 «Ontological Relativity and Relative Identity», *Logic and Ontology* ed. Munitz pp. 287-302, (New York, New York University Press).

HINTIKKA, K.J.J.

1959 «Existential Presuppositions and Existential Commitments» «*Journal of Philosophy*, 56, pp. 125-137.

1963 «The Modes of Modality» *Acta Philosophica Fennica*, 16 pp. 65-82.

HUHES, G.E. and CRESSWELL, M.J.

1968 *An Introduction to Modal Logic* (London, Methuen).

KRIPKE, S.A.

1963 «Semantical Considerations on Modal Logic» *Acta Philosophica Fennica*, 16, pp. 83-94.

LEWIS, D.

1972 «General Semantics», *Semantics of Natural Language* ed. Harman and Davidson, pp. 169-218, (Dordrecht, Reidel).

STEVENSON, L.F.

1972 «Relative Identity and Leibniz's Law» *Philosophical Quarterly*, 22, pp. 155-158.

1974 «Dummett on Frege», *Philosophical Quarterly*, 24, pp. 349-359.

1975 «A Formal Theory of Sortal Quantification» *Notre Dame Journal of Formal Logic*, 16, pp. 185--207.

1976 «On what Sorts of Thing There Are», *Mind* 85, pp. 503-521.

STRAWSON, P.F.

1959 *Individuals* (London, Methuen).

WIGGINS, D.

1967 *Identity and Spatio-Temporal Continuity* (Oxford, Blackwell).