RUSSELL'S THEORY OF DESCRIPTION AS A VEHICLE FOR A TRANSITION FROM «OUGHT» TO «IS» AND *VICE VERSA**

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Let us consider the following two arguments:

(A1) The most perfect being ought to be loved.
Therefore, the most perfect being exists.

(A2) The most perfect being ought to be loved.
Therefore, there is a most perfect being.

These two arguments are, in a certain way, equivalent to the following two arguments:

(A3) The most perfect being does not exist.
Therefore, it is not the case that the most perfect being ought to be loved.

(A4) There is no perfect being at all.
Therefore, it is not the case that the most perfect being ought to be loved.

These four arguments A1 - A4 have the following logical forms (in the following, standard notation will be used, and 'O' represents 'it ought to be the case that ...'):

(AF1) $O\chi(\iota x)(\varphi x) \quad \therefore \sim O\chi(\iota x)(\varphi x)$

(AF2) $O\chi(\iota x)(\varphi x) \quad \therefore (\exists x)\varphi x$

(AF3) $\sim E!(\iota x)(\varphi x) \quad \therefore \sim O\chi(\iota x)(\varphi x)$

(AF4) $\sim (\exists x)\varphi x \quad \therefore \sim O_{\chi}(\iota x)(\varphi x)$

(*) I am indebted to Josef Vascovitz for improvements.

Let us now compare these four argument forms AF1 - AF4 with the following two families of argument forms AF1' - AF4' and AF1'' - AF4'', respectively:

(AF1') $\psi\,(\imath x)\,(\varphi x)$
$\therefore\ E\,!\,(\imath x)\,(\varphi x)$

(AF2') $\psi\,(\imath x)\,(\varphi x)$
$\therefore\ (\exists x)\,\varphi x$

(AF3') $\sim E\,!(\imath x)\,(\varphi x)$
$\therefore\ \sim \psi\,(\imath x)\,(\varphi x)$

(AF4') $\sim (\exists x)\,\varphi x$
$\therefore\ \sim \psi\,(\imath x)\,(\varphi x)$

(AF1'') $f\,\{\chi\,(\imath x)\,(\varphi x)\}$
$\therefore\ E\,!\,(\imath x)\,(\varphi x)$

(AF2'') $f\,\{\chi\,(\imath x)\,(\varphi x)\}$
$\therefore\ (\exists x)\,\varphi x$

(AF3'') $\sim E\,!\,(\imath x)\,(\varphi x)$
$\therefore\ \sim f\,\{\chi\,(\imath x)\,(\varphi x)\}$

(AF4'') $\sim (\exists x)\,\varphi x$
$\therefore\ \sim f\,\{\chi\,(\imath x)\,(\varphi x)\}$

The conclusion of AF3' and AF4', the premise of AF1'' and AF2'', and the conclusion of AF3'' and AF4'' are according to *Principia Mathematica* ambiguous (1). With respect to the conclusion of AF3' and AF4', i.e. $\sim \psi\,(\imath x)\,(\varphi x)$, we have to distinguish between $\sim \{[(\imath x)\,(\varphi x)]\,.\,\psi\,(\imath x)\,(\varphi x)\}$, i.e. $\sim \{(\exists c) : \varphi x\,.\equiv_x.\,x = c : \psi c\}$, and $[(\imath x)\,(\varphi x)]\,.\sim \psi\,(\imath x)\,(\varphi x)$, i.e. $(\exists c) :$

(1) *Cf. Principia Mathematica,* Vol. I, pp. 69, 182, 185f. With respect to the following distinctions it is important to note: «When $f\{(\imath x)\,(\varphi x)\}$ ... forms part of some other proposition, we shall say that $(\imath x)\,(\varphi x)$ has a *secondary* occurrence. When $(\imath x)\,(\varphi x)$ has a secondary occurrence, a proposition in which it occurs may be true even when $(\imath x)\,(\varphi x)$ does not exist.» (*Principia Mathematica,* Vol. I, p. 68). «When $(\imath x)\,(\varphi x)$ does not exist, there are still true propositions in which '$(\imath x)\,(\varphi x)$' occurs, but it has, in such propositions, a *secondary occurrence* ..., *i.e.* the asserted proposition concerned is not of the form $\psi\,(\imath x)\,(\varphi x)$, but of the form $f\{\psi\,(\imath x)\,(\varphi x)\}$, in other words, the proposition which is the scope of $(\imath x)\,(\varphi x)$ is only part of the whole asserted proposition.» (*Principia Mathematica,* Vol. I ,p. 182).

$\varphi x \,.\!\cdot\!\!\equiv_x . \, x = c : \sim \psi c$ (the notation used is that of *Principia Mathematica* [2], whose objective is, to exclude ambiguities in sentences containing '$(\iota x)\,(\varphi x)$' by indicating its scope).
With respect to the premise of AF1'' and AF2'', i.e. $f\{\chi\,(\iota x)\,(\varphi x)\}$, we have to distinguish between

$f\{[(\iota x)\,(\varphi x)] \,.\, \chi\,(\iota x)\,(\varphi x)\}$, i.e. $f\{(\exists c) : \varphi x \,.\equiv_x .\, x = c : \chi c\}$, and $[(\iota x)\,(\varphi x)] \,.\, f\{\chi\,(\iota x)\,(\varphi x)\}$, i.e. $(\exists c) : \varphi x \,.\equiv_x .\, x = c : f\,(\chi c)$.

With respect to the conclusion of AF3'' and AF4'', i.e. $\sim f\{\chi\,(\iota x)\,(\varphi x)\}$, we have to distinguish among $\sim f\{[(\iota x)\,(\varphi x)] \,.\, \chi\,(\iota x)\,(\varphi x)\}$, i.e. $\sim f\{(\exists c) : \varphi x \,.\equiv_x .\, x = c : \chi c\}$, and $\sim \{[(\iota x)\,(\varphi x)] \,.\, f\{\chi\,(\iota x)\,(\varphi x)\}\}$, i.e. $\sim \{(\exists c) : \varphi x \,.\equiv_x .\, x = c : f\,(\chi c)\}$, and $[(\iota x)\,(\varphi x)] \,.\, \sim f\{\chi\,(\iota x)\,(\varphi x)\}$, i.e. $(\exists c) : \varphi x \,.\equiv_x .\, x = c : \sim f\,(\chi c)$.

These distinctions allow the following variations of the above mentioned argument forms:

AF1'	remains unchanged
AF2'	remains unchanged
(AF3a')	$\sim E\,!\,(\iota x)\,(\varphi x)$
	$\therefore \sim \{[(\iota x)\,(\varphi x)] \,.\, \psi\,(\iota x)\,(\varphi x)\}$
(AF3b')	$\sim E\,!\,(\iota x)\,(\varphi x)$
	$\therefore [(\iota x)\,(\varphi x)] \,.\, \sim \psi\,(\iota x)\,(\varphi x)$
(AF4a')	$\sim (\exists x)\,\varphi x$
	$\therefore \sim \{[(\iota x)\,(\varphi x)] \,.\, \psi\,(\iota x)\,(\varphi x)\}$
(AF4b')	$\sim (\exists x)\,\varphi x$
	$\therefore [(\iota x)\,(\varphi x)] \,.\, \sim \psi\,(\iota x)\,(\varphi x)$
(AF1a'')	$f\{[(\iota x)\,(\varphi x)] \,.\, \chi\,(\iota x)\,(\varphi x)\}$
	$\therefore E\,!\,(\iota x)\,(\varphi x)$
(AF1b'')	$[(\iota x)\,(\varphi x)] \,.\, f\{\chi\,(\iota x)\,(\varphi x)\}$
	$\therefore E\,!\,(\iota x)\,(\varphi x)$
(AF2a'')	$f\{[(\iota x)\,(\varphi x)] \,.\, \chi\,(\iota x)\,(\varphi x)\}$
	$\therefore (\exists x)\,\varphi x$

[2] *Cf. Principia Mathematica,* Vol. I, pp. 69ff., 173ff. For the whole theory of description contained in *Principia Mathematica cf.* Vol. I, pp. 30f., 66-71 and 173-186.

(AF2b″) $[(\iota x)(\varphi x)] . f\{\chi(\iota x)(\varphi x)\}$
$\therefore (\exists x)\varphi x$

(AF3a″) $\sim E!(\iota x)(\varphi x)$
$\therefore \sim f\{[(\iota x)(\varphi x)] . \chi(\iota x)(\varphi x)\}$

(AF3b″) $\sim E!(\iota x)(\varphi x)$
$\therefore \sim \{[(\iota x)(\varphi x)] . f\{\chi(\iota x)(\varphi x)\}\}$

(AF3c″) $\sim E!(\iota x)(\varphi x)$
$\therefore [(\iota x)(\varphi x)] . \sim f\{\chi(\iota x)(\varphi x)\}$

(AF4a″) $\sim (\exists x)\varphi x$
$\therefore \sim f\{[(\iota x)(\varphi x)] . \chi(\iota x)(\varphi x)\}$

(AF4b″) $\sim (\exists x)\varphi x$
$\therefore \sim \{[(\iota x)(\varphi x)] . f\{\chi(\iota x)(\varphi x)\}\}$

(AF4c″) $\sim (\exists x)\varphi x$
$\therefore [(\iota x)(\varphi x)] . \sim f\{\chi(\iota x)(\varphi x)\}$

According to Russell's theory of description the following two theorems of *Principia Mathematica* (8) are valid:

*14∴21. $\vdash : \psi(\iota x)(\varphi x) . \supset . E!(\iota x)(\varphi x)$
*14∴201. $\vdash : E!(\iota x)(\varphi x) . \supset . (\exists x) . \varphi x$

From these two theorems we can derive the following by Hypothetical Syllogism:

$\vdash : \psi(\iota x)(\varphi x) . \supset . (\exists x) . \varphi x$

which may be called *14∴21-201.

The following argument forms can be proved valid because of theorems *14∴21 and *14∴21-201:

AF1′, AF2′, AF3a′, AF4a′, AF1b″, AF2b″, AF3b″, and AF4b″.

From this we can draw certain conclusions:
1. Provided that AF1 and AF2 can be treated as substitution instances of AF1′ and AF2′ or of AF1b″ and AF2b″, respectively, we can establish that:

(8) *Cf. Principia Mathematica*, Vol. I, p. 181, also pp. 68, 174f.

(i) the argument forms AF1 and AF2 are valid, and consequently
(ii) the arguments A1 and A2 are also valid. Therefore,
(iii) an «ought» entails an «is».

2. If AF3 and AF4 are substitution instances of AF3a' and AF4a' or of AF3b'' and AF4b'', respectively, then we can prove that:

(iv) the argument forms AF3 and AF4 are valid, and consequently
(v) the arguments A3 and A4 are also valid;

and under the additional assumption that the negation of an «is» is itself an «is» and the negation of an «ought» is itself an «ought», we can prove logically that:

(vi) an «is» entails an «ought».

And this is the moral of these considerations: Anyone who denies the deducibility of «is» from «ought» or the deducibility of «ought» from «is» must reject Russell's theory of description.

Instead of this, our investigations might lead us to conclude that some sentences, which at first glance appear normative, are really not ([4]).

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([4]) If we treat AF1 and AF2 as instances of AF1' and AF2' or of AF1b'' and AF2b'', the premise of AF1 and AF2 is to be interpreted (according to *Principia Mathematica*) as a conjunction with an «is» component and an «ought» component, the negation of which, i.e., the conclusion of AF3 and AF4, is a disjunction of an «is» and an «ought». But to derive such a «mixed» sentence from an «is» by AF3 and AF4 or to derive an «ought» from a «mixed» conjunction by AF1 or AF2 does not disprove (or establish a counterexample of) Hume's thesis.