ON STRONGLY CREATIVE DEFINITIONS: A REPLY

to V.F. RICKEY

In a recent article in this Journal [5] V.F. Rickey discusses our paper [3] in which we proved that as the axioms are written in *Principia Mathematica* (from now on abbreviated as PM) the definition for material implication is not a proper definition. V.F. Rickey says in his article that out paper contains an incorrect citation, a valuable insight which we win by misconstruing PM, and a logical error. In this paper we intend to answer this criticism, and introduce the distinction between strongly creative and weakly creative definitions, which may have some interest in its own right.

Let us now take V.F. Rickey's points in turn:

1. *Incorrect citation:* V.F. Rickey thinks that we have incorrectly attributed to Leśniewski the so-called condition of *eliminability* (a defined symbol should always be eliminable) and that of *non-creativity* (a definition should not permit the proof of previously unprovable relationships among the old symbols). He observes that although these conditions (which we shall call condition (1) and condition (2) respectively) have been also credited to Leśniewski by P. Suppes [6] and G. Kelley [1], «he never formulated these conditions as conditions a definition should satisfy either in his published work or in his lectures». ([5] p. 175). And a little later he adds that «not only did Leśniewski not formulate these conditions but also he would not accept it as a condition which all definitions should satisfy.»

These quotations show that V.F. Rickey fuses and confuses two separate issues:

i) Conditions (1) and (2) can be attributed to Leśniewski.

ii) Leśniewski accepted the view that all definitions should satisfy these conditions.

We certainly did not attribute (ii) to Leśniewski and our paper did not say this. What we said in our paper was that the idea that a definition should not strengthen the theory in any significant way finds expression in those two conditions of Leśniewski and thus could be tested by them. But we did think, and still think, that one can truly hold (i). He was the first, at least in modern times, to discuss and use definitions that play a creative role in a system whose logical status is therefore that of a «thesis» rather than a mere abbreviation (or «typographical convenience»). Leśniewski's report on experimenting with two converse conditionals as a definition, can be found in his «Grundzüge eines neuen Systems der Grundlagen der Mathematik» [2]. The fact that Leśniewski discussed and used creative definitions in his systems shows that he had a clear idea between «creative role» and a «mere abbreviative role» of a definition, which finds expression in conditions (1) and (2). But our main concern was not what view Leśniewski, or any other logician, had on definitions, or which view is preferable, but what role the definition of material implication plays in PM. And we have proved, using these two conditions, that as the axioms are written the definition for material implication plays a creative role in PM, and thus is not a proper definition. In a second paper in *Mind* [4] we have made our position quite clear concerning this matter. Unfortunately, V.F. Rickey fails to refer to this second paper.

II. Valuable Insight. V.F. Rickey says that we win our valuable insight by misconstruing what is said in PM about definitions; and this is because we presented the axioms of PM not in terms of the primitive symbols '$\sim$' and '$\vee$' but in terms of '$\supset$' and '$\vee$'. Now, if V.F. Rickey had read our paper a little more carefully he should have noticed that we have not fallen into the trap he accuses us of. In our brief paper we explicitly stated three times that our aim was to investigate PM *«as the axioms are written»*. Of course, we could have written the axioms of PM in terms of the primitive symbols '$\sim$' and '$\vee$' and eliminate the defined symbol '$\supset$' from the whole system. But then the question about the creative role (or any other role for that matter) of the definition for '$\supset$' would not arise. But if, for some reason, one has decided to use defined symbols in the axioms, as the authors of PM *de facto* did, then it is an interesting question to ask whether the definition for '$\supset$', though eliminable, does or does not play a creative role. In fact, our original paper can be understood as an argument in favour of writing the axioms of PM in terms of the primitive symbols '$\sim$' and '$\vee$', or at least, writing them in such a way that the logical status of the definition for '$\supset$' is not obscured. Thus V.F. Rickey's criticism here is totally unfounded.

III. Logical Error. First of all, we have to point out that the «logical error» V.F. Rickey speaks of, *does not affect* our thesis or proof but concerns only an example. Our newly written axiom system was given as an example where the definition for '$\supset$' is not creative. In this, however, we were mistaken. As V.F. Rickey showed in his model, the definition for '$\supset$' is still creative in this newly written system with respect to certain formulae and we are grateful to V.F. Rickey for drawing attention to this fact. But his remark only strengthens our thesis. We made our mistake because we assumed a stronger creativity which in fact obtains in regard to the definition for '$\supset$' in the system of PM «as the axioms are written», but does not in our newly written system. To give an explicit formulation of the distinction between what we may call *strongly creative* and *weakly creative* definitions we may say

Def. 1. A set of definitions D is *strongly creative* with respect to a proper fragment F of the classical calculus CL if and only if $F \subset (F + D) = CL$, where '$\subset$' indicates proper inclusion.

In other words, D is strongly creative if and only if the addition of D to F extends the fragment F to the classical calculus CL.

Def. 2. A set of definitions D is weakly creative with respect to a fragment F if and only if $F \subset (F + D) \subset CL$.

In other words, D is *weakly* creative if and only if the addition of D to F properly extends F but (F + D) still remains a proper fragment of CL.

Using this distinction, we may say that «as the axioms are written» in PM the set of definitions specified there is *strongly creative*; in our newly written system the set of definitions is not strongly but, as V.F. Rickey has pointed out, *weakly creative*; and if we write the axioms of PM in terms of the primitive symbols then the definitions are neither strongly nor weakly creative.

We can put our conclusion in a different way, which may be more illuminating. Since the logical connectives in a calculus are defined by the axiom system in which they occur, in any proper fragment F of CL, their syntactical role (or their meaning if we wish to give interpretation to them) must somehow be different from that of CL. A set of definitions D (concerning the connectives) is *strongly* creative if the addition of D to F changes the meaning of the logical connectives of F into that of the classical connectives. Roughly speaking, and in particular, the definition of '$\supset$' is *strongly creative* if D introduces the classical '$\supset$'. If however the set of definitions is such that it introduces a stronger '$\supset$' but not yet the classical one then we may talk about a *weakly* but not *strongly* creative set of definitions. In fact, if we prescind from the meaning of the connectives, then the axioms of PM as they are written are all theorems of Johannsson's minimal calculus. This was the original observation which led us to the insight that as the axioms are written, the definitions of PM are creative, indeed, *strongly creative*.

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REFERENCES

[1] G. KELLEY, General Topology, Van Nostrand, 1955, p. 251.

[2] S. LESNIEWSKI, Grundzüge eines neuen Systems der Grundlagen der Mathematik, in *Fundamenta Mathematicae*, vol. 14 (1929) p. 50. The relevant passage runs as follows: «Man kann ein System der Protothetik auf Grund eines einzigen Axioms aufbauen wenn man neben den Direktiven α_1, β_1, γ_1, η^*_1, des Systems Σ_4, die zwei Definitions direktive — δ^*_1, und ε^*_1, — annimmt, die statt der Definitionen, welche auf eine in den Direktiven δ_1, und ε_1, des Systems Σ_4 vorhergesehene Wiese konstruiert sind, Definitionen in Gestalt zweier einander reziproker und deshalb die entsprechende *eine* Äquivalenz vertretender Konditionalsätze (Direktive δ^*_1) *respective* zweier solcher Konditionalsätze mit den ihnen vorangehenden universalen Quantifikatoren (Direktive ε^*_1) einführen. Als ein Axiom dieser Art kann z.B. das Axiom 2 das *sub A* erwähnten Systems gelten.»
We are grateful to Owen Le Blanc for giving us this reference.

[3] E.Z. NEMESSZEGHY and E.A. NEMESSZEGHY, Is $(p \supset q) = (\sim p \vee q)$ Df. a proper definition in the system of *Principia*? in *Mind* (April 1971) pp. 282-283.

[4] E.Z. NEMESSZEGHY and E.A. NEMESSZEGHY, On the creative role of the definition $(p \supset q) = (\sim p \vee q)$ Df. in the system of *Principia:* Reply to V.E. Dudman (I) and R. Black (II), in *Mind* (October 1973) pp. 613-616.

[5] V.F. RICKEY, On creative definitions in the *Principia Mathematica, Logique et Analyse* (Mars-Juin 1975) pp. 175-187.

[6] P. SUPPES, Introduction to Logic, Van Nostrand, 1957, p. 153.