

PETITIO AND RELEVANT MANY-PREMISSSED ARGUMENTS

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We want to examine the question whether it is possible for arguments with more than one premiss, and no superfluous premisses, ever to beg the question. In order properly to formulate this question, it is necessary (i) to set out a general account of the conditions under which an argument may be said to beg the question, (ii) to explicate the concept of a «superfluous» premiss, and (iii) to adumbrate a certain historical dispute which throws interesting light on the nature of the question.

1. *Petitio Principii*

That arguing in a circle is an essential *informal* fallacy is easy to establish. Arguments of the form 'p, therefore p' always or nearly always beg the question, yet their formal validity is impeccable reflected in standard truth-functional logic. Thus there is a kind of dissonance, especially palpable in the paradigm 'p, therefore p', that pervades all arguments in which the conclusion appears as a premiss or conjunct of a premiss. Such arguments are both valid and fallacious.

We are recapitulate here from [7], our survey of what is standardly conceived to be the fallacy. In writings on *petitio* there are, broadly speaking, two main conceptions of this fallacy. According to the EQUIVALENCE CONCEPTION, an argument is circular where the conclusion is tacitly or explicitly assumed as one of the premisses, that is, where the conclusion is equivalent to a premiss. Copi [1], for example, writes (p. 83),

If one assumes as a premiss for his argument the very conclusion he intends to prove, the fallacy committed is

that of *petitio principii*, or begging the question. If the proposition to be established is formulated in exactly the same words both as premiss and conclusion, the mistake would be so glaring as to deceive no one. Often, however, two formulations can be sufficiently different to obscure the fact that one and the same proposition occurs both as premiss and conclusion.

The problem with the equivalence conception is that strict identity of premiss and conclusion is too narrow a criterion, reflecting only the obvious cases, and equivalence is too broad a criterion, ascribing circularity to many arguments which clearly are not circular. De Morgan [2] preferred the narrower criterion that, «strictly speaking, there is no formal *petitio principii* except when the very proposition to be proved, and not a mere synonyme of it, is assumed.» (254) Yet Sidgwick [5] was of the opinion that nothing «appears to be really gained by restricting the name to so small a compass as this; and there is no doubt that such a restriction would be very much at variance with the popular acceptance of the term.» (194) On the other hand, the broader criterion, demanding only that a premiss be logically equivalent to the conclusion, would appear to condemn single-premissed arguments such as

All philosophers are fragile

Therefore, no philosophers are not fragile

to hopeless circularity in every context. Of course, it is sometimes said that an argument is circular when to state a premiss is to state the conclusion (in appropriately epistemically rich sense of «state»). The trouble is, *does* one, in here stating the conclusion, in an appropriately rich sense of «state» also state the premiss?

According to what we call the DEPENDENCY CONCEPTION, an argument is circular when the conclusion is presupposed by a premiss or where some premiss actually depends on the conclusion, in the sense that in order to accept

the premiss one need first accept the conclusion. Normally the «flow of inference» in an argument is from the premiss to the conclusion.



Figure 1

But where it is also required that an inference be made in the other direction, from the conclusion to the premiss,

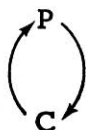


Figure 2

the argument, schematized in Figure 2, is circular. According to this conception, an argument is non-circular only if one may know that each premiss is true without having to infer it from the conclusion, or from some other statement that can be known only by an inference from the conclusion.

Two additional general points bear remarking on. First, some writers are inclined to think that begging the question is a purely *alethic* question, pertaining to a relation of logical entailment (as dependency) or logical equivalence between the conclusion and a premiss. Others (notably Whately [6], p. 179) explicitly recognize that circularity of argument is essentially an *epistemic* phenomenon, and that whether or not an argument is said to beg the question will depend critically on the informational or epistemic circumstances of the person to whom it is directed. Secondly, we have never seen in the logic texts an allegation of *petitio* where two or more premisses are said to jointly or severally beg the question. Whether cases of this sort are *possible* is of course a separate question that we return to below.

2. Mill: *The Syllogism is a Petitio*

Mill was concerned to differentiate between the mental operation of inference whose medium is belief, and the purely logical concepts of deductive proof and valid argument which are deemed timelessly to map truths onto truths in the doxastically neutral, characteristic way soon to be elevated to mathematical logic by Boole and Frege. The *System of Logic* is laden with Mill's attempts to grapple with this important distinction. Perhaps the most celebrated and provocative offshoot of Mill's preoccupation is his *dictum* that there is a *petitio principii* in every valid syllogism. The thrust of Mill's argument [4], p. 120f., is easily felt. Consider the archetypal syllogism,

- (1) All men are mortal.
 Plato is a man.

 Plato is mortal.

Clearly the first premiss presupposes the conclusion in the sense that we cannot be assured that all men are mortal, «unless we are already certain of the mortality of every individual man.» Thus if it is doubtful that Plato is mortal, or any other individual for that matter, then it is at least as doubtful whether all men are mortal. The point is that the general statement 'All men are mortal' cannot be offered without circularity as evidence of the particular statement 'Plato is mortal' when the latter is part of the evidence for the former.

Mill's *aporia* is no superficial puzzler. This can be seen from his ensuing discussion in which he observes that the typical reply of logicians involves an appeal to a distinction between what is *asserted* by the premisses and what is *implied* in the premisses. But is this distinction really all that clear? When Whately, for example, tries to deal with Mill's problem by claiming that when you admit the major premiss you assert the conclusion, but assert it by implication merely, does he mean that you asserted it unconsciously? This

is a fascinating preview of contemporary disputes concerning rationality-assumptions in epistemic and doxastic modal logics, but we will not here pursue it further. We turn instead to the response of DeMorgan.

3. DeMorgan's Rejoinder

DeMorgan, that staunch defender of formalistic methods, was inclined to argue for what we will now call the Equivalency Reduction Thesis, namely, that all cases of *petitio* can be ultimately reduced to the case where the «very proposition to be proved, and not a mere synonyme of it, is assumed.» ([2], p. 254) His preference for the equivalence conception is not merely aesthetic—we will see further below that DeMorgan offers what amounts to an ingenious and deeply based defence of this apparently implausible position.

DeMorgan opens his discussion of Mill's conundrum by pointedly ruling that the definition of *petitio* refers to what is assumed in one premiss (257): «The most fallacious *pair* of premisses, though expressly constructed to form a certain conclusion, without the least reference to their truth, would not be assuming the question, or an equivalent.» This thesis of a connection between *petitio* and the number of premisses is not today commonly known or recognized, even in standard treatments of the fallacies, but it had been clearly stated before DeMorgan [2]. Whately [6], p. 179, writes: «[P]etitio principii ... takes place when one of the Premises ... is either plainly equivalent to the conclusion, or depends on that for its own reception. I have said «one of the Premises,» because in all correct reasoning the two premisses taken together must imply and virtually assert the conclusion.» Yet because Whately only mentions the thesis in passing, and because DeMorgan was apparently the first to attempt to establish it in a general way ⁽¹⁾, we formulate the following principle

(¹) Though there is no doubt that the point (or something very like it) is adumbrated by Aristotle in the *Posterior Analytics*, 73a 10.

DeMorgan's Thesis: No syllogism begs the question.

A syllogism is classically required to have exactly two premisses, and it follows that if it has exactly three distinct terms each of which occurs twice, one in both premisses and the remaining two in one premiss and in the conclusion, then neither premiss will be superfluous. Of course, the restriction of *DeMorgan's Thesis* to specifically syllogistic arguments is a bit archaic, and one is tempted to formulate the thesis more generally. We will offer a generalized version below, so let us dub the above statement, '*DeMorgan's Weaker Thesis*'.

DeMorgan's defence of his Thesis against Mill's objection^(*) that all syllogisms are dependency-circular bears careful scrutiny. The argument is interesting, but lengthy and involved, and we merely sketch the thrust of it here. The objection, writes DeMorgan, «tacitly assumes the superfluity of the minor; that is, tacitly assumes we know Plato to be a man, as soon as we know him to be Plato.» In other words, Mill's claim that 'Plato is mortal' is part of the evidence for 'All men are mortal' assumes that Plato is a man and not, for example, a dog. But this very assumption is in fact the minor premiss. As DeMorgan puts it: «Grant the minor to be superfluous, and no doubt we grant the necessity of connecting the major and the conclusion to be superfluous also. Grant any degree of necessity, or want of necessity, to the minor, and the same is granted to the connection of the major and the conclusion.» (p. 259) This is an extraordinarily interesting argument. If correct, it may serve generally to establish that multi-premissed arguments without superfluous premisses never beg the question. Accordingly, we turn now to the question of establishing a condition that will allow us to rule on the question of when a premiss is «superfluous». Of course, in a properly constructed syllogism, there can be no superfluous premiss, but for the contemporary reader the

(*) Mill is not mentioned by name. DeMorgan concedes that there is «much ingenuity» in the argument, but he does not credit it to a specific source.

real interest of DeMorgan's Thesis lies in the possibility of its application to first-order logic.

4. *Relevant Arguments*

We propose, following Lehrer [3], that a *relevant deductive argument* is a valid argument in which knowledge of the truth of each and every premiss is required to establish the truth of the conclusion by deduction from the premisses. A *minimally inconsistent set* of statements is an inconsistent set (a set from which a contradiction may be deduced in standard logic), every proper subset of which is consistent. Some inconsistent sets remain inconsistent even when some statement is removed from the set. Others, the minimally inconsistent sets, are such that once any statement is deleted, the remainder is consistent. An argument is a *relevant deductive argument* if the premisses and the denial of the conclusion is a minimally inconsistent set. On the basis of the above pair of definitions, Lehrer shows in [3] that: (i) arguments with contradictory premisses are not relevant, (ii) valid arguments with superfluous premisses are not relevant, and (iii) no argument with a valid conclusion is relevant (or, at any rate, no such argument *with premisses* is relevant). In effect, (i), (ii), and (iii) exclude all arguments except deductively valid ones such that (1) the premisses can be true together, (2) each premiss can be false, and (3) the conclusion can be true and can be false. Hence we have that relevant arguments may contain only contingent statements. On the basis of the foregoing definitions, we now set out the generalized form of De Morgan's Thesis.

DeMorgan's Thesis: No relevant argument with more than one premiss begs the question.

We think that this thesis has an interesting degree of initial plausibility, and that it may even be true. There are relevant one-premissed arguments that beg the question, such as at least some of the form, 'p, therefore $p \vee q$ ' or 'p, therefore

p¹. And there are multi-premissed irrelevant arguments that beg the question, such as 'p, q, therefore p'. But it is not so clear that there are not relevant multi-premissed arguments that beg the question, given the adequacy of DeMorgan's reply to Mill. However, let us look at two possible counter-instances.

5. *Disjunctive Syllogism*

One counter-instance to the general formulation of De Morgan's Thesis seems to arise from the possibility of the circular use of the disjunctive syllogism. Suppose some sophist makes it plain that he wishes his victim to infer the disjunctive premiss from the conclusion, in order to evade the obligation of providing independent evidence for that premiss, as schematized below.

$$\begin{array}{rcl}
 (2) & p & \vee \quad q \\
 & \sim p & \\
 \hline
 & q &
 \end{array}$$

Transparently evasive, such an argument would perhaps not be very convincing to a sophisticated audience, but if such an argument were proffered, it would clearly constitute a classical *petitio* of the dependency variety. Here we seem to have a counter-instance that evades DeMorgan's defence of syllogism. In the syllogism discussed by DeMorgan, the remaining premiss was required to establish the fallacy-producing dependency relation, whereas here it is not, since ' $\sim p$ ' is not required to complete the entailment from q to ' $p \vee q$ '. Apparently, what we have here is a two-premissed relevant argument that begs the question, contrary to De Morgan's Thesis.

The tenor of DeMorgan's remarks indicate quite clearly how he might well have responded to this kind of case, employing what we will call *DeMorgan's Defence*, as follows.

The counter-instance posits, in effect, not one but *two* arguments.

$$(3) \quad \begin{array}{c} p \vee q \\ \sim p \\ \hline q \end{array} \quad \begin{array}{c} q \\ \hline p \vee q \end{array}$$

The conclusion of the argument on the right is identical to a premiss of the argument on the left. Thus what we have amounts to a combined pair of arguments into a *sequence of arguments*, where the conclusion of the first argument appears as a premiss in the second.

$$(4) \quad \begin{array}{c} q \\ \hline p \vee q \\ \sim p \\ \hline q \end{array}$$

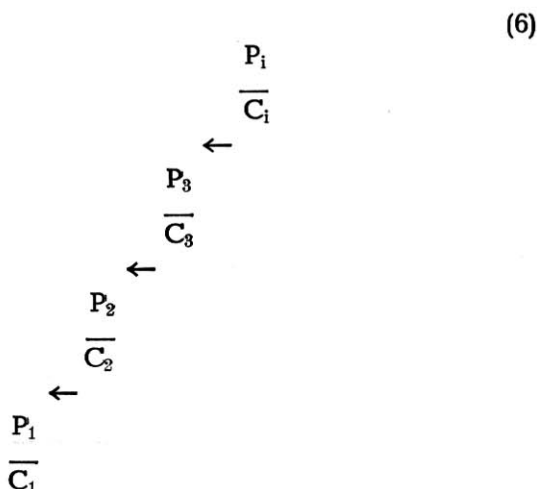
Essentially we have one extended argument with three premisses.

$$(5) \quad \begin{array}{c} q \\ p \vee q \\ \sim p \\ \hline q \end{array}$$

But the second and third premisses are superfluous. The second premiss follows deductively from the first, and the third premiss is superfluous for establishing the conclusion, given the first. Indeed, any premiss would here be superfluous, given the first, and in fact (5) is obviously a classical case of straightforwardly logical, explicit equivalence-circularity, DeMorgan's preferred type.

The Defence makes an assumption worth stating, namely, that in an array of arguments such as (3) the ones on the right

can be absorbed within the ones on the left. The general scheme that emerges is that the «evidence» for a premiss can take us back to the conclusion of a previous argument, and that this step may be repeated.



In any argument, the premisses may drive us back to further argument. The required ruling to ban *petitio* in such a framework is that no C_i must appear as a P_j ($j \geq i$). That is, the conclusion of the initial argument, C_1 , must never appear as a premiss, as the argument expands out to the right and upwards.

The prior question that De Morgan's Defence turns on is whether a sequence of arguments, such as (6) above, is an *argument*, or whether it is not, properly speaking, a single argument but a plurality of arguments. To introduce some helpful vocabulary, let us distinguish between an *atomic argument* and a *molecular argument*. An atomic argument is an ordered pair $\langle P, C \rangle$, where C is a statement (the conclusion) and P is a set of statements (the premisses). A molecular argument is a finite sequence of atomic arguments, which may also be called the component arguments of a molecular argument. In a molecular argument, a premiss in the compo-

nent argument may also appear as the conclusion of a previous argument. Given a reasonable-seeming and obvious extension of the notion of relevance to molecular arguments, we have it that no molecular arguments are relevant, for at least one premiss (the conclusion of the next argument to the right) follows deductively from some other premisses (the premisses of that very argument on the right). Thus no molecular argument can ever constitute a counter-example to De Morgan's Thesis. We also have it that if the disjunctive syllogism under-represented by (2) «really» is a molecular argument, it can constitute no threat to DeMorgan's Thesis. Thus the general framework suggested by (6) serves to reveal a third thesis that provides a linkage between the Equivalency Reduction Thesis and DeMorgan's Thesis: any circular argument, if expanded out far enough to the right, will eventually reveal a premiss identical to the conclusion, and thus will become single-premissed when all superfluous intervening premisses are deleted.⁽³⁾ The two parts of DeMorgan's treatment of *petitio*, although separately perhaps not very plausible, jointly interlock to form a perspective that is both ingenious and deep.

6. *Conjunctive Conclusions*

Another apparent counter-instance to DeMorgan's Thesis may be found in an argument of the form below that begs the question.

$$\begin{array}{rcl}
 (7) & & p \\
 & & q \\
 & \hline
 & & p \ \& \ q
 \end{array}$$

Arguments of this form need not always beg the question,

⁽³⁾ Of course, whether the resulting argument is single-premissed will depend on which superfluous premisses are deleted. For example, argument (5) has two non-superfluous premisses when its first premiss has been deleted.

we suspect, but it is extremely plausible that often they do. Yet such an argument is both relevant and multipremised. The appropriate strategy for preserving the Thesis against the threat of (7) and its kind might be called *DeMorgan's Zwischenzug* (in-between-move): no (single) argument may have a conjunctively molecular conclusion. According to De Morgan's *Zwischenzug*, (7) becomes the pair of arguments,

(8)

$$\begin{array}{cc} p & p \\ q & q \\ \hline p & q \end{array}$$

No harm appears to accrue from this move, since of course the pair above are both valid, just as the original two-in-one argument was valid. Some perplexity is generated by the *Zwischenzug* in other cases, however, for it appears that an argument of the form,

(9)

$$\begin{array}{c} p \\ \hline p \ \& \ q \end{array}$$

is not simply an invalid argument but really a pair of arguments, one of which is valid (and circular), the other invalid.

(10)

$$\begin{array}{cc} p & p \\ \hline p & q \end{array}$$

This consequence is hard to swallow. We can, and customarily do, tolerate the interchangeability of the following pair of forms,

$$\begin{array}{rcl}
 & & (11) \\
 p_1 & p_1 \ \& \ p_2 \ \& \ \dots \ p_i & \\
 p_2 & \hline & C & \\
 & \cdot & \\
 & \cdot & \\
 & \cdot & \\
 p_i & & \\
 \hline & C &
 \end{array}$$

But the same conjunctive flexibility with respect to the conclusion of an argument must not be allowed. One conclusion per argument is the standard upper limit, and thus it would seem that DeMorgan's *Zwischenzug* leaves logic bereft of (7) and its kind altogether. If it is not feasible to deal with (7) as more than one argument, the only option that remains is to declare it no argument at all. This alternative is not very palatable either, and hence the *Zwischenzug* is blocked.

Of course, if DeMorgan's Thesis were not restricted to multi-premissed arguments, the way out would be obvious. (7) could be recast as

$$\begin{array}{rcl}
 & & (12) \\
 p \ \& \ q & \\
 \hline p \ \& \ q &
 \end{array}$$

And the *petitio* here is manifest. But if the multiple-premiss restriction is dropped, relevant arguments such as 'p, therefore p' and 'p, therefore p v q' destroy any semblance of plausibility such a strengthened Thesis might seem to possess.

7. Concluding Remarks

We conclude that DeMorgan's Thesis is an extremely interesting object for further study. It could be over-sanguine to suppose that either of the two counter-instances we have de-

veloped constitutes a conclusive refutation of the Thesis, but it would be cavalier to dismiss them as easily surmountable. We commend the problem of the confirmation or refutation of DeMorgan's Thesis to serious students, if any there be, of that sadly understudied but pedagogically important domain, the informal fallacies. ⁽⁴⁾

Let us add that even if DeMorgan's Thesis turns out to be true, it would be incorrect, if not fallacious, to conclude that a purely alethic criterion of question-begging would automatically be vindicated. For significantly, the concept of relevance, as we have outlined it following Lehrer, is essentially epistemic. We would also maintain, as we have done in [7], that the weight of evidence supports the hypothesis that the notion of argument appropriate to the *petitio* is essentially epistemic in character. But that is another story.

⁽⁴⁾ Not to overlook the welcome and useful efforts of C.L. Hamblin to reverse this trend. See Hamblin [8].

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