LOGICS

William Sacksteder

Our talk *about* logic, or our theories and justification for logic itself, cannot be «logical» in the same direct sense which that logic stipulates as a requirement for any arguments or contents which are to be ruled by it. It is for this reason that Aristotle's *Organon* is neither syllogistic nor inductive in procedure, but rather dialectical like the *Metaphysics;* and that Dewey distinguishes between the proximate and the ultimate subject-matters of logic; and Kant, between Transcendental and General Logic. It is also perhaps for a related reason that modern logic cycles around problems of self-reference or depends on what one commentator calls its «political philosophy;» that for Hegel, logic generates an entire universe in endeavoring to become more than *an sich;* and that Plato both supplies no logic and is also, so to speak, logic's grandfather.

I wish in these pages to contribute to «talk about logic,» to «meta-logic,» to the «philosophy of logic,» or to whatever — according to your preferred title — stands outside of logic and looks at it, thereby failing to exhibit its canons simply or to employ it strictly. My effort will be to define it, and that in old fashioned senses whereby I shall try to locate it among other sorts of concerns, to suggest delimitations from kindred efforts, and to capture its ruling characteristics and variants. Such aims are blatantly prescriptive and programmatic. For present expository limits, they require arrogantly doctrinaire and pointed presentation. Accordingly, I shall propose a series of «positions,» supported with rather sparse and merely explanatory commentary. These positions are intended to set forth starkly one viewpoint as to what logic is, its separation from other concerns, and how it rules other things or is ruled internally or in its variations. Each of them is itself, despite rigid statement, merely a proposed view-

point, a stance from which we might look in a certain way at some aspect of logic. Nor — despite cross-references and stylistic connections — is the interdependence among them intended to be rigorous or to be displayed as proof. I hope that they cohere, and that my citations from one to another are helpful. But their interweaving could be made «deductive,» or in some sense logically necessary, only after the fact, so to speak: after we had decided among ourselves just what a logic is and what it is good for; whereas here, I invite you to pretend for the moment that we do not yet have such insight at all. It is thus that you might entertain my positions as one viewpoint, seeing what can be seen by trying their adoption. It is only thus that they might contribute to our talk about logic.

Aside from niceties of expression, or suggestive loading, the only contribution to be proposed here which might seem novel is the 's' in the title. For, whether by naivete or sophistication, we more naturally speak of logic in the singular, as I have in these opening pages. We presume that there is only one logic, could we but find it, which properly governs our discourse. But to the contrary, I shall argue that a proper understanding of what a logic is, as I shall announce in my first position, coheres suggestively with the position, which will form such climax as this paper can muster, that there are many logics.

1. *By a LOGIC I understand a structure permitting new arrangements connecting elements in interesting ways to be derived from given arrangements according to some repeatable rule of transformation.*

Any item in our experience can be an ELEMENT in a logic, if it can be so connected with other items that we take some interest in the connection by way of interpretation. For most practical purposes, the elements are words which are given a reference; or sounds and print standing in turn for words. But they might be points, for example, in a non-verbal logical

structure such as a map. They might also be gestures and movements, as in bodily communication. They might also be numbers or electrical impulses or beads on an abacus. It is the fact that we can manipulate the elements as tokens which makes them amenable to logical uses in our discourse. We control their connections, in the expectations that others can thence interpret them. We are disinterested in their own qualities as things, or we require only that they be observable by others and that they are such that we can push them around with relative freedom. It is the arrangements, not the elements, which concern us.

In a logic, we are interested in those CONNECTIONS among elements which can be understood to correspond with relations in the world in some accordance with the arrangements we are able to impose on them. For standard logical purposes, the usual sorts of connections are those which can be set forth in a proposition, because it connects verbal forms which are subject to arrangement and rearrangement by rule, and which we are able to understand as saying something about the state of the world. But notice that we form connections also by relative location on a map, say with points higher on a page corresponding to a northerly direction. Arrows, pictograms and gestures like pointing have the same connective function. So also does location in an equation. Connections interest us because they locate in this way and because they are ingredients in arrangements which can be altered by us without losing their correlation with arrangements among facts.

The elements and connections which enter into a logical inference are only those which are susceptible to certain kinds of arrangement. A map, since it consists of spatial arrangements, connects shapes and points according to alterations of direction or distance. We do not make a map of love, unless very figuratively, say in a poem. A botanical classification, since it consists of including arrangements, connects types and membership according to sequences and properties. Even an analytic geometry is only possible by pursuing some added metaphor which relates points and coordinates.

Hence both elements and connections are taken for pur-

poses of logical construction according to the arrangements into which they can enter. An ARRANGEMENT is a set of connected elements which is alterable according to rule without loss of some anticipated referential identity. Any inferential rule connects such arrangements and may accordingly be understood as being itself an arrangement which we might in turn alter according to another rule. The De Morgan transformation, for example, notes two arrangements, $— (p \text{ v } q)$ and $(— p \,.\, — q)$ to be equivalent despite the different connectives and elements, so that we may transform formulae having either arrangement or make substitutions within formulae containing either. The whole rule itself employs the connective, '≡,' and is an arrangement which can itself be subjected to rearrangement or transformation according to rule. Syllogistic formulae are likewise arrangements, although not so flexible. Mathematical operations may be understood similarly. So also may the shape of Africa on a map, that shape being an arrangement of lines, points, and mutual locations. It is likewise subject to rearrangement according to rule, which we do when we enlarge the scale for an inset map, or when we make a new map having an alternative projection system. Any calculator or computer likewise arranges mechanical gimmicks and their push-pull (or electrical information and their recurrence) in ways which duplicate or substitute, altering connections among the elements it has been programmed to receive. A first grader surreptitiously counting on his fingers is engaging in the same arranging and rearranging of counters and the ways they are connected. He has invented a minimum computer.

A logic is thus formed from arrangements subject to rearrangement according to rule. Elements and connections are ingredients in these arrangements. The former two serve to the extent that they enter into the latter and are subject to their manipulations. But the arrangements which constitute a logic are those which we can transform according to rule.

2. *A logic is characterized by its mode of transformation.*

The indications given concerning the ingredients of a logic allow for the traditional reduction of such factors to terms, propositions, and inferential rules. However they do so with a reordering whereby the third factor is taken as including the first and second, so that these may be regarded as whatever might be instrumental within it. Thus the other two can be understood more flexibly than is usual, in order to include anything whatsoever which might enter into the arrangements and rearrangements by rule, as that third requires. Elements are whatever can enter as tokens into an argumentative communication; connections are whatever juxtaposition of elements can constitute interpretable aspects of arrangements; arrangements are those organizations which are subject to transformation by rule without loss of identity.

The same inversion follows concerning any materials or interpretations to which these ingredients are applied, whether by way of arguments to be assessed, objects thought to correspond, or calculative schemes we invent. And conversely, it is consequent to this inverted order that I am able to cite as examples of a logic various systematic orders which are non-verbal and which we might be hesitant to call a «logic» in any strict sense. Thus a map is composed of arrangements of elements connected diagrammatically. The arrangement among these has some correspondence to select elements within a range of fact, that is, it has «meaning.» We are able to communicate and argue concerning that meaning by understanding the factors — points, diagrammatic connections, and shapes — according to a rule of transformation within the structure and consequent rules for translation and identification relating to the portion of the world which is mapped. A map embodies its own logic, which we utilize in argumentative fashion. The same might be said, although without so close an approximation to rigor, of the arguments of gesture; and also, although without so rich or direct a reference to experience, of the devices of computation. Other inclusions will emerge later. These include modern «relational» logics, important di-

mensions of mathematics and sciences, and the current talk about the «logic» of a familiar expression as we ordinarily use it. In the present sense, I also argue that there is a logic in a slot machine, a Sears Roebuck catalogue, a pecking order, and a legal codification. We may further inquire what profundity might underlie writing concerning «The Logic of God» or calling the plot-sequence of a drama «the argument of time.»

In all of these cases, at least to the extent that they are properly called logics, we are concerned with structures within which new arrangements connecting elements in interesting ways can be derived from given arrangements (1). Our purpose on its logical side is to make such derivations. We concoct elements, connections, and arrangements such that we can transform them by our manipulations. Such an action is accomplished according to what we take to be a rule. A way of acting is taken as a rule when we suppose that we might act according to it repeatedly or that we might mistakenly fail to act according to it. There is thus an exact correlation between modes of transformation and the strictly logical ingredients which they are understood to rule.

A TRANSFORMATION is an act whereby an arrangement is altered in a manner which can be repeated as if according to a rule exemplified by the alteration. By such an inferential act, we convert one arrangement into another, a rearrangement. But we do so neither randomly nor willfully, but rather «logically». We proclaim that the derivation is justified, being either required or permitted by the rule. Hence a given transformation is an inference exemplifying a rule, or at least we pretend that it is one. Or a transformation taken as a proper way of acting on arrangements is itself a rule. A rule is a mode of transforming arrangements of the kind proper to it.

If the logical factors or their applications differ in kind, the rules for transforming them will differ accordingly. When we act by transformations taken as a rule, the kind of rearrangements in which we engage can be only such as accord with transformation rules of that certain kind. For example, when I take arithmetic as governing my check book, my calculations set aside my desires or interesting qualitative characteristics

of my purchases. I am limited for these manipulations to the countable factors which are subject to just those kinds of transformations, namely addition and subtraction of numerical and quantitative units. My logic has its systematic character according to the modes of transformation I have adopted. Its included arrangements and its external applications are just those which correspond to the operational rules I employ. The items and relations in that financial world to which I apply my calculations are selected indirectly by the structure of rules I am employing; and so also are the kinds of accuracy and error I might detect by consulting these rules.

Elements and connections and their interpretations and uses are acted on indirectly, according to the arrangements into which they enter. It is arrangements which are directly altered by a transformation rule: we make rearrangements, or we derive new arrangements from given arrangements. Hence it is that the arrangements in question are just those which are permitted by the rule; and the rule in question is just that which governs such arrangements. The structures ordering both rule and instance are correlated. When we take the structure as logical, we understand them to define each other mutually. That is, a logic (and any contents or applications, internal or external) is characterized by its mode of transformation.

3. *A material logic is characterized by its prevailing mode of transformation together with an interpretation of it.*

Any ARGUMENT is an inferential act whereby we derive something in which we are interested from something else which is given, doing so in a manner which we profess corresponds to a rule. Being an act, it is a motion, so to speak, from this to that, a replacement of one condition or thing with another. Presumably in any specific instance of argument we have a special interest, in that case, in whatever is derived after this fashion: we want to get to that result. However, we do not wish to get there by any random route. In an argu-

ment, we wish to make the derivation from just the beginning points we propose, and by a manner of action which we claim corresponds to a rule. An argument is inferential in replacing something given with something derived in a manner claiming to be correct.

Such justification as an argument attains is supplied to it by a logic. For a logic is a structure permitting derivation according to a rule (1). Our arguments claim to conform to such a structure, so that the result which interests us derives from the given in a way which duplicates the manner of replacement supplied by the logic. Our act of replacement pretends to repeat a transformation which is a rule. The logic is characterized by its mode of transformation (2). That is, the manner or type of replacement is taken as the rule which we hope that that logic supplies to a given argument.

We usually anticipate that the inferential acts in which we engage as we pose arguments concern themselves with items outside our logic, in the world about which we argue. A logic's uses and bases are composed of such items, and we usually pride ourselves that our arguments are not merely verbal, or manipulations of symbols. We hope that its devices carry with them both the furnishings of the world and our thoughts about the world. In manipulating them, we suppose that we are referring to locations, quantities, and characteristics of things, and to our own mental tools for assimilating ourselves to them. In creating a logic, such reference is taken for granted in two sorts of justification for the logic itself, that whereby we take it as applicable to further contents, and that whereby (in the first place) we form it in anticipation of such use.

Any logic may propose several rules of transformation, according to the derivations we wish to attain or the data we have at hand. Hence it may supply several methods, each with its appropriate kind of exemplary contents. Often its methods are inverse to each other, or approximately so, in order that we may move in inverse directions between two opposite but commensurable kinds of given and derived contents. We so move in the case of inverse operations such as

addition and substraction, inverse methods may also permit us to move in inverse directions between distinct but corresponding levels of presumed objects, as it is sometimes said that induction and deduction move in inverse directions between universals and particulars.

It is usually employment of a logic by way of application which interests us, whether we manipulate contents or assess arguments. By the APPLICATION of a logic, I understand our consideration of some range of materials other than the logic itself to be instances which its transformations can be taken to govern as rules. I shall call a logic so applied a material logic, to distinguish it from the merely self-ruling aspect of it, which I shall call a formal logic. By a MATERIAL LOGIC I understand a formal logic together with an interpretation of its factors such that its rules are taken to govern matters other than its own structure alone. Any use of a logic by way of application to some materials outside itself is a material logic. As such, it contains two aspects. It includes a formal logic, as a structure ruled solely by transformations, of which more later. But it contains also interpretive understandings whereby that logic is applied, for a logic can rule matters beyond its structure only by means of a certain commensurability between that order and the structure of whatever matters we wish to treat as contents for it. This commensurability is the isomorphism between the two structures and their included arrangements, connections, and elements. It is indicated by an interpretation of these ingredients. By an INTERPRETATION I understand citations showing that the arrangements, connections, and elements of one structure which is taken as a rule correspond with those of another containing contents or instances of that rule.

A material logic supplies a rule to many sorts of argument. By it our arguments are enabled to cope with many very different sorts of objects, making replacements by derived results in which we have various sorts of interest. It also supplies various ways in which those replacements can be understood as exemplary of a rule. Nonetheless, to the extent that the argument claims to be logical, or according to rule, it demands

that whatever contents it may take as given or derivable, these contents must conform to the arrangements of a material logic. Our argument claims to make replacements among them by some manner of derivation, some method, which is supplied by that logic. Our arguments are thus justified by appeal to a material logic as ruling them. It rules by means of a METHOD, or a way of acting proposed as a rule for arguments; it is exemplified by application to a variety of contents which are assimilated to GIVEN and DERIVED arrangements corresponding to that method.

Whatever the contents, we replace given arrangements with arrangements according to a method. The given and derived arrangements must be so interpreted that they can be correlated with proposed contents. The modes of transformation must be so interpreted that they supply ruling methods or one method prevailing among a set of methods. These correspondences are assured by an interpretation showing that the structure of the logic is isomorphic to the structure of the contents, at least in respects permitting an interpretation to be implemented. Hence a material logic is not sufficiently characterized by its mode of transformation alone. A complete characterization includes also the interpretation whereby that mode of transformation is understood as a methodical rule, and its arrangements are understood to be given and derived contents exemplifying that rule. Accordingly, the material logic is characterized by its mode of transformation together with an interpretation which explicates its structure and that of any contents to which it might be supplied.

4. *Many maerial logics are possible.*

I shall not pause long over this position. It does assert the plurality of logics which is to be the principal burden of this paper. But it does not thereby refer to the case respecting which that position is either surprising or interesting, namely formal logics. It is formal logic that we hope or presume should be one, at least in some ultimate canonical sense. For

we cite it as ruling and assessing our arguments, or as that one format to which any argument must correspond. Whether formal logics are plural in more than accidental senses is the question to which I direct these remarks. I cite the many material logics only to leave them aside as a case which is readily granted.

In both the application and the formation of a material logic, we look outside the logic to the interpretive uses which may be made of it and to possible contents which suggest its arrangements. It is characterized by its prevailing mode of transformation together with an interpretation of it (3). Such interpretation enters into both the methods which are to be followed as rules and the given and derived contents which can exemplify them. That is, they are additions enabling application of a formal logic to materials outside itself.

But such interpretations are as manifold as our various interests in forming logics together with the various contents to which they may be applied. When we turn from merely formal considerations, we are interested in proofs or refutations with which they might guide our controversies and inquiries; or we are interested in deriving new facts by reasonings designed to take them into account according to some relatively exact aspect of their interconnections; or we are interested in multiplying schemes of possible inter-relations among objects imagined in some limited aspect to permit inferences from rules. Moreover, we may undertake any or all of these studies with further interests: sometimes in extending our knowledge, sometimes in forming coherent guidance for our actions, sometimes in perfecting the tidiness, richness and accessibility of our presentations.

The materials to which we might wish to apply a logic are quite diverse also. We may be concerned with the conformity to rule of some aspect of events and objects which we find in experience; or we may be concerned with the assessment and sharpening of the arguments we use in controversy and proof; or we may be concerned with the exact organization of some imagined kind of calculator or an imaginary order of possible objects. But in all of these cases, irrespective of

the diversity of materials, we have an interest in something other than the logic itself. The more inclusive considerations which we raise include some extraneous materials.

This plurality of material logics, however, affects only its many interpretations and employments by way of application. A material logic is two-sided. It looks in the direction of such applications, and the interpretive correspondence which makes them possible. But it looks also in the inverse direction, toward the structure which governs it. It is with reference to this structure that we call it a logic at all. Part of its characterization is the mode of transformation which constitutes its formal part. For it is in virtue of that formal part that it forms a rule, or a logic in a proper sense. Hence it is that included formal part which we expect to be one and to form a rule.

5. *Any material logic contains a formal logic as a ruling part.*

Whenever we make use of a logic by way of applying it to some materials outside itself, we take that prior logic as given. We use or apply or interpret it as itself being characterized by its mode of transformation (2). That is, it has a structure, and we presuppose or pursue the analogy between that structure and another which we think is an aspect of some range of materials. However, in taking it as given, or as pre-formed, we are supposing that its structure and rules are already contained within it, irrespective of any further uses and application. That is, we take it to be a formal logic. By a FORMAL LOGIC I understand a structure solely as formed by indefinite repetition of transformations taken as rules formative of its own arrangements. A formal logic has a sort of an «application,» namely to itself. But such application is reflexive, or internal, insofar as we consider a logic as formal merely. Yet it is a logic. It is a structure permitting transformations according to rule. That structure we may cite as resembling the structure of some range of external or other materials, when we form a material logic which includes it.

In forming a material logic, in whatever way, we not only take its formal part as given: we also take it as supplying the rules which govern both the formal and the interpretive structures. Thence we apply the material logic as a rule to any matters to whose structure we can assimilate it. It might be that the transformations which generate he formal logic have been originally suggested to us by matters outside it. Or we may have made them up in a fit of inspiration. Or we may have invented them by metaphorical ingenuities. But irrespective of such origins, our formation of a material logic consists in supposing that from such suggestions we formulate rules which govern both the formal structure generated and the organization of matters to which we apply it. By taking them as such rules, we may engage in the manipulations of a formal logic, and our manipulations may carry for us such external matters as have structures conforming to the same rules. That is, we take the included formal logic as a ruling part.

6. *We form a formal logic by ARTICULATING the structure of transformations whereby arrangements within it are both formed and ruled.*

On very rare occasions, we attend to a formal logic for its own sake, rather than providing or using some application of it. Or — perhaps with greater frequency — we intermittently consider the logic merely as a formal rule, alternating an effort to perfect its formal manipulations with trials of their applications or tests of the refinements we have introduced into them. On those rare occasions or during these interludes, we are concerned with the articulation of a formal logic. By the ARTICULATION of a logic, I understand our consideration of its formal part alone as showing transformations whereby arrangements within it are both formed and ruled. The result of such consideration is a formal logic, which in turn might (or might not) form the ruling part of a material logic.

It is important to notice that in such consideration the arrangements are taken to be *both* formed and ruled by the transformations. Indeed these are the same supposition, for the arrangements which are formed by the transformation as a rule are identical with those arrangements which are governed by the rule. At least in the formation of a logic we are limiting ourselves to such identical arrangements. We do not take others into account, or we do so only as a trial effort to show that they too are formed and ruled by the transformation. For it is in isolating or presenting that transformation both as ruling and as formative that we are engaged.

A logic is characterized by its mode of transformation (2), and hence we form any logic by showing its transformations to be rules governing (directly, its own arrangemets and (indirectly) any contents to which it might be applied. Articulation is thus the way in which a formal logic is formed. It is accomplished by two processes, namely that whereby we disperse its rules into those arrangements (and only those) which we think presuppose its transformations as rules; and the inverse process whereby we gather and express its rules, pursuing them from among the arrangements which are taken to be generated by them. In this elaboration, we are interested both in finding as many arrangements as possible which we can regard as produced by the logic alone and in as adequate as possible an expression of it as a set of rules generating these arrangements. Under neither heading do we attend to any actual contents or assessments to which the logic might be applied. That is, we are forming a formal logic, and only that.

We generate arrangements by indefinite repetition of the activity of transformation which we are taking as the rule. By this indefinite repetition, we display many transformations as being made possible by the rule, and we disclose those elements and connections which are consequent to the arrangements and ingredient to them. We disperse rules into arrangements and we gather rules from arrangements. We set the machinery in operation, so to speak, in order to see what products it will crank out. We do something of this sort, al-

though not purely formally, when we are learning mathematics, performing manipulations (or simply working problems) for the sake of learning to act according to the rules and accustoming ourselves to the sorts of new arrangements which they generate. A somewhat more sophisticated case occurs when we seek to exhaust the possible consequences of a rule by constructing a complete inventory of resultant cases, noting, for example, that triangles on a plane may be acute, obtuse, or right only, in virtue of the rule concerning the sum of the angles. We often indulge such elaboration of possible arrangements presupposing the rule in learning a formal logic already formulated. More sophisticated efforts to invent one which is complete and consistent often do so systematically, though usually with an eye to expressing the rule more adequately. Often we pursue the rule to that limit at which repetition of the operation becomes impossible or is completed to perfection. We also scrutinize the conflict between two or more rules in pursuit of a more fundamental rule which unifies a set of them and establishes the range of each. In these latter cases, our attention has shifted from an interest in generating arrangements from rules toward gathering and codifying the rules governing arrangements.

The two inverse processes whereby we articulate a logic are both conerned with the structure of that logic. They articulate that structure in the sense that they locate the joints connecting its members together as one organization, thereby showing the limbs and sockets whereby its processes are enacted and reenacted. We display it as a structure, activated by transformations, when we gather these transformations into rules directing their manner of operation. We display the structure as a set of arrangements generated by transformations, when we disperse them into resultant arrangements. In both cases, although they are inverse, it is the structure which is articulated as transformations whereby arrangements within it are both formed and ruled.

7. A formal logic is characterized solely by its prevailing mode of transformation.

A formal logic as such, and apart from its use as the ruling part of a material logic, has no interpretation. It is *only* the structure formed by articulating transformations whereby arrangements within it are both formed and ruled (6). It has no contents, other than its own structure or the set of its own arrangements. Accordingly, what is derived and what is given cannot be arrangements interpreted as contents: the beginnings and endings of inferences are only uninterpreted arrangements. They are not examples of the rules or better, we might say somewhat circuitously, that a formal logic exemplifies itself in its own arrangements. Similarly, a formal logic supplies no methods, other than its own rules of transformation. The inferential acts it contains may be used as methods to justify or refute proposed arguments. But its own inferences are solely movements on the structure formed by articulating its transformations. That structure is formed by repetition (as if by a rule) of ways of transforming its arrangements. They are not methods or rules for argumentation; or better, we might say, again somewhat circuitously, that a formal logic rules itself by its own modes of transformation.

It is the peculiarity of a formal logic that it lacks an interpretation: it is self-referring and self-generating. Hence its formation consists in articulating the structure of arrangements wherein its transformations rule and are exemplified. The summary or prevailing mode of transformation in the sole criterion employed, and hence it is sufficient to characterize the logical structure formed by it.

Since a formal logic governs and exemplifies itself, it has no contents other than the arrangements generated by its rules. Its rules are only those modes of transformation which generate its structure. The items «given» and «derived» in a formal logic are only arrangements peculiar to it. Its «method» is solely those acts of transformation. Having no other reference or source, it is formed by two procedures which are perfectly inverse to each other: either we gather its possible

transformations together as rules, or we disperse ways of transforming arrangements into resultant re-arrangements. Its characteristic inferential acts are gathering, whereby we move from arrangements to rules; and dispersion, whereby we move from rules to arrangements. In expression, these are inverse acts. But they are reversible transformations within the same structure. That structure is formed solely by the manner or mode of transformation. Accordingly a formal logic is characterized only by its own prevailing mode of transformation.

8. *A formal logic is the only adequate logic.*

By an ADEQUATE logic, I understand one whose structure is exhausted in the act of forming it. That a formal logic has this virtue is owing solely to its limitations. Its structure is constituted solely by those modes of transformation which rule it; it is exemplified solely by those arrangements which are included in it. Consequently, it exhausts its own structure, reaching to no further contents or arguments, even imaginary or constructed ones. Or, when we decide to pursue it toward such ramifications, we add an interpretation, so that it becomes a material logic, rather than one merely formal.

No material logic can be an adequate logic, since it includes an interpretation referring its structure to contents of some sort and employing it to assess further arguments. It is interpreted as including an additional structure which must be correlated to the structure of the ruling formal part. That such an extension is involved should be clear concerning any logic which refers to some actual kind of objects or to arguments we might propose or refute. Mathematics provides material logics which might seem also to be adequate, particularly in view of the fact that all of the procedures of a mathematics are proclaimed to be rigorous. However, the structure of a mathematics is not exhausted in the act of forming it. Geometry, for example, makes reference to a possible space-like organization, although not necessarily to the actual space of our world. It requires a correspondence between the logistic structure

and the organization of structures forming a continuum within which such imagined objects as triangles and circles are constructed. Were there no such reference, the only structure is question would be solely that formed and ruled by the mode of transformation. That is, our seeming mathematics would be adequate. We would make it so by removing the interpretive reference to a continuum within which possible objects might be constructed. But thereby we convert it into a formal logic. We articulate a single structure only, and omit to interpret it as correlating with another structure of possible objects. For we articulate a structure which is ruled and formed only by its prevailing mode of transformation. Lacking an interpretive reference to some structure not thereby created, it is not a material logic.

The adequacy of a formal logic is expressed as the exhaustion of its structure in the act of forming it. For that structure, which includes both rule and example, is one. There is nothing more to it than this articulation whereby it is created.

9. *We may form various formal logics by articulating differing modes of transformation.*

We find many very different transformations occurring in our changing world. We can invent many more which we do not so find, so that our imagination can toy indefinitely among their possible permutations. Suggestions of either sort we can employ as ways of looking at further events in the world. In so doing, we test the extent to which transformations we discover in some cases can be taken to govern some range of other cases. Or we use our own constructions to calculate about the world. Or we require that behaviors or occurrences conform to rules we decide to legislate.

We often endeavor to codify such suggested transformations, with various success and attainment of completion. That is, we assume that some of the transformations we find or imagine can be regarded as relating rules to instances. In sorting out types of transformation and typical arrangements

governed by them, we form a logic. Many different material logics are thus possible (4), being characterized by modes of transformation thus suggested together with interpretations we append to them (3). For we elaborate the corresponding structures between the mode of transformation taken as ruling and the contents we take to be instances of it. Many such modes of transformation can be derived from suggestions from our experience or from our imaginative ingenuity.

We may by similar processes codify suggestions and applications, in order to form a formal logic. In doing so, we may isolate one mode of transformation (whatever the origin of it as a suggestion) and articulate the transformations whereby arrangements within it are both formed and ruled (5). That is, we take that mode of transformation as solely characterizing the formal logic (7) by articulating its structure. We rule and form one structure as sole resultant of one mode of transformation: or at least we seek to do so. We articulate the logic of that mode of transformation.

Sometimes in forming a formal logic we combine several types of transformation for either of two reasons: under pressure, we find that more than one is needed, perhaps for some anticipated interpretation; or else (for whatever reason) we wish to explore the conjunction of two or more transformations, seeking their coincidence, their mutual limitation, or their supplementary spheres of rule. However, we do not thereby evade the pursuit of one ruling logic. We only postpone that requirement pending a further quest for consistency or mutual support among transformations. For we may pursue two of them repeating the operation they supply until they reach a point of conflict. Or we may find it convenient to reduce one mode of transformation to another, so that arrangements generated by either are equivalent and both taken as rules may form exactly duplicating structures. Or we may formulate a prior rule whereby their respective orders are distinguished but interconnected. But in any of these quests one of our major interests is in unity and in consistency of ruling modes of transformation.

Each of three seemingly different modes of transformation

has at some time served as a prevailing model generating a relatively complete formal logic. When any one of these three is so taken, the other two are reduced to derivative or special cases under it. In such a case, that one is taken as sufficient to articulate the formal logic characterized solely by it. Such a dominant mode of transformation derives from or suggests many ways whereby we organize our thinking and discourse about the world. It might also profess to correspond to inclusive and preëminent principles of our philosophies.

A formal logic is characterized solely by its prevailing mode of transformation (7). Thus differing formal logics may be formed by articulating the structure of transformations whereby arrangements within it are both formed and ruled (6). If three modes of transformation have so served, three logics result. Hence, whenever any one mode of transformation is thought to be generative in this way, one formal logic is thereby proposed for our assessment.

We may articulate a formal logic which takes modes of transformation as prevailing when they are transitive but not symmetrical. Such structures may be extrapolated from relations like «...is taller than...» and «...is north of...» and from the formula for the hypothetical syllogism. They are richly suggestive, in application, of temporal orders and of causal dependence, neither of which is obviously reducible to spatial or interactive terms. Inference tends to be predominately chain-like, and to engender quests for foundations providing certainty. The prevalance of such transformations in a formal logic leads to controversies defending the logical significance of strict implication and class inclusion.

We may articulate a formal logic which takes modes of transformation as prevailing when they are symmetrical but not transitive. Such structures may be extrapolated from relations like «...is married to...» or «...is excluded from...» and from the formulae for disjunctive and conjunctive syllogisms. In application, they are richly suggestive of spatial orders and of coexistence, neither of which is obviously reducible to temporal terms or to independent priority. Inference tends to be predominantly diagrammatic, and to engender quests for

completeness of organization. The prevalance of such transformations in a formal logic leads to controversies asserting the logical significance of material implication and truth-tabular derivations.

We may articulate a formal logic which takes modes of transformation as prevailing when they are both transitive and symmetrical. Such structures may be extrapolated from relations like «...is equivalant to...» or «...is identical with...» and from the axiom that «equals are equal to equals.» In application, they are richly suggestive of individualization in space and time, and of a theory of natures, neither of which are obviously reducible to dependence or co-existence. Inherence tends to be predominantly by identities and to engender a quest for richness of content .The prevalance of such transformations in a formal logic leads to controversies defending instantiation or mathematical equivalences.

Each of these three types of transformation has in the past prevailed in articulating a whole formal logic, and deriving thereform various applications consequent to a material use of it. Modes of transformations which are transitive or symmetrical or both each generate a formal logic which is at least verbally distinct from the other two. Each of them is rich in suggestive applications and defenses which seem on the surface to be very different from each other, if not incompatible. Certainly as their proponents have elaborated them into full sciences, moral guides, or ruling philosophic principles, they have generated acrimonious conflict and mutual denials of importance or completeness. Each of them may be an adequate formal logic, so far as it refers to itself. For an adequate logic is one whose structure is exhausted in the act of forming it (7). But it is only proposed as such. So long as other logics are also proposed, that is so long as there are disagreeing proponents and opponents, it is one among many. In itself, it may be an adequate logic. But its sufficiency in broader senses is challenged, and its interpretive accompaniments are disputed. Thereby also is frustrated our naïve or sophisticated expectation that one logic, and only one, ultimately supplies the rule to arrangements.

Insofar as any mode of transformation or the articulations resultant thereon enables the rule of formal logics which are different in expression and not obviously reducible to each other, we may form many different formal logics. In fact we have done so, according to the suggestion of many modes of transformation which articulate differing structures.

Each of these may be suggested by many different experiences. And it may in turn be formed into a material logic which elaborates interpretations having many different applications.

10. *We do not possess one acknowledged articulation of a ruling formal logic.*

We expect diversity and multiplicity as we enlarge logics in the direction of application. When so engaged, we employ material logics encompassing many different contents of experience, in many differing uses, and to expand many imaginative ingenuities. But conversely, we are inclined to expect that there should be only one formal logic. For some formal logic supplies methods to a material logic, being its ruling part (5). Our supposition that there is only one such rule would seem to be required by any application of a formal logic to some contents. We also seem often, whether with naïveté or sophistication, to project for ourselves a hope that we might find some one logic which applies to all possible contents as a rule. Such expectation of a single ruling logic also seems to motivate our efforts to reduce any given formal logic to fewer and more inclusive rules. Also, we feel obliged to show that the formal logic which we take to be governing constitutes one consistent and complete set of rules adequate to govern all the arrangements it generates within a single structure.

These hopes and efforts are enlivened by some of the most ultimate and comprehensive endeavors of the human spirit. Their pursuit reaches toward that single presupposition which has sometimes been called the *Logos*. This one being, it is occasionally claimed, comprehends whatever there may be in

the universe within one single generative principle which is also the goal of all our intellectual activities. We may accordingly insist that our pursuit of that one logic which is the ruling part of all other logics is an approximation to this *Logos,* or even that we attain to it in some preëminent argument than which no greater can be conceived.

Yet, whatever ultimate unity may exist or we may profess, we speak thereof in many tongues. That is, our exposition or enforcement of it for our communications with each other disperses it into arrangements ordering the structure of some logic which we favor for ourselves, and which we urge on others. Although prayer may consist in uttering one Word, our discourse disperses our speech into many elements, connected in many ways, and forming many arrangements. We communicate with presumed hearers by arguments which transform these ingredients in special ways, so that we may convey our message or signal our understanding. We accomplish such communication by forming a logic whereby we are enabled, different persons among us, to repeat transformations which we take to be rules governing our arguments. We may use such a structure by way of application to some materials outside itself, for which purpose we presuppose or pursue many different structural correspondences. These often locate units which are distinguishable parts of a larger whole. Even if the structure of the world constitutes one whole, to which we hope the structure of our logic corresponds, we form and govern arguments about that whole according to the structure of those contained arrangements into which our mode of transformation disperses it. Thus our material logics are many (4). So far as our communicative discourse is concerned, they have no application to any One which cannot be so dispersed.

We may also articulate that formal logic which we take to be the ruling part of a material logic. To do so, we articulate transformations whereby arrangements within it can be displayed as being both formed and ruled within a common structure. That formal structure also may be adequate, being single and a capacious whole. Indeed it might be understood as the model or paradigmatic example of such wholeness. But

we articulate it in the very process of creating it. Our presentation gathers its many arrangements into rules or disperses its rules into many arrangements. The one structure consists of multiple arrangements. We might attempt to reduce its many rules to a single one. But even were we successful, that one is enabled to govern only inasmuch as it forms the rule for many instances. A formal logic is articulated in presenting this correlation of structure between governing rule and resultant arrangements. Hence multiplicity is involved in any logic which is to govern many arrangements, however single its rule. For such multiplicity is necessary to any formation of a structure.

In addition to the lack of unity inherent in any discourse, our agreements are confounded by the fact that we may differently articulate our formal logic in elaborating differing modes of transformation (9). I have recalled three types of formal logic as proposed by elaborating three differing modes of transformation. I take them to be distinct. The three formal logics formed by their respective prevalence are accordingly different formal logics. They articulate different structures. In purely formal terms, many of the rules and arrangements generated by one can be translated into equivalents to those generated by another. It has also been repeatedly claimed that one or another can serve for eliminating the remaining two by completely reducing them to that one which prevails. By such a move, the former two can be called special but subordinate cases. Or as they are projected onto the contents of a material logic they appear increasingly fallacious or deceptive. By such a presupposition, one of the three modes of transformation is conceived so that it is inclusive of the others and exclusive rule to them. Such a gambit is eased by proclaiming that such a prevailing mode of transformation alone articulates that one structure which is true, or which is rooted in the nature of things, or which can be adopted in common among friends or proper scientists or men of sound sense.

Such claims confute each other, giving rise to massive ideological conroversy irrespective of merely formal equivalence or reducibility. As long as they are not arbitrated, no one

formal logic is sufficient to rule all arguments, no matter what its adequacy. To that extent, we do not possess one acknowledged articulation of a ruling formal logic. For in the various cases, we articulate different structures. Even were we to share such an agreed upon articulation, or to enforce it on our opponents, it would be sufficient only to articulate rules for discourse. Although we might pursue some single *Logos* by means of it, or hope thereby to persuade others to attend to it, our arguments only exemplify the methods of our logic, or the rules of its governing formal part. They do so by means of transformations among arrangements, to which all of our communication is ultimately reduced. They may point to some more ultimate unity to be pursued, or they may presuppose it. But none of our arguments speak of it according to the rules of an adequate logic. Concerning any ultimate single *Logos,* the articulations of our logics remain silent.

University of Colorado William Sacksteder